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ENTANGLEMENT AND QUANTUM TELEPORTATION CONDITIONS FROM NON-LOCAL QUANTUM COMPUTING THEORY

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Entanglement and quantum teleportation are two inter-related subjects and can be explained through four fundamental non-local quantum computing diagonal operator-state relations, which model the conservative phase-interaction between two adjacent atoms of an entangled atomic chain. Each model atom possesses four eigen-states. There exists a minimum focus distance between the two atoms for the entanglement to exist. Consequently, there exist perpetual elastic phase-changing events on the entire entangled chain at every discrete time interval that is evaluated and shown here. It is analogous to Newtons first law of motion in discrete time. Quantum teleportation means every extra 2π phase between the atoms can be translated in Fourier space into an additional separation distance, a quantized value.

Keywords: Non-local quantum computing theory, time crystals, entanglement and quantum teleportation, physical foundation of cellular automaton

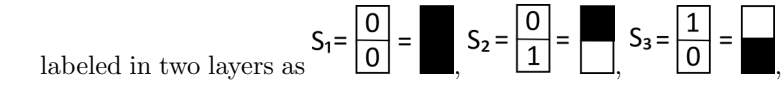
1. Introduction

Quantum computing from an entangled atomic chain require the superposition of the electron states within each atom and the entanglement between any two adjacent atoms of an atomic chain [1-3]. In a model atom with four free-electron eigen-states, the superposition means the discrete Fourier transform to further vectorize and transform the four eigen-states into four computation states (or four corresponding states in the momentum space). Entanglement between two atoms is needed to further settle the relative phase relations of all the eight electron states between the two atoms. That is to say: there is a need to further making a Fourier transform between any pair of atoms, the entanglement procedure. Once those two procedures are prepared and a desired initial computing configuration is provided, all the phase relations of the electron eigen states in the entire atomic chain are accounted for at that given moment. But those phase relations will then undergo perpetual elastic phase changes at every time interval, τ, that is the center discussion of this work. First, we will show there exists a minimum entanglement distance between the two atoms, the focus length to execute the Fourier transform between two atoms. Once that minimum distance, a, is established, any addition of four times of that distance is equally valid. This is to argue that any added integer of 2π phase associated with the eigen-function can be translated in Fourier space into a further separation distance between the two atoms. This provides the foundation

of quantum teleportation and for the transmission of quantum information to a long distance away without requiring the deletion of the original copy. Secondly, the existence of this minimum distance of entanglement implies there also exists a time, τ, where all the electron eigen-states will undergo the rule-based phase changes simultaneously, because all the electron states are elastically coupled with proper relative phase relations and are activated by the non-local operator-state transition mechanism we described earlier [4, 5] and is restated and revised in the section here. Thus, quantum computing from an entangled atomic chain, which is simply a fully Fourier-transformed chain, is to perform the phase computing perpetually. Those phase changes are periodic with a Poincare cycle and hence generate a time crystal [6, 7, 8, 9], through the discrete-time phase-changing mechanism. In addition, there are birth-and -death features, the foundation for a true artificial intelligence. These perpetual phase-changes in an entangled atomic chain are like Newtons first law of motion, except it is not in a continuous time frame. The fact that discrete-time conservative phase changes can exist permanently shows we have an entangled atomic chain as a one-dimensional cellular automaton without the need from any further external inputs, if the above two Fourier-transform procedures are executed and maintained. The foundation of the above arguments come from the earlier establishment of four non-local operator-state relations [4, 5].

