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NON-MARKOVIAN DECOHERENCE DYNAMICS OF ENTANGLED COHERENT STATES

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We microscopically model the decoherence dynamics of entangled coherent states of two optical modes under the influence of vacuum fluctuation. We derive an exact master equation with time-dependent coefficients reflecting the memory effect of the environment, by using the Feynman-Vernon influence functional theory in the coherent-state representation. Under the Markov approximation, our master equation recovers the widely used Lindblad equation in quantum optics. We then investigate the non-Markovian entanglement dynamics of the two-mode entangled coherent states under vacuum fluctuation. Compared with the results in Markov limit, it shows that the non-Markovian effect enhances the disentanglement to the initially entangled coherent state. Our analysis also shows that the decoherence behaviors of the entangled coherent states depend on the symmetrical properties of the entangled coherent states as well as the couplings between the optical fields and the environment.

Keywords: Non-Markovian dynamics; Decoherence theory; Exact master equation

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1 Introduction

Optical fields are widely used in quantum communication since quantum information is almost invariably transmitted using photons. Experimental quantum teleportation has been realized using the discrete two-photon polarization entanglement states [1] or the continuous two-mode squeezed entanglement states [2, 3] as quantum channels [4]. Another important type

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of continuous variable entanglement states, entangled coherent states [5, 6, 7, 8, 9, 10], has also been proposed as a potential quantum channel to teleport unknown quantum states [11, 12, 13]. In this paper, we shall investigate the non-Markovian decoherence dynamics of the continuous variable quantum channel in terms of entangled coherent states.

As well known, a realistic analysis of quantum systems for quantum information processing must take into account decoherence effect. There has been an increasing interest in describing various continuous variable quantum channel under noise [14, 15, 16, 17, 18, 19]. Conventional approaches treat the interaction between the quantum system and its environment perturbatively, which yield equations of motion such as Redfield or master equations under the Born-Markov approximation [20, 21, 22]. Although the approximation has been widely employed in the field of quantum optics, where the characteristic time of the environmental correlation function is short compared with that of the system investigated [22], its validity is experiencing more and more challenges in facing new experimental evidences [23]. Moreover, the Born-Markov approximation is in general invalid in dealing with most condensed-matter problems, for example, a quantum system hosted in a nanostructured environment [24, 25, 28, 26, 27], because possible large coupling constants and long correlation time scales of the environment both require a non-perturbative treatment. Thus, how to develop a general non-perturbative microscopic description of open quantum systems has attracted much attention recently [29, 30, 31, 32].

In the present work, we shall focus attention on the influence of vacuum fluctuation on quantum channels in terms of entangled coherent states. To this end, we model the system as two optical modes coupled to a bosonic environment at zero temperature. We shall then develop non-perturbatively a microscopic description to the decoherence dynamics of such systems. We have noticed that most of previous theoretical works to explore the decoherence dynamics of the optical field system relied on Born and/or Markov approximation [14, 15, 16, 17, 18, 19]. To derive non-perturbatively the decoherence dynamics of an open quantum system, we will employ the Feynman-Vernon influence functional theory [33, 34, 35] in the coherent state path integral formalism [36], which enables us to treat both of the back-actions from the environment to the system and from the system to the environment self-consistently. After a careful evaluation of the coherent state path integrals, we obtain an operator form of the exact master equation with time-dependent coefficients describing the full non-Markovian dynamics of the back-actions between the system and the environment.

We then investigate the non-Markovian decoherence dynamics of the entangled coherent states [5] using the exact solution of the reduced density matrix, where the entanglement is measured by the concurrence [37]. The non-Markovian effect is manifested in the short time peak of the time dependent coefficients in the master equation, which results in an enhancement of the disentanglement to the entangled coherent states. Indeed, in a recently published paper [32], we have already used the exact non-Markovian master equation derived in this paper to study the decoherence dynamics of another type of the continuous variable entangled states, i.e. entangled squeezed states, where the entanglement is determined by the logarithmic negativity [38] rather than the concurrence since the later is not applicable to entangled squeezed states.

The paper is organized as follows. In Sec. II, we introduce the model of two optical modes interacting with a common environment in the coherent-state representation. In Sec. III, we show the detailed derivation of the influence functional theory to the model. The exact master equation is derived in Sec. IV. Sec. V is devoted to the study of entanglement dynamics and the decoherent properties of quantum channels in terms of the entangled coherent states. Finally, a brief summary is made in Sec. VI.

2 The Hamiltonian of two optical modes in an environment

Our system includes two separated optical modes subject to a common vacuum fluctuation, which is relevant to quantum network and has been widely investigated [39, 40]. Since we are interested in the decoherence of the two optical modes mediated by a vacuum electromagnetic field after the two-mode entangled coherent state is prepared [7, 8], we can omit the terms regarding the atoms in [39, 40]. The Hamiltonian of the whole system is then given by

$$H = H_S + H_E + H_I, \tag{1}$$

where

$$H_S = \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar\kappa(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{2}$$

$$H_E = \sum_k \hbar\omega_k b_k^\dagger b_k, \tag{3}$$

$$H_I = \sum_{l,k} \hbar(g_{lk} a_l^\dagger b_k + g_{lk}^* a_l b_k^\dagger), \tag{4}$$

are, respectively, the Hamiltonians of the two optical modes, the environment (vacuum fluctuation), and the interaction between them. The operators a_l and $a_l^\dagger$ ($l = 1, 2$) are the corresponding annihilation and creation operators of the l-th optical mode with frequency ω_l. The parameter κ is a coherent tunneling rate of photons between the two optical systems, such as two cavities [41, 42], which is proportional to the overlap of the two wave packets of the optical fields. Such coupled optical array system recently attracts much attention [41, 42, 43] for the possible materialized in a variety of physical systems, for example, fiber coupled micro-toroidal cavities [44], arrays of defects in photonic band gap materials [45] and superconducting qubits coupled through microwave stripline resonators [46].

The environment is modeled, as usual, by a set of harmonic oscillators identifying the vacuum electromagnetic field with the annihilation and creation operators b_k and $b_k^\dagger(k = 1, 2, \cdots)$, g_{lk} are the coupling constants between the optical modes and the environment. In Eq. (1) we have also suppressed the polarization of the fields for both the systems and the environment. Since most quantum optical experiments are made currently in low temperature and under vacuum condition, the vacuum fluctuation should be a main source of decoherence. Therefore, we take the environment to be at zero temperature throughout this paper.

To apply the influence functional method to an open quantum system, the first step towards the dynamics of the reduced system is to compute the forward and backward propagators between certain initial and final states of the full system by choosing a convenient representation. In the present work we use the coherent-state representation [36], in which the basis of the Hilbert space for the environment consists of multi-mode bosonic coherent states

$$|\mathbf{z}\rangle = \prod_k |z_k\rangle, \quad |z_k\rangle = \exp(z_k b_k^\dagger)|0_k\rangle, \tag{5}$$

and that for the two optical modes is the two single-mode bosonic coherent states

$$|\boldsymbol{\alpha}\rangle = \prod_{l=1}^{2} |\alpha_l\rangle, \ |\alpha_l\rangle = \exp(\alpha_l a_l^\dagger)|0_l\rangle, \tag{6}$$

where the shortened notations for the complex variables, $\mathbf{z} = (z_1, z_2, \cdots)$ and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$, are introduced.

The coherent states defined above are eigenstates of annihilation operators,

$$b_k|z_k\rangle = z_k|z_k\rangle, \ a_l|\alpha_l\rangle = \alpha_l|\alpha_l\rangle. \tag{7}$$

As these coherent states are over-complete, they obey the resolution of identity,

$$\int d\mu(\mathbf{z})|\mathbf{z}\rangle\langle\mathbf{z}| = 1, \ \int d\mu\left(\boldsymbol{\alpha}\right)|\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}| = 1, \tag{8}$$

where the integration measure is defined by $d\mu(\mathbf{z}) = \prod_k e^{-z_k^* z_k} \frac{dz_k^* dz_k}{2\pi i}$ and a similar form for $d\mu\left(\boldsymbol{\alpha}\right)$. As it is shown, the bosonic coherent states we used here are not normalized, and the normalization factors are moved into the above integration measures, which corresponds to the Bargmann representation of the complex space. Moreover, these coherent states are also nonorthogonal,

$$\langle\mathbf{z}|\mathbf{z}'\rangle = \exp(\sum_k z_k^* z_k'), \ \langle\boldsymbol{\alpha}|\boldsymbol{\alpha}'\rangle = \exp(\sum_l \alpha_l^* \alpha_l'). \tag{9}$$

The use of the coherent-state representation makes the evaluation of path integrals extremely simple. In the coherent-state representation, the Hamiltonians of the two optical modes, the environment (vacuum fluctuation), and the interaction between them are expressed as

$$H_S(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}) = \hbar \sum_{l=1}^{2} \omega_l \bar{\alpha}_l \alpha_l + \hbar\kappa(\bar{\alpha}_1\alpha_2 + \bar{\alpha}_2\alpha_1), \tag{10}$$

$$H_E(\bar{\mathbf{z}}, \mathbf{z}) = \sum_k \hbar\omega_k \bar{z}_k z_k, \tag{11}$$

$$H_I(\bar{\boldsymbol{\alpha}}, \ \boldsymbol{\alpha}, \bar{\mathbf{z}}, \mathbf{z}) = \sum_{lk} \hbar(g_{lk}\bar{\alpha}_l z_k + g_{lk}^* \bar{z}_k \alpha_l), \tag{12}$$

where $\bar{\mathbf{z}}$ and $\bar{\boldsymbol{\alpha}}$ denote the complex conjugate of $\mathbf{z}$ and $\boldsymbol{\alpha}$, respectively. With the above coherent-state representation, we will present in the next two sections a detailed derivation of the exact master equation for the reduced density matrix of the two optical fields that we have simply outlined in our early work [32].

3 The influence functional theory

3.1 The influence functional in coherent-state representation

We follow the influence functional method of [47] by expressing the density matrix of the composite system as a double-path coherent state path integral. After eliminating the degrees of freedom of the environment, we can incorporate all the environmental effects on the reduced system in a functional integral named influence functional [33]. Then the dynamics of the reduced system will be governed by an effective action retaining all the influences from the environment in the influence functional.

The total density matrix of the system plus the environment obeys the quantum mechanical equation $i\hbar\partial\rho_{\rm tot}(t)/\partial t = [H, \rho_{\rm tot}(t)]$, which yields the formal solution:

$$\rho_{\rm tot}\left(t\right) = e^{\frac{-iHt}{\hbar}}\rho_{\rm tot}\left(0\right)e^{\frac{iHt}{\hbar}}. \tag{13}$$

Different from the coordinate representation in [34, 35], the coherent-state representation leads to,

$$\begin{aligned}\langle \boldsymbol{\alpha}_f, \mathbf{z}_f|\rho_{\rm tot}\left(t\right)|\boldsymbol{\alpha}'_f, \mathbf{z}_f\rangle = \int d\mu(\mathbf{z}_i)d\mu(\boldsymbol{\alpha}_i)d\mu(\mathbf{z}'_i)d\mu(\boldsymbol{\alpha}'_i)\langle \boldsymbol{\alpha}_f, \mathbf{z}_f; t|\boldsymbol{\alpha}_i, \mathbf{z}_i; 0\rangle \\ \times\langle\ \boldsymbol{\alpha}_i, \mathbf{z}_i|\rho_{\rm tot}(0)|\boldsymbol{\alpha}'_i, \mathbf{z}'_i\rangle\langle \boldsymbol{\alpha}'_i, \mathbf{z}'_i; 0|\boldsymbol{\alpha}'_f, \mathbf{z}_f; t\rangle,\end{aligned} \tag{14}$$

where the resolutions of identity, Eq.(8), has been used. The density matrix given by Eq. (14) describes the behavior of the two optical modes plus the environment as a whole. As we are only interested in dynamics of the two optical modes, we will work with the reduced density matrix by integrating over the environmental variables. We also assume that the initial density matrix could be factorized into a direct product of the two-mode state and the environment state $\rho_{\rm tot}(0) = \rho(0)\otimes\rho_E(0)$, namely, assuming no correlation between the environment and the system at $t \leq 0$ [48]. Then the reduced density matrix fully describing the dynamics of the two optical modes is given by

$$\rho(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t) = \int d\mu(\boldsymbol{\alpha}_i)d\mu(\boldsymbol{\alpha}'_i)\rho(\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0)\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t|\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0), \tag{15}$$

where $\rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; \tau) \equiv \int d\mu(\mathbf{z})\langle\boldsymbol{\alpha}, \mathbf{z}|\rho_{\rm tot}(\tau)|\boldsymbol{\alpha}', \mathbf{z}\rangle$, and

$$\begin{aligned}&\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t|\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \\ &\quad\int d\mu(\mathbf{z}_f)d\mu(\mathbf{z}_i)d\mu(\mathbf{z}'_i)\langle \boldsymbol{\alpha}_f, \mathbf{z}_f; t|\boldsymbol{\alpha}_i, \mathbf{z}_i; 0\rangle\rho_E(\bar{\mathbf{z}}_i, \mathbf{z}'_i; 0)\langle \boldsymbol{\alpha}'_i, \mathbf{z}'_i; 0|\boldsymbol{\alpha}'_f, \mathbf{z}_f; t\rangle,\end{aligned} \tag{16}$$

is the propagating function of the reduced density matrix, which contains two propagators for the total system: the forward and backward propagators, $e^{\mp\frac{iHt}{\hbar}}$, plus the initial density matrix of the environment as a matrix element in the coherent-state representation.