1.1. *Four non-local operator-state relations between two entangled atoms*

First, the proper number of states used for computing needs to be settled. The amount of superposition of states to be used for computing can be easily overlooked from the start. It becomes the center concept for the quantum parallel computing. The choice of this number alone reveals the computing scheme that can be possibly employed later. The model used here is from a chain of atoms having a set of four free-electron eigen-states associated with each atom. The standard qubit approach uses only two superposition states and the Fourier transform of two states, the Hadamard transform, will lose the clockwise and counterclockwise rotation distinction. As we have shown earlier [4, 5, 10, 11, 12, 13], a four-state approach provides an upper nutrition layer for quantum computing that is suitable for the general-purpose type (that is the addition-rule based computing) to the biological type and to the quantum phase-computing type from a chain of entangled atoms. That is to say that, in general-purpose computing, for an addition operation of two numbers, say $5 + 3 = 8$, one layer theory requires two separated Euclidean lengths of 5 and 3 units to be joined together along the same line to produce the length of 8 units. In the Turing machine, this scheme requires moving the length of 5 towards that of 3 and thus both front and back ends of length 5 are to be operated on successively and it requires 8 instructions [12, 15]. Such a processor for this scheme has never been constructed by Turing or others. In the two-layer theory, the number, 5, is the nutrition existed at the top layer, to be poured into the lower layer of length 3. When the nutrition layer is totally disappeared from the top, the lower layer of 3 is then grown into 8 units, using the correct 16-rule of parallel addition [4, 5, 12, 13]. It requires only four instruction capability from the processor and such a quantum parallel processor has been constructed (US Patent # 8,525,544), using Aharonov-Bohm effect on a proper quantum network [14]. Biological and quantum computing also require a nutrition layer to preserve the 16 equal probability transition rules. Note that the 16 addition-operation rules are not of equal probability of survival [4, 5, 11, 12]. Therefore, the four computation states,

labeled in two layers as  , are superpositions (discrete Fourier transform) of the four energy eigen-vectors of the free electron in an isolated atom and are labelled as $\bar{S_1}, \bar{S_2}, \bar{S_3}$, and $\bar{S_4}$ corresponding to the four eigen-energies of E_1, E_2, E_3, and E_4 respectively [Fig. 1(e), also Fig. 4 example]. Then, a minimum focus distance between the two neighboring electrons is set when $k_1 a = \pi/2$, the Fourier transform condition. Here, a, is the smallest entanglement distance derived through the lowest electron energy E_1 and momentum value k_1. At this condition, $k_2 a = \pi$, $k_3 a = 3\pi/2$, and $k_4 a = 2\pi$ are also properly Fourier transformed as a set [Fig. 1(c)]. The free electron energy is related the wave vector by $E_m = (\hbar k_m)^2/(2m_e)$, $m = 1, 2, 3, 4$. A chain of N entangled atoms is shown in Fig. 1(a) and labelled with positions as $1, 2, l-1, l, l+1, N-1, N$. Thus, a non-local operator-state relation can be established, if an operator $\bar{S}_i(l-1)$, located at position $l-1$, is brought to operate on the state $\bar{S}_j(l)$ at position l. The orthogonality condition means $i = j$ only [Fig. 1(c)].

When the lowest-energy eigen-vector, $\bar{S}_1(l-1)$, is brought to the location at l, it will acquire an additional phase of $\pi/2$ before it operates on the state $\bar{S}_1(l)$ when the distance, a, is traveled. Thus, the Fourier transform requires the establishment of a focus length, a, like a beam of photons through a pinhole and a proper distance behind the hole is where the Fourier pattern is established. Associated with that minimum distance, a, there is a corresponding time, τ, for this non-local operator-state phase interaction to be accomplished and $\tau = (am_e)/(\hbar k_1)$, where m_e is the electron mass. For an electron energy $E_1 \approx 0.1ev$, we have $a \approx 10\mathring{A}$ and $\tau \approx 5 \cdot 10^{-15} s$. This is compared to our earlier estimate from the uncertainly principle, $\Delta t = \hbar/(\Delta E_1) \approx 6.6 \cdot 10^{-15} s$ [4].

Once τ exists, the elastic phase changes can occur at every such discrete time interval perpetually as an equivalent of Newtons first law in discrete time frame. Hence the one-dimensional cellular automaton for piece-wise Euclidean geometry exists perpetually and a new type of non-local mathematics emerges at the fundamental level.

Furthermore, in this non-local operator-state transition, a quantum teleportation is also established because $e^{(ik_1 a)} = e^{(i\pi/2)} = e^{[i(\pi/2+2M\pi)]} = e^{[ik_1(a+4Ma)]}$.