In the following we will show how to calculate the forward propagator in terms of the coherent state path integral [49, 36]. The similar calculation could be done for the backward one. To evaluate the forward propagator operator $e^{\frac{-iHt}{\hbar}}$ between the initial ($|\boldsymbol{\alpha}_i, \mathbf{z}_i\rangle$) and the final ($\langle\boldsymbol{\alpha}_f, \mathbf{z}_f|$) coherent states, one can generally divide the time interval $t_f - t_i$ into N subintervals. Then by inserting the resolution of identity $(N-1)$ times between each subintervals and taking the limit of large N, we have the forward propagator in terms of the coherent state path integral,

$$\langle\boldsymbol{\alpha}_f, \mathbf{z}_f; t|\ \boldsymbol{\alpha}_i, \mathbf{z}_i; 0\rangle = \int D^2\mathbf{z}D^2\ \boldsymbol{\alpha}\exp\left\{\frac{i}{\hbar}\big(S_S[\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}] + S_I[\bar{\mathbf{z}}, \mathbf{z}, \bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}] + S_E[\ \bar{\mathbf{z}}, \mathbf{z}]\big)\right\}, \tag{17}$$

where S_S, S_E, and S_I are the actions corresponding to the two optical modes, the environment, and the interaction Hamiltonian H_S, H_E, and H_I, respectively,

$$S_S[\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}] = \sum_l \Big\{ - i\hbar\bar{\alpha}_l\alpha_l\,(t) + \int_0^t d\tau[i\hbar\bar{\alpha}_l\dot{\alpha}_l(\tau) - H_S(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha})]\Big\}, \tag{18}$$

$$S_E[\bar{\mathbf{z}}, \mathbf{z}] = \sum_k \Big\{ - i\hbar\bar{z}_k z_k(t) + \int_0^t d\tau[i\hbar\bar{z}_k\dot{z}_k(\tau) - H_E(\bar{\mathbf{z}}, \mathbf{z})]\Big\}, \tag{19}$$

$$S_I[\bar{\mathbf{z}}, \mathbf{z}, \bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}] = - \int_0^t d\tau H_I(\bar{\boldsymbol{\alpha}},\ \boldsymbol{\alpha}, \bar{\mathbf{z}}, \mathbf{z}). \tag{20}$$

All the functional integrations are carried out over paths $\bar{\mathbf{z}}(\tau)$, $\mathbf{z}(\tau)$, $\bar{\boldsymbol{\alpha}}(\tau)$, and $\boldsymbol{\alpha}(\tau)$ with endpoints $\bar{\mathbf{z}}(t) = \bar{\mathbf{z}}_f$, $\mathbf{z}(0) = \mathbf{z}_i$, $\bar{\boldsymbol{\alpha}}(t) = \boldsymbol{\alpha}_f$, and $\boldsymbol{\alpha}(0) = \boldsymbol{\alpha}_i$. Substituting Eq. (17) and a similar expression for the backward propagator into Eq. (16), we obtain

$$\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t|\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \int D^2\boldsymbol{\alpha} D^2\boldsymbol{\alpha}' \exp\Big\{\frac{i}{\hbar}(S_S[\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}] - S_S^*[\bar{\boldsymbol{\alpha}}',\ \boldsymbol{\alpha}'])\Big\}\mathcal{F}[\bar{\boldsymbol{\alpha}},\ \boldsymbol{\alpha}, \bar{\boldsymbol{\alpha}}', \boldsymbol{\alpha}'], \tag{21}$$

where

$$\begin{aligned}\mathcal{F}[\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}, \bar{\boldsymbol{\alpha}}', \boldsymbol{\alpha}'] = &\int d\mu(\mathbf{z}_f) d\mu(\mathbf{z}_i) d\mu(\mathbf{z}'_i) D^2\mathbf{z} D^2\mathbf{z}' \rho_E(\bar{\mathbf{z}}_i, \mathbf{z}'_i; 0) \\ &\times \exp\Big\{\frac{i}{\hbar}(S_E[\bar{\mathbf{z}}, \mathbf{z}] - S_E^*[\bar{\mathbf{z}}', \mathbf{z}'] + S_I[\bar{\mathbf{z}}, \mathbf{z}, \bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}] - S_I^*[\bar{\mathbf{z}}', \mathbf{z}', \bar{\boldsymbol{\alpha}}', \boldsymbol{\alpha}'])\Big\}\end{aligned} \tag{22}$$

is defined as the Feynman-Vernon influence functional in the coherent state representation, which contains all the environmental effects on the two optical modes.

3.2 Evaluation of the influence functional

Now we can calculate explicitly the influence functional of our model using the coherent-state path-integral formalism presented above. Substituting the model Hamiltonian into the actions of Eq. (18-20), we obtain the explicit form of the forward propagator. The path integral of the environmental part of the propagator can be done by the stationary phase method [49, 36] with the boundary conditions $z_k(0) = z_{ki}$ and $\bar{z}_k(t) = \bar{z}_{kf}$, which results in the equations of motion,

$$\dot{z}_k + i\omega_k z_k = -i\sum_l g_{lk}^*\alpha_l, \quad \dot{\bar{z}}_k - i\omega_k\bar{z}_k = i\sum_l g_{lk}\bar{\alpha}_l, \tag{23}$$

where the paths regarding $\bar{\boldsymbol{\alpha}}$ and $\boldsymbol{\alpha}$ are taken as external sources. The solution to the stationary path equation (23) are

$$z_k(\tau) = z_{ki}e^{-i\omega_k\tau} - i\sum_l g_{lk}^* \int_0^\tau d\tau' e^{-i\omega_k(\tau-\tau')}\alpha_l(\tau'), \tag{24}$$

$$\bar{z}_k(\tau) = \bar{z}_{kf}e^{i\omega_k(\tau-t)} - i\sum_l g_{lk} \int_\tau^t d\tau' e^{i\omega_k(\tau-\tau')}\bar{\alpha}_l(\tau'). \tag{25}$$

Note that the prefactor under the contribution of stationary path in the coherent-state path integral is unity, and the stationary phase method to treat the environmental part here is exact for the action being only a quadratic function of the dynamical variables. The path integral of the environmental part for the backward propagator $\langle \boldsymbol{\alpha}'_i, \mathbf{z}'_i; 0|\boldsymbol{\alpha}'_f, \mathbf{z}_f; t\rangle$ can be evaluated in the same way.

Since we only consider the vacuum fluctuation, the environment is initially in the equilibrium state at zero temperature, we then have $\rho_E(\bar{\mathbf{z}}_i, \mathbf{z}'_i; 0) = 1$. Substituting the solution (24-25) for $z_k(\tau)$, $\bar{z}_k(\tau)$ and a similar solution for $\bar{z}'_k(\tau)$, $z'_k(\tau)$ together into Eq. (22), and using the Gaussian integral identity $\int \frac{d^2z}{\pi} e^{-\gamma\bar{z}z+\lambda z+\nu\bar{z}} = \frac{1}{\gamma} e^{\frac{\lambda\nu}{\gamma}}$ repeatedly for the integral over $\mathbf{z}_i, \mathbf{z}'_i, \mathbf{z}_f$, we reach the final form of the influence functional that we have used in [32],

$$\begin{aligned}\mathcal{F}[\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}, \bar{\boldsymbol{\alpha}}', \boldsymbol{\alpha}'] = \exp\Big\{ \int_0^t d\tau \int_0^\tau d\tau' \Big[\sum_{l,m} (\bar{\alpha}'_l(\tau) - \bar{\alpha}_l(\tau))\mu_{lm}(\tau-\tau')\alpha_m(\tau') \\ +(\alpha_l(\tau) - \alpha'_l(\tau))\mu^*_{lm}(\tau-\tau')\bar{\alpha}'_m(\tau')\Big]\Big\}, \end{aligned} \tag{26}$$

where $\mu_{lm}(x) = \sum_k e^{-i\omega_k x} g_{lk} g^*_{mk}$ is the dissipation-noise kernel.

4 The exact non-Markovian master equation

4.1 The propagating function of the reduced density matrix

In the above derivation of the influence functional, the back-actions between the two optical modes and the environment have been treated self-consistently. All the effects from environment on the two optical modes are incorporated in the influence functional which leads to a modification to the action of the two optical modes,

$$\begin{aligned}\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t|\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \int D^2\boldsymbol{\alpha} D^2\boldsymbol{\alpha}' \exp\Big\{ \sum_{l=1}^2 \left(\bar{\alpha}_l\alpha_l(t) + \bar{\alpha}'_l\alpha'_l(t)\right) \\ - \int_0^t d\tau \Big[\sum_{l=1}^2 \left(\bar{\alpha}_l\dot{\alpha}_l + \dot{\bar{\alpha}}'_l\alpha'_l\right) + iH_S(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}) - iH_S(\bar{\boldsymbol{\alpha}}', \boldsymbol{\alpha}')\Big]\Big\} \mathcal{F}[\bar{\boldsymbol{\alpha}}, \ \boldsymbol{\alpha}, \bar{\boldsymbol{\alpha}}', \boldsymbol{\alpha}'].\end{aligned} \tag{27}$$

To execute the path integral of Eq. (27), again we resort to the stationary phase method and obtain the equations of motion as ($l \neq l'$)

$$\dot{\alpha}_l + i(\omega_l\alpha_l + \kappa\alpha_{l'}) = -\int_0^\tau d\tau' \sum_{m=1}^2 \mu_{lm}(\tau-\tau')\,\alpha_m(\tau'), \tag{28}$$

$$\dot{\bar{\alpha}}'_l - i(\omega_l\bar{\alpha}'_l + \kappa\bar{\alpha}'_{l'}) = -\int_0^\tau d\tau' \sum_{m=1}^2 \mu^*_{lm}(\tau-\tau')\,\bar{\alpha}'_m(\tau'). \tag{29}$$

with the boundary conditions $\alpha_l(0) = \alpha_{li}$ and $\bar{\alpha}'_l(0) = \bar{\alpha}'_{li}$.

The integro-differential equations render the reduced dynamics non-Markovian, with the memory of the environmental dynamics registered in the time-nonlocal kernels. To simplify the discussion, we further assume that the two optical modes are identical, i.e., $\omega_1 = \omega_2 \equiv \omega_0$. Then the coupling strength to the common environment should also be the same: $g_{1k} = e^{i\phi}g_{2k} \equiv g_k$, where the phase factor $e^{i\phi} \equiv \lambda$ models the phase difference between the two optical modes coupled with the environment. In the present work we will consider two special cases: the two optical modes couple with the environment in phase (a constructive interference coupling with $\phi = 0 \to \lambda = 1$) and out of phase (a destructive interference coupling with $\phi = \pi$ so that $\lambda = -1$). By introducing the new variables

$$\alpha_l(\tau) = \alpha_{li} u(\tau) - \alpha_{l'i} v(\tau), \quad \bar{\alpha}'_l(\tau) = \bar{\alpha}'_{li} \bar{u}(\tau) - \bar{\alpha}'_{l'i} \bar{v}(\tau), \tag{30}$$

and using the equations of motion (28-29), we obtain the propagating function of the reduced density matrix as

$$\begin{aligned}\mathcal{J}(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t | \bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \exp\Big\{ & \sum_{l=1}^{2} \big[u \bar{\alpha}_{lf} \alpha_{li} + \bar{u} \bar{\alpha}'_{li} \alpha'_{lf} - (\bar{u}u + \bar{v}v - 1) \bar{\alpha}'_{li} \alpha_{li} \big] \\ & - \sum_{\langle l,l' \rangle}^{2} \big[v \bar{\alpha}_{lf} \alpha_{l'i} + \bar{v} \bar{\alpha}'_{li} \alpha'_{l'f} - (\bar{u}v + \bar{v}u) \bar{\alpha}'_{li} \alpha_{l'i} \big] \Big\}. \end{aligned} \tag{31}$$

where u, v are solutions of Eq. (30) at time $\tau = t$. The exact reduced density matrix is then easy to be obtained by substituting the above solution of the propagating function into Eq. (15) and integrating over the initial state.

4.2 The exact non-Markovian master equation

Eq. (31) is an exact result. In this section, we will deduce the master equation from Eqs. (15) and (31). From Eq.(31), we obtain

$$\alpha_{li} \mathcal{J} = \frac{u \frac{\delta \mathcal{J}}{\delta \bar{\alpha}_{lf}} + v \frac{\delta \mathcal{J}}{\delta \bar{\alpha}_{l'f}}}{u^2 - v^2}, \quad \bar{\alpha}'_{li} \mathcal{J} = \frac{\bar{u} \frac{\delta \mathcal{J}}{\delta \alpha'_{lf}} + \bar{v} \frac{\delta \mathcal{J}}{\delta \alpha'_{l'f}}}{\bar{u}^2 - \bar{v}^2}, \tag{32}$$

which will be used to eliminate the dependence on the initial values $\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i$ in Eq. (15). Combining Eqs. (31) and (15) together, and using the identities of Eq. (32), the evolution equation of the reduced density matrix is given by

$$\begin{aligned}\dot{\rho}(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t) = & \sum_{l=1}^{2} \Big\{ - i\Omega(t) \big[\bar{\alpha}_l \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \bar{\alpha}_l} - \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \alpha_l} \alpha_l \big] \\ & \qquad + \Gamma(t) \big[2 \frac{\delta^2 \rho(\bar{\boldsymbol{\alpha}}, \ \boldsymbol{\alpha}'; t)}{\delta \alpha_l \delta \bar{\alpha}_l} - \bar{\alpha}_l \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \bar{\alpha}_l} - \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \alpha_l} \alpha_l \big] \Big\} \\ & + \sum_{\langle l,l' \rangle}^{2} \Big\{ - i\Omega'(t) \big[\bar{\alpha}_l \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \bar{\alpha}_{l'}} - \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \alpha_l} \alpha_{l'} \big] \\ & \qquad + \Gamma'(t) \big[2 \frac{\delta^2 \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \bar{\alpha}_l \delta \alpha_{l'}} - \bar{\alpha}_l \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \bar{\alpha}_{l'}} - \frac{\delta \rho(\bar{\boldsymbol{\alpha}}, \boldsymbol{\alpha}'; t)}{\delta \alpha_l} \alpha_{l'} \big] \Big\} \end{aligned} \tag{33}$$

where

$$\Gamma(t) + i\Omega(t) = -\frac{u\dot{u} - v\dot{v}}{u^2 - v^2}, \quad \Gamma'(t) + i\Omega'(t) = -\frac{v\dot{u} - u\dot{v}}{u^2 - v^2}. \tag{34}$$

Eq. (33) is the exact master equation of the reduced density matrix for the dynamics of the two optical modes in the coherent-state representation, in which $\Omega(t)$ plays the role of a shifted time-dependent frequency of the two modes, $\Omega'(t)$ accounts for a shifted time-dependent coherent interaction between the two modes, $\Gamma(t)$ represents a time-dependent individual decay rate of each mode, and $\Gamma'(t)$ is for a correlated decay rate of the two modes induced by the environment.