That means every extra 2π in the exponential phase factor will be translated into the $4Ma$ distance, where M is any integer. Note the distance of a possible entangled distance between two neighboring electrons must be satisfied by the commensuration of the phase relations between all the eigen states. When the phase of the lowest energy is extended with 2π, the phase of the second (third, fourth) lowest energy is extended with $4\pi(6\pi, 8\pi)$. Thus, the commensuration condition of the phases is translated to a set of quantized distances, $4Ma$. There exist four complete diagonal non-local operator-state rules, labelled as Rules I, II, II and IV from left-right and time reversal symmetry considerations (Fig. 2). The two mix-conjugates of the operator-state relations of Rule II and Rule III can only exist when four computation states are used and thus cannot exist in the Qubit approach or from a local theory. The four corresponding computation rules are the four possible cyclic rules shown in Fig. 3.

2. Conclusions

Quantum computing derived from an entangled atomic chain requires two prepared processes.

When the discrete Fourier transform (the eigen-state superposition process) within each atom is performed through the proper optical transition procedures and executed on the atom, it still requires a minimum separate distance, a, between any two neighboring atoms to further execute the discrete Fourier transform (the entanglement process) between any two adjacent atoms. This minimum focus distance is to settle all the phase relations of all eigen-vectors from one end of the atomic chain to the other end in order to perform the parallel quantum computing, the phase computing, since all eigenvectors in the entire chain are accountable with their relative phase relations, the superconducting condition.

In the model, each atom possesses four free electron eigen-vectors. At this minimum focus distance, the lowest state eigen-vector of an atom can be brought over to interact with the adjacent atom at every interval time, τ, the electron phase-interaction or traveling time required over the distance, a. We show that the time, τ, is the order of uncertainty time. During this time interval of τ, the three other higher-order eigen-states are also completed for the similar phase-interactions with two, three and four times of the interaction rate. Once those two sets of Fourier transform are maintained on the chain, there now exists a discrete-time perpetual elastic phase alterations on all the eigen-vectors in the chain. Thus, for a given initial computation state on the chain, represented by a sequence of one of the four possible superposition state on each atom, the computation result is a perpetual time crystal growth through this elastic phase-bonding mechanism in time and it exhibits a well-defined Poincare cycle, the period of the time crystal. This mimics Newtons first law of motion, but from a conservative phase-coupled atomic chain in a discrete-time performance. This establishes a new physics-based cellular automaton chain that can exist through its internal elastic iteration mechanism permanently if the two sets of Fourier transform are maintained.

All the results here are directly originated from the four sets of four non-local operator-state tensor relations of the eigen-states between two adjacent atoms [4, 5]. They are fundamental in nature and lead to the four complete cyclic sequences of the computation state transition rules (Rule I, II, III and IV) and the physical foundation of the time crystal generations in quasi-planar as well as in spherical time crystals for the quantum computing and the memory storage [4].

Furthermore, the minimum entanglement distance, a, is shown to be extendable to a set of quantized larger distances to explain the quantum teleportation. This is because the disposal mathematical exponential 2π factor is translated into the larger separating distance between the two atoms with the same entanglement condition in the Fourier space without extra phase-interaction time added as if the 2π phase is an actual direct interconnection. In other words, the 2π phase factor in an exponent attached to an eigen-vector may not be disposable for a mathematical convenience in momentum space. When the multiple 2π phasor factors from all four eigen-vectors are commensurate, it establishes the identical phase-relations at that location as compared to the relations established at the minimum distance, a. That means a proper large distance will maintain the same Fourier transform relations, the entanglement condition. Thus, quantum information existed in one end of the chain, represented by a string of initial states, will then be duplicated over a long distance to the other end of the chain without deleting the original copy. This is due to the birth-and-death capability of a longer entangled chain.