If we define a new variable $F_\pm(\tau) = u(\tau) \pm v(\tau)$, then Eqs. (28-29) is reduced to

$$\dot{F}_\pm(\tau) + i(\omega_0 - \lambda\kappa)F_\pm(\tau) + (1 \mp \lambda)\int_0^\tau d\tau'\mu(\tau-\tau')F_\pm(\tau') = 0, \tag{35}$$

with $\mu(x) = \sum_k e^{-i\omega_k x}|g_k|^2$ and $\lambda = \pm 1$. The explicit forms of $\Omega(t)$, $\Omega'(t)$, and $\Gamma(t)$ in the master equation are given by

$$\begin{aligned} &\Omega(t) = \omega_0 + \mathrm{Im}\left[G_\lambda(t)\right], \\ &\Omega'(t) = \kappa + \lambda\,\mathrm{Im}\left[G_\lambda(t)\right], \\ &\Gamma(t) = \lambda\Gamma'(t) = \mathrm{Re}\left[G_\lambda(t)\right], \end{aligned} \tag{36}$$

where

$$G_\lambda(t) = -\frac{1}{2}\left[\frac{\dot{F}_{-\lambda}(t)}{F_{-\lambda}(t)} + i(\omega_0 + \lambda\kappa)\right] = \frac{1}{F_{-\lambda}(t)}\int_0^t d\tau\mu(t-\tau)F_{-\lambda}(\tau). \tag{37}$$

This result has the similar form as the coefficients in the non-Markovian master equation of a two-level atom derived in [29].

To obtain the operator form of the master equation, we should introduce the following functional differential relations in the coherent-state representation (i.e., the Bargmann representation of operators [51], and it is also called the D-algebra in coherent state representation [36]),

$$\begin{aligned} &\bar{\alpha}_l\frac{\delta\rho(\bar{\boldsymbol{\alpha}},\ \boldsymbol{\alpha}';t)}{\delta\bar{\alpha}_m} \longleftrightarrow a_l^\dagger a_m\rho(t), \\ &\frac{\delta\rho(\bar{\boldsymbol{\alpha}},\boldsymbol{\alpha}';t)}{\delta\alpha_l}\alpha_m \longleftrightarrow \rho(t)a_l^\dagger a_m, \\ &\frac{\delta^2\rho(\bar{\boldsymbol{\alpha}},\boldsymbol{\alpha}';t)}{\delta\bar{\alpha}_l\delta\alpha_m} \longleftrightarrow a_l\rho(t)a_m^\dagger, \end{aligned} \tag{38}$$

with which we arrive at an operator form of the master equation shown below,

$$\begin{aligned} \dot{\rho}(t) = &-\frac{i}{\hbar}[H'(t),\rho(t)] + \Gamma(t)\sum_{k=1}^{2}[2a_k\rho(t)a_k^\dagger - a_k^\dagger a_k\rho(t) - \rho(t)a_k^\dagger a_k] \\ &+ \Gamma'(t)\sum_{k\neq k'}[2a_k\rho(t)a_{k'}^\dagger - a_k^\dagger a_{k'}\rho(t) - \rho(t)a_k^\dagger a_{k'}], \end{aligned} \tag{39}$$

where

$$H'(t) = \hbar\Omega(t)(a_1^\dagger a_1 + a_2^\dagger a_2) + \hbar\Omega'(t)(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{40}$$

is the renormalized Hamiltonian of the two optical modes. From Eq. (39), we can see that besides the spontaneous decay of the individual mode, the environment, even only the vacuum fluctuation is considered, will result in a coherent interaction and a correlated spontaneous decay between the two modes. More importantly, our derivation of the master equation is fully non-perturbative, which goes beyond the Born-Markov approximation and contains all the back-actions between environment and the optical modes. The non-Markovian character resides in the time-dependent coefficients of the exact master equation. These formulae have been used to study the non-Markovian entanglement dynamics of two squeezed states [32].

The time-dependent coefficients in the exact master equation, determined by Eq. (35), crucially depend on the so-called spectral density, which characterizes the coupling strength of the environment to the system with respect to the frequencies of the environment. It is defined as $J(\omega) = \sum_k |g_k|^2 \delta(\omega - \omega_l)$. In the continuum limit the spectral density may have the form

$$J(\omega) = \eta\omega\left(\frac{\omega}{\omega_c}\right)^{n-1} e^{-\frac{\omega}{\omega_c}}, \tag{41}$$

where ω_c is an exponential cutoff frequency, and η is a dimensionless coupling constant. The environment is classified as Ohmic if $n = 1$, sub-Ohmic if $0 < n < 1$, and super-Ohmic for $n > 1$ [48]. Different spectral densities manifest different non-Markovian dynamics.

It is worth mentioning that an exact master equation has also been obtained very recently for the system of two harmonic oscillators bilinearly coupled with a thermal environment [31], where the master equation is derived in the Wigner representation rather than the operator form of Eq. (39). Also the bilinear coupling in [31] is defined in terms of the coordinate variables of harmonic oscillators which is different from the interacting Hamiltonian we used in Eq. (4). In terms of quantum optics language, the coupling between the system and the environment used in [31] involves simultaneously photon-photon scattering process and two-photon creation and annihilation process with the same coupling strength. Note that photon-photon scatterings are linear optical processes while two-photon creation and annihilation processes are non-linear optical processes, they cannot have the same coupling strength in quantum optics. Therefore, the model used in [31] might describe a physical system quite different from the optical system we considered in the present work.

4.3 The Markov approximation

It is interesting to see that one can reproduce the conventional Markov solution from our exact non-Markovian master equation under certain approximation. By redefining the dynamical variables of the system as $\alpha_l(\tau) = x_l(\tau)e^{-i\omega_0\tau}$, and $\bar{\alpha}'_l(\tau) = x'_l(\tau)e^{i\omega_0\tau}$, we can recast Eq. (28-29) into

$$\dot{x}_l + i\kappa x_{l'} + \int_0^\infty d\omega J(\omega)\int_0^\tau d\tau' e^{i(\omega_0-\omega)(\tau-\tau')}[x_l(\tau') + \lambda x_{l'}(\tau')] = 0, \tag{42}$$

$$\dot{\bar{x}}'_l - i\kappa\bar{x}'_{l'} + \int_0^\infty d\omega J(\omega)\int_0^\tau d\tau' e^{-i(\omega_0-\omega)(\tau-\tau')}[\bar{x}'_l(\tau') + \lambda\bar{x}'_{l'}(\tau')] = 0. \tag{43}$$

Then, we take the so-called Markov approximation,

$$x(\tau') \cong x(\tau), \quad \bar{x}'(\tau') \cong \bar{x}'(\tau), \tag{44}$$

namely, approximately taking the dynamical variables to the ones that depend only on the present time so that any memory regarding the earlier time is ignored [52].

The Markov approximation is mainly based on the physical assumption that the correlation time of environment is very small compared with the typical time scale of system evolution.