Here a new mathematical foundation is shown through this example of non-local non-

Euclidean (or piece-wise Euclidean) operator-state relations, which can also be viewed more generally as a pattern- transformation process. The consequence of those four sets of computation rules (Rule I, II, III, and IV) is that they are not addition-rule compatible. For example, $5 + 3 = 8$ can not be satisfied here on those non-local computation states. The cellular automaton example shown here represents part of our new understanding into a new kind of science which is not Euclidean. This example here is provided with a physical foundation by showing that the conservative phase-changing relations that can exist in an atomic chain through the internal Fourier transform on the atoms and the external Fourier transform between two atoms.

Finally, a remark for the standard Qubit approach is addressed here [1, 2, 3]. Qubit method employs Fourier transform on a chain of atoms with two eigen-states only, through Hadamard gate, for example. Then the two Fourier transformed states on each atom are used as the computation states. It is a one-layer theory without the nutrition layer on the top to form a 4-state theory that is shown here. Biological and quantum phenomena require a nutrition layer for birth-and-death phenomena in order to reduce the instruction capabilities required on the quantum processor needed for the quantum computing. If the non-local theory here is reduced to one layer theory, then the four non-local operator-state relations are reduced to only two relations. That means there are no two mix-conjugate relations shown here and there is no distinction between clockwise and counterclockwise rotations for the cyclic computation states [Fig. 4(a)]. The two-layer (four-states) approach presented here establishes a concrete cellular automaton computing architecture which is rule-based (Rules I, II, II and IV) for quantum parallel computing, in sharp contrast with the qubit approach which is described through truth-table-based (logic gate-based) by others [1, 2, 3]. Pattern transformation (or non-local operator-state relations) is the general scheme to classify various groups of parallel computing from Euclidean-based general-purpose computing to helical-based biological and to piece-wise-Euclidean based quantum phase computing.

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Appendix A

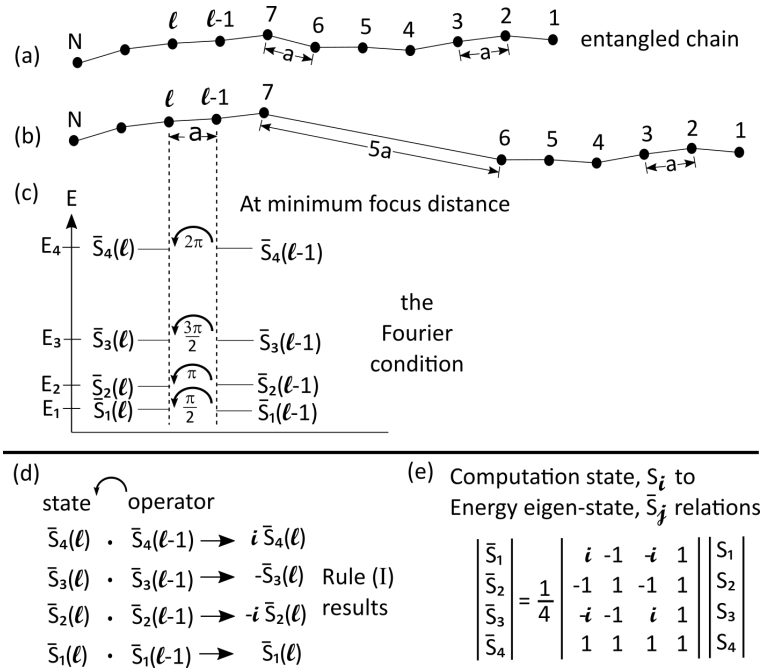

Fig. A.1. (a) Quasi-linear one-dimensional entangled atomic chain of size N, each atom is separated by a minimum focus distance of a; (b) one of the many possible equivalent geometries of (a), where atoms #6 and #7 are separated by $5a$; (c) Four pairs of eigen-energies and eigen-vectors of the two atoms between location $l-1$ and l, when a minimum focus distance is established for the fundamental diagonal non-local operator-state relations to exist. That is: for an operator $\bar{S}_j(l-1)$ at location, $l-1$, to operate on the state $\bar{S}_i(l)$ at location, l, on the chain. Those non-local diagonal results are shown in (d) for Rule I only; (e) The discrete Fourier transform between the four computation states and the four eigenvectors on each atom. In quantum parallel computing, the eigen-state at l for the operator at $l-1$ is also simultaneously served as an operator for the state located at $l+1$. The 16-non-local computation-state rules are in four complete cyclic form as shown in the four sequences in the circles and arrows in Fig. 3. The inner arrows come first, then the outer arrows. The four 16-rule sets are shown in Fig. 3 as Rule I, II, III and IV.

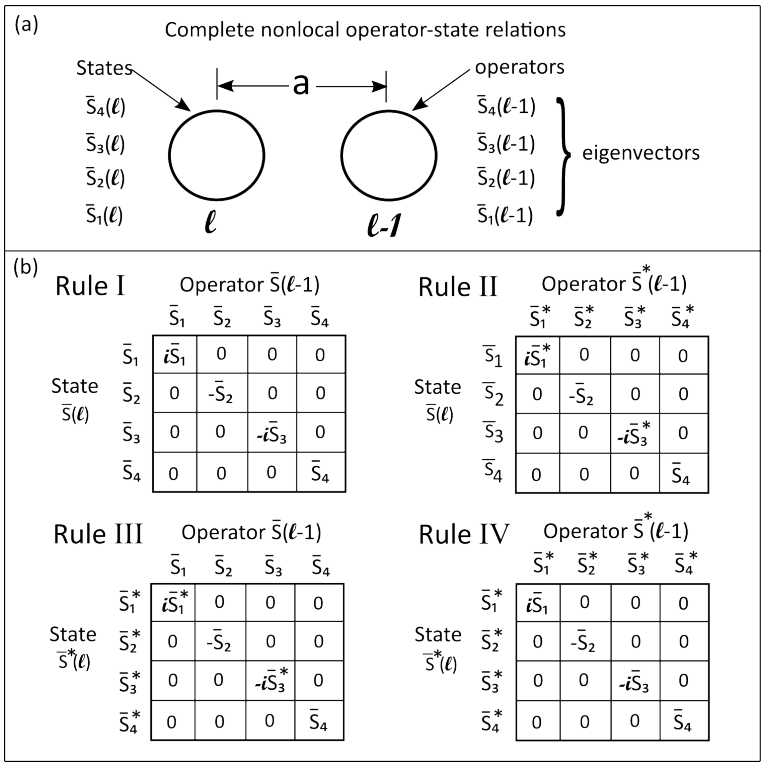

Fig. A.2. Complete 4 sets of non-local operator-state relations for the four pairs of electron eigenvectors between two adjacent atoms at a minimum focus distance, a. They exist concurrently. Rule II and Rule III are for the mix-conjugate pairs that do not exist in qubit theory. The equivalent configurations for an entangled atomic chain are shown in two examples in Fig. 1(a) and 1(b). Therefore, in quantum mechanics, the exponential phase factor of an integer of 2π, associated with the eigen-vectors, is not a disposal factor, but it is translated further into the separation distance in Fourier domain and that forms the foundation of entanglement at a large distance away to establish the quantum teleportation capability. The quantum phase- computing from an entangled atomic chain is thus illustrated in Fig. 4 between a pair of atoms having a computation state $S_3(l-1)$ at the right side and $S_2(l)$ state at the left. The superposition of states in each atom is shown by the four vectors with their associated phase values. The transition of $S_2(l)$ computation state, due to the presence of $S_3(l-1)$ computation state at location $l-1$, into $S_4(l)$, after an interval of time, τ, is explained in Fig. 4. This illustrates one of 16 rules, rule #7 in Rule I in Fig. 3. There are four such rule sets (Rule I, II, II and IV) and they exist simultaneously. Thus, the quantum parallel computing described here is rule-based and not logic-gate-based.