Also under this assumption we can extend the upper limit of the τ' integration in Eqs. (42-43) to infinity and use the equality

$$\lim_{\tau\to\infty}\int_0^\tau d\tau' e^{\pm i(\omega_0-\omega)(\tau-\tau')} = \pi\delta(\omega-\omega_0) \mp i\mathscr{P}\Big(\frac{1}{\omega-\omega_0}\Big), \tag{45}$$

where $\mathscr{P}$ and the delta-function denote the Cauchy principal value and the singularity, respectively. The integro-differential equations in (42-43) are thus reduced to a couple of linear ordinary differential equations. The solutions of x_l and $\bar{x}'_l$, as well as α_l and $\bar{\alpha}'_l$ can then be easily obtained, which result in

$$u = \frac{e^{-i(\omega_0-\lambda\kappa)\tau} + e^{[-i(\omega_0+\lambda\kappa)-2(\pi J(\omega_0)-i\delta\omega)]\tau}}{2}, \tag{46}$$

$$v = \frac{e^{-i(\omega_0-\lambda\kappa)\tau} - e^{[-i(\omega_0+\lambda\kappa)-2(\pi J(\omega_0)-i\delta\omega)]\tau}}{2\lambda}, \tag{47}$$

where $\delta\omega = \mathscr{P}\int_0^\infty \frac{J(\omega)d\omega}{\omega-\omega_0}$. Using the solutions (46-47), one can verify from Eqs. (34) that,

$$\begin{aligned} \Omega(t) &= \omega_0 - \delta\omega, \\ \Omega'(t) &= \kappa - \lambda\delta\omega, \\ \Gamma(t) &= \lambda\Gamma'(t) = \pi J(\omega_0), \end{aligned} \tag{48}$$

which is exactly the coefficients in the Markov master equation of the optical system [22]. This result can also be obtained easier by directly applying the Markov approximation Eqs. (44-45) to Eqs. (36-37).

As shown above, all the coefficients in the master equation have become time-independent, and the non-Markovian master equation (39) is reduced to the Markov master equation under the Markov approximation. This Markov approximation is valid to all kinds of spectral densities, including Ohmic, super-Ohmic and sub-Ohmic cases, while a different spectral density does produce the frequency shift, $\delta\omega = \mathscr{P}\int_0^\infty \frac{J(\omega)d\omega}{\omega-\omega_0}$, and decay rate, $\Gamma = \pi J(\omega_0)$, differently. As a result, we conclude that our exact non-Markovian master equation can not only explore more complicated situation where Markov approximation is unreachable, but also examine different spectral densities between the system and the environment even in the Markov limit. This actually provides a simple way to reveal the underlying mechanism of quantum decoherence.

5 Decoherence dynamics of entangled coherent states

There are two different types of continuous variable entangled states. One is the entangled squeezed states, and the other is the entangled coherent states[5]. We have used the exact non-Markovian master equation derived here to study the non-Markovian entanglement dynamics of two squeezed states in a recent published paper [32]. In this section, we will analyze the decoherence properties of the entangled coherent states. The decoherence dynamics of the two coherent modes is also fully described by the master equation (39) with the non-Markovian character residing in its time-dependent coefficients. The time-dependent coefficients in the master equation are determined by $F_\pm(t)$ as the solution of Eq. (35) for a specific environmental spectral density. In the present work, we will consider the Ohmic spectral density, i.e., $n = 1$ in Eq. (41), which is often the case for optical communication.

In Figs. 1 and 2, we plot the numerical results of the frequency shift $\delta\omega(t)$ and decay rate $\Gamma(t)$ of the individual optical field as well as their corresponding Markov values. It shows that the non-Markovian dissipation-noise dynamics is characterized by two time scales: $\tau_1 = 1/\omega_c$ (the shortest time scale of the environment) and $\tau_2 = 1/\omega_0$ (the time scale of the optical modes). When $t < \tau_1$, both coefficients, $\delta\omega(t)$ and $\Gamma(t)$, grow very quickly, while after τ_1, $\delta\omega(t)$ and $\Gamma(t)$ approach to the corresponding Markov values, given by Eq. (48), gradually when the time approaches to the time scale τ_2. It clearly evidences that the non-Markovian effect has a huge deviation from the Markov effect within the time scale τ_2. This deviation will influence the dynamics later on significantly as a historical memory effect. The time dependent coefficients in the exact master equation (39) contain all the back-action effects between the system and the environment. The non-Markovian decoherence dynamics of the quantum optical field system thus becomes transparent due to the sensitive time dependence of these coefficients within the time scale τ_2.

In the following, we will investigate the decoherence dynamics of the entangled coherent states under the influence of the vacuum fluctuation. The entangled coherent states are defined as

$$|\psi_\pm\rangle = \frac{1}{\sqrt{N_\pm}}\big(|\alpha,-\alpha\rangle \pm |-\alpha,\alpha\rangle\big), \quad |\phi_\pm\rangle = \frac{1}{\sqrt{N_\pm}}\big(|\alpha,\alpha\rangle \pm |-\alpha,-\alpha\rangle\big), \tag{49}$$

which were studied as quasi-Bell states [5, 53], where $N_\pm = 2(e^{2|\alpha|^2} \pm e^{-2|\alpha|^2})$ are the normalization constants. Many schemes to generate such states have been proposed in optical systems and also in other systems [7, 8, 9, 54, 55]. It has also proposed to use these entangled coherent states for teleporting the superposed coherent states [11, 56].

The time evolutions of these entangled coherent states are given by

$$\begin{aligned}\rho_{\psi_\pm}(t) =& \frac{1}{N_\pm}\Big[e^{A_+(t)}\big(|a_+(t),-a_+(t)\rangle\langle\bar a_+(t),-\bar a_+(t)| + |-a_+(t),a_+(t)\rangle\langle-\bar a_+(t),\bar a_+(t)|\big)\\ &\pm e^{-A_+(t)}\big(|a_+(t),-a_+(t)\rangle\langle-\bar a_+(t),\bar a_+(t)| + |-a_+(t),a_+(t)\rangle\langle\bar a_+(t),-\bar a_+(t)|\big)\Big],\\ \rho_{\phi_\pm}(t) =& \frac{1}{N_\pm}\Big[e^{A_-(t)}\big(|a_-(t),a_-(t)\rangle\langle\bar a_-(t),\bar a_-(t)| + |-a_-(t),-a_-(t)\rangle\langle-\bar a_-(t),-\bar a_-(t)|\big)\\ &\pm e^{-A_-(t)}\big(|a_-(t),a_-(t)\rangle\langle-\bar a_-(t),-\bar a_-(t)| + |-a_-(t),-a_-(t)\rangle\langle\bar a_-(t),\bar a_-(t)|\big)\Big],\end{aligned} \tag{50}$$