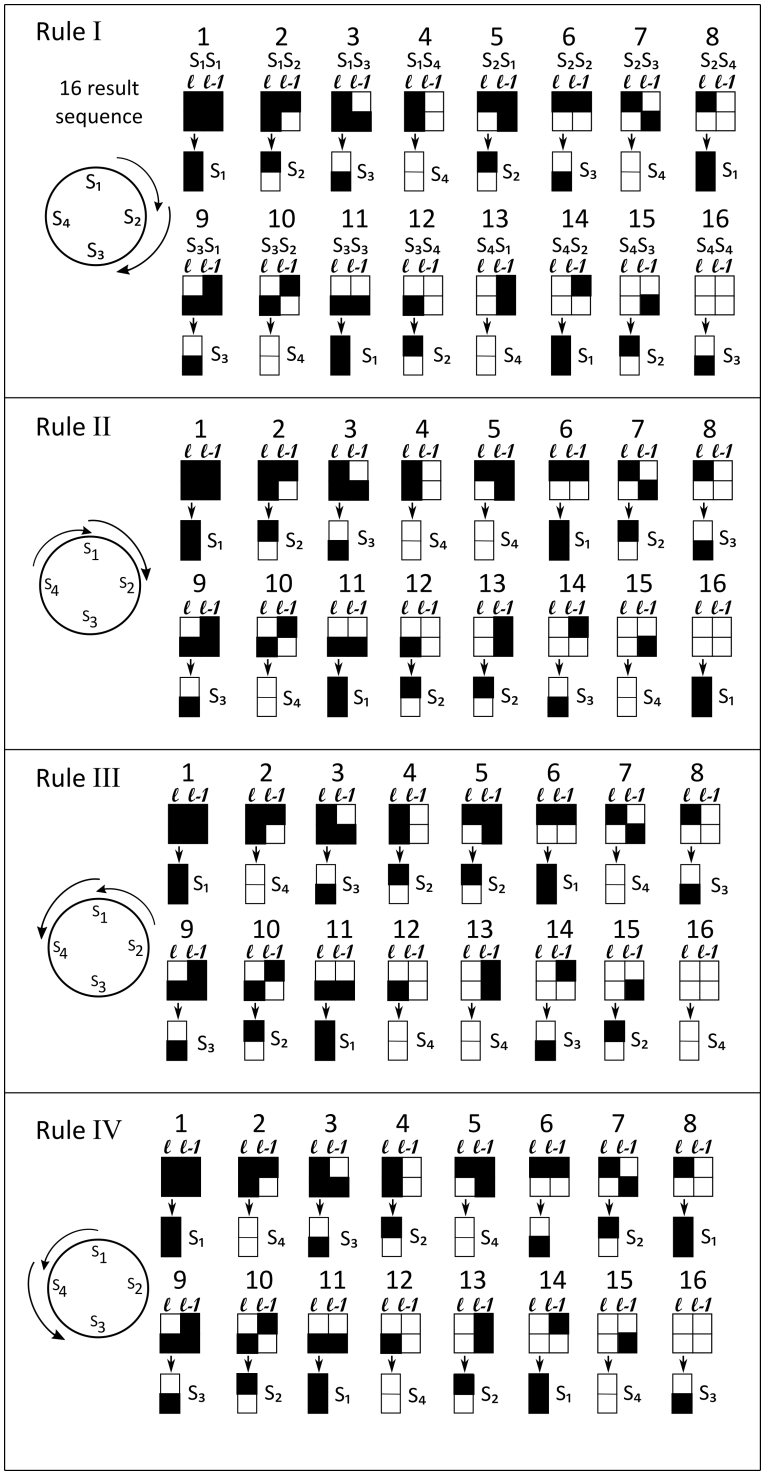

Fig. A.3. Four complete computation-state transition rule sets: Rule I, II, III, and IV of the 16 computation-state transitions rules with four separate cyclic sequences. Those are obtained through discrete Fourier transform relations between the computation states and the eigen vectors from Fig. 2. For example, in Rule I set, the resultant sequence for the new states is in the order of 1234, 2341, 3412 and 4123 as indicated by the left circle.