respectively, where $A_\pm(t) = 2(|\alpha|^2 - |a_\pm(t)|^2)$, and $a_\pm(t) = \alpha F_\pm(t)$. Eqs. (50) can be obtained directly from the exact solution of the reduced density matrix, Eq. (15) plus Eq. (31) by integrating over the initial variables. From Eq. (35) one can verify that for $\lambda = 1$, the entangled coherent states $|\psi_\pm\rangle$ remain in pure states (decoherence free states) because $a_+(t) = \alpha F_+(t) = \alpha e^{-i(\omega_0-\kappa)t}$, namely the time evolution of $|\psi_\pm\rangle$ is independent of the decay rate $\Gamma(t)$ and the shift frequency $\delta\omega(t)$ which are determined by $F_-(t)$ when $\lambda = 1$ [see Eqs. (36) and (37)] and only affect on the time evolution of the other two entangled coherent state $|\phi_\pm\rangle$. Similarly, when $\lambda = -1$, the entangled coherent states $|\phi_\pm\rangle$ becomes decoherence free states since $a_-(t) = \alpha F_-(t) = \alpha e^{-i(\omega_0+\kappa)t}$, while the decay rate $\Gamma(t)$ and the shift frequency $\delta\omega(t)$ are determined by $F_+(t)$ which only affects the states $|\psi_\pm\rangle$. Since $\lambda = 1$ (or -1) corresponds to the case of the two optical modes coupling to the environment in phase (or out of phase), the above result indicates that two of the four entangled coherent states in Eq. (49) become decoherence-free states [15] if the two optical modes couple to the environment in phase (a constructive interference coupling) or out of phase (a destructive interference coupling).

The reason that the ψ-type and ϕ-type entangled coherent states in Eq. (49) have different decoherence behaviors comes from different symmetric properties of these entangled coherent states. The ψ-type and ϕ-type coherent states correspond to the center-of-mass and relative motions of two-field coherent states, respectively. This property becomes clear by defining the center-of-mass and relative motional variables of the two subsystems as $A^\dagger = (a_1^\dagger + a_2^\dagger)$ and $a^\dagger = (a_1^\dagger - a_2^\dagger)$. As one can find, $|\psi_\pm\rangle$ consist of only the relative motion, while $|\phi_\pm\rangle$ lie only on the center-of-mass motion. When the two optical modes couple to the environment in phase, namely, $g_{1k} = g_{2k} = g_k$, the interaction between the optical modes and the environment only affects the center-of-mass motion so that the entangled coherent states of the relative motion, $|\psi_\pm\rangle$, become decoherence-free states. On the other hand, if the two optical modes couple to the environment out of phase, i.e. $g_{1k} = -g_{2k} = g_k$, the interaction between them only affects the relative motion but leaves the entangled coherent states of the center-of-mass motion, $|\phi_\pm\rangle$, free from decoherence. This is indeed a consequence of the sufficient condition for the decoherence-free space protected by symmetry[57].

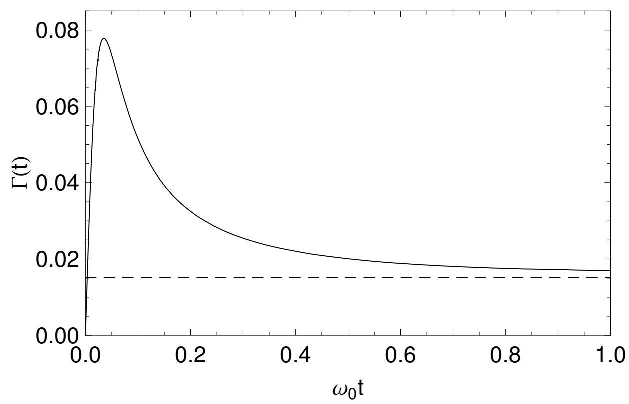

Fig. 1. Comparison of the decay rate $\Gamma(t)$ $[= \lambda\Gamma'(t)]$ between the non-Markovian (solid line) and Markov (dashed line) results. The parameters $\kappa/\omega_0 = 0.5$, $\omega_c/\omega_0 = 30.0$, and $\eta = 0.005$ used in the numerical calculation.

We shall quantify the entanglement degree of the entangled coherent states by the familiar concept of concurrence usually used in a discrete basis [37]. To do so, we may rewrite Eq. (50) in terms of the orthogonal basis [53],

$$|\mathbf{0}\rangle = e^{-\frac{|a_\pm(t)|^2}{2}}|a_\pm(t)\rangle, \qquad |\mathbf{1}\rangle = \frac{e^{-\frac{|a_\pm(t)|^2}{2}}|-a_\pm(t)\rangle - p_\pm(t)|\mathbf{0}\rangle}{\sqrt{1-p_\pm(t)^2}}, \tag{51}$$

with $p_\pm(t) = e^{-2|a_\pm(t)|^2}$. Above change from the coherent state basis to the $|\mathbf{0}\rangle$ and $|\mathbf{1}\rangle$ basis is equivalent to a local unitary transformation of the states, which does not modify the entanglement degree in the original states. In this discrete basis, the concurrence can be calculated as usual. It is not difficult to find that the concurrence $C_{\phi_-}(0) = C_{\psi_-}(0) = 1$, which

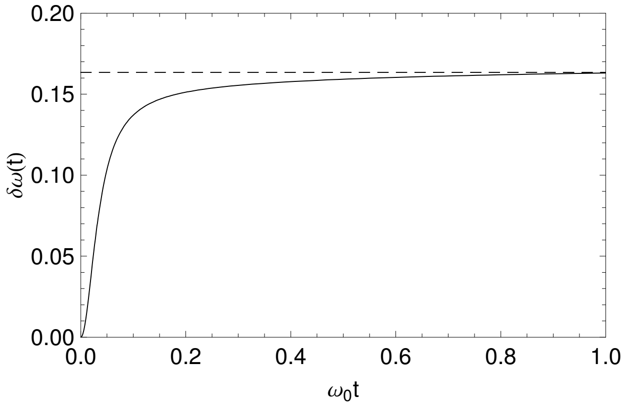

Fig. 2. Comparison of the frequency shift $\delta\omega(t)$ between the non-Markovian (solid line) and Markov (dashed line) results. The parameters used in the numerical calculation are the same as that in Fig. 1.

is maximally entangled irrespective of the amplitude α, while $C_{\phi_+}(0) = C_{\psi_+}(0) = \tanh 2\,|\alpha|^2$, which imply that the ϕ_+ and ψ_+ states are not initially maximally entangled. One can also show that when the two optical modes couple with the environment in phase, i.e. $\lambda = 1$, the concurrence $C_{\psi_-}(t) = 1$, and $C_{\psi_+}(t) = \tanh 2\,|\alpha|^2$, namely, the entanglement of $|\psi_\pm\rangle$ remain unchanged during the time evolution. While $|\phi_\pm\rangle$ are sensitive to decoherence. In contrast, if the two optical modes interact with the environment out of phase, i.e. $\lambda = -1$, $C_{\phi_-}(t) = 1$, and $C_{\phi_+}(t) = \tanh 2\,|\alpha|^2$, while $C_{\psi_-}(t)$ and $C_{\psi_+}(t)$ will decay (disentanglement) due to the decoherence.

In Fig. 3, we show the concurrence evolution in time for the entangled coherent states $|\phi_\pm\rangle$ and $|\psi_\pm\rangle$. With the in-phase coupling between the optical modes and the environment ($\lambda = 1$), our numerical results verify that the entanglement degrees of $|\phi_\pm\rangle$ (given by the solid and dot-dashed lines in Fig. 3) suffer from a fast decay during the time evolution while the entanglement degrees of $|\psi_\pm\rangle$ remain unchanged (the dot-dot-dashed and dot-dot-dot-dashed lines in Fig. 3). To compare the non-Markovian entanglement dynamics with the Markov dynamics, we also plot the concurrence evolution for $|\phi_\pm\rangle$ under the Markov approximation, denoted by the dashed and dotted lines, respectively, in Fig. 3. As one can see, the non-Markovian effect accelerates the disentanglement. This is mainly a contribution of the short time peak in the decay rate $\Gamma(t)$ as a memory effect. It is also worth noting that no entanglement oscillator is observed in above solution even there has coherent coupling $\Omega'(t)$ presented. This is because the two-optical-field coupling $\Omega'(t)$ contributes only a global phase to the entangled coherent states during the time evolution, which has no influence on the entanglement degree of the states. While as expected, $|\psi_\pm\rangle$ are decoherence-free irrespective of Markov or non-Markovian dynamics being considered.

For the case of out-of-phase coupling between the optical modes and the environment, namely $\lambda = -1$, the roles of the decoherence effect on the ϕ-type and the ψ-type entangled coherent states are exchanged. The ϕ-type entangled coherent states remain unchanged, while the ψ -type entangled coherent states are disentangled by decoherence. The numerical results of the entanglement evolution for $|\psi_\pm\rangle$ and $|\phi_\pm\rangle$ are given by the same curves in Fig. 3 with

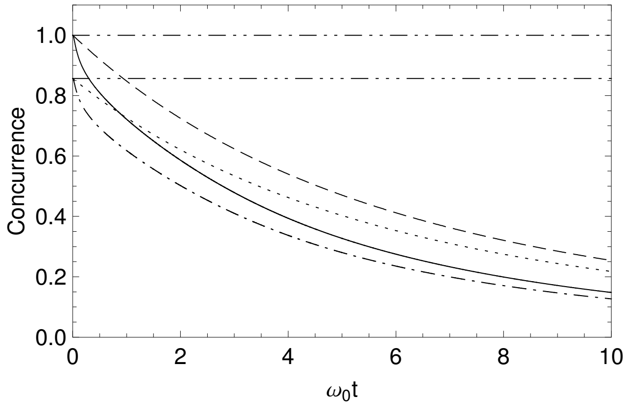

Fig. 3. Time evolution of the concurrences for different initial states. The solid and dot-dashed lines show the non-Markovian time evolution of the concurrences for $|\phi_-\rangle$ and $|\phi_+\rangle$ with $\lambda = 1$ (or $|\psi_-\rangle$ and $|\psi_+\rangle$ with $\lambda = -1$), respectively. Their corresponding Markov behaviors are shown as the dashed and dotted lines, respectively. The dot-dot-dashed and dot-dot-dot-dashed lines are the concurrences for $|\psi_-\rangle$ and $|\psi_+\rangle$ with $\lambda = 1$ (or $|\phi_-\rangle$ and $|\phi_+\rangle$ with $\lambda = -1$), respectively, which remain unchanged during the time evolution. The initial coherent state parameter $\alpha = 0.8$, and other parameters are the same as in Fig. 1.

the exchange between $|\psi_\pm\rangle$ and $|\phi_\pm\rangle$ states as in the in-phase coupling case.

From the above analysis, one can find that when the two identical optical modes couple to a common environment in an arbitrary phase difference, no decoherence-free entangled coherent state can exist among the four entangled coherent states. All the four entangled states could be disentangled by the vacuum fluctuation in time. The non-Markovian dynamics will speed up the disentanglement process with respect to the Markov approximation. However, the parameter λ models the phase difference between the two optical modes coupled with the environment. Physically, it is always possible to adjust the two optical modes such that the couplings of the two optical fields with the environment are either in phase ($\lambda = -1$) or out of phase ($\lambda = -1$). Then two decoherence free states among the four entangled coherent states can always be constructed in principle. It is certainly interesting in seeing experimental evidences on the preservation of two decoherence free entangled coherent states as well as the non-Markovian disentanglement enhancement to the other two entangled coherent states.

6 Summary and Discussions

In summary, we have studied the detrimental effects of environment on the entangled coherent states. We microscopically modeled the decoherence dynamics of entangled coherent states under the influence of vacuum fluctuation. An exact master equation with time-dependent coefficients reflecting the full memory effect of the reduced system has been derived by using the Feynman-Vernon influence functional theory in the coherent-state path-integral representation, which enables us to treat both of the back-actions from the environment to the system and from the system to the environment self-consistently. In addition, we have also explicitly deduced the well-known Markov dynamics for the optical modes from our exact non-Markovian master equation in the Markov approximation. The analytical analysis of the difference between the non-Markovian dynamics and its Markov approximation presented in this paper may provide a quantitative way to experimentally explore the non-Markovian effect as well as the spectral densities between the system and the environment even in the Markov limit.

We then investigated the non-Markovian dynamics of the entangled coherent states, one of two typical continuous variable entanglement states often used in quantum information processing. The other type of continuous variable states, the entangled squeezed states, has already be studied by two of us based the same master equation derived here [32]. Our first-principle analysis shows that the non-Markovian effect accelerates the disentanglement compared with the results based on Markov approximation. It is the short time peak of the time dependent coefficients in the master equation, which is incorporated with the system's dynamics as a historical memory effect, that contributed to this acceleration. Although the Born-Markov approximation has been widely employed in the field of quantum optics, we argue that our investigation might be helpful for understanding decoherence in nanoscale cavity devices and ultrafast optical processes. For example, we have noticed the rapid development of optical cavity technology, which has been employed to confine a single atom [58] or a single quantum dot [59, 60] in strong coupling regime, the prerequisite of quantum network. The strong interaction occuring in nanometer size subject to vacuum fluctuation suffers from some unpredictable incoherence errors [58], which should involve the non-Markovian effects. In this sense, our study of non-Markovian dynamics, although with a simplified model, paves a way toward clarification of the mechanism regarding those incoherence sources.

We have also shown how the decoherence behaviors of the different entangled coherent states depend on the symmetrical properties of these entangled coherent states as well as the interference properties of couplings between the two optical modes with the vacuum electromagnetic environment. Since the exact non-Markovian master equation has been derived non-perturbatively and exactly, decoherence dynamics subject to different spectral densities of environment would be naturally available by our treatment. In fact, the non-Markovian master equation (i.e., Eq. (39)) derived in this paper has been used in treating the decoherence dynamics of entanged squeezed states with sub-Ohmic and super-Ohmic spectral densities of the environment [32]. More complicated cases, e.g., the environment at finite temperature, would be hopefully figured out by the similar way to the derivation of Eq. (31). As a final remark, we would like to mention a very recent experiment for distinguishing different coherent states [61], which shows a potential of using entangled coherent states for quantum communication. The entangled coherent states have significantly different properties from the entangled squeezed states and have been proposed as another type of continuous variable quantum channels. As robustness of the quantum channel is essential in view of decoherence, we expect that our consideration of decoherence dynamics of entangled coherent states would be useful for understanding quantum communication experiments with realistic ultrafast optical processes in nanocavities and nanophotonic systems.

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