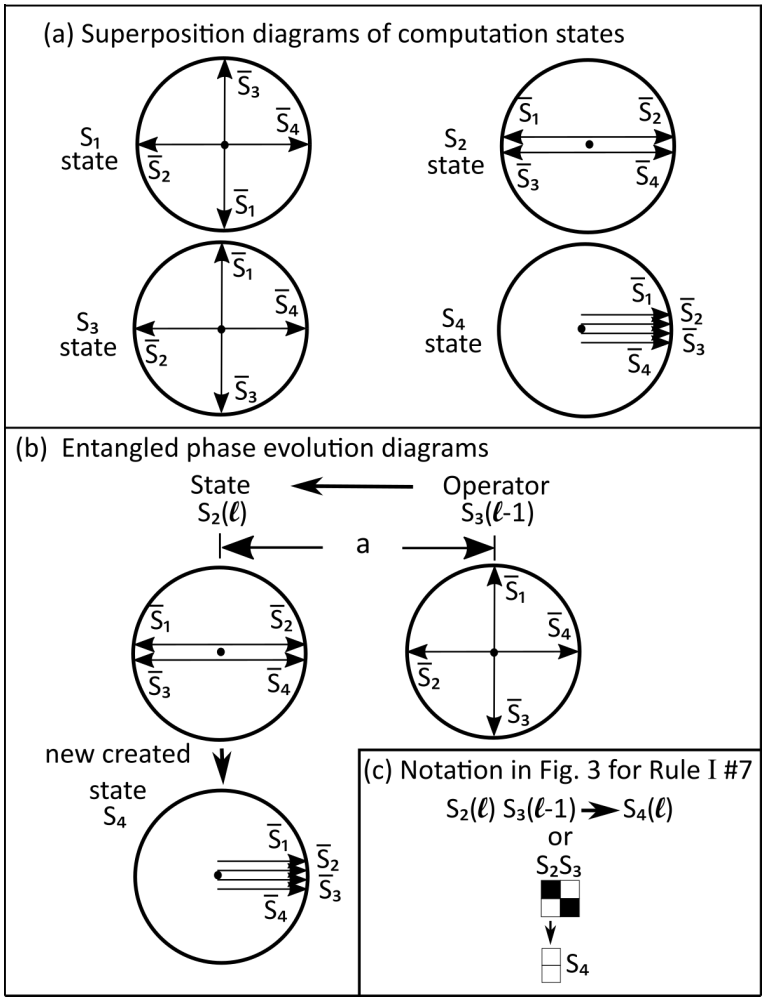

Fig. A.4. Example of quantum phase computing from an entangled atomic chain with the illustration of $S_2(l)S_3(l-1) \rightarrow S_4(l)$ as shown by Fig. 3, Rule I, #7 rule. For the Rule I set, the superpositions of four computation states, S_1,S_2,S_3 and S_4 in terms of the four eigen-vectors of the atom (the circle) are represented by the four-arrows to indicate the four phase relations, in 4 possible, $(i,-1,-i,1)$, directions. For example, $S_1 = -i\bar{S}_1 - \bar{S}_2 + i\bar{S}_3 + \bar{S}_4$, is shown in the phase diagram on the upper left corner and similarly for S_2,S_3, and S_4. That means S_1 computation state is constructed from the four eigen-vectors, spaced clockwise equally with 90-degree phase difference apart. Similarly, $S_2(S_3, S_4)$ are constructed with eigen-vectors spaced $180(270, 360)$ -degrees apart. The quantum phase computing is illustrated with the evolution of computation state $S_2(l)$ at l, due to the presence of a computation state $S_3(l-1)$ at $l-1$, into a new state of S_4 at l. Here $S_3(l-1) = i\bar{S}_1 - \bar{S}_2 - i\bar{S}_3 + \bar{S}_4$ and $S_2(l) = -\bar{S}_1 + \bar{S}_2 - \bar{S}_3 + \bar{S}_4$ are shown in the phase diagrams from part (a). For the first term operator-state computation, the 90-degree phase $\bar{S}_1$ at $l-1$ is to interact with 180-degree phase of $\bar{S}_1$ at l, it will acquire additional 90-degree phase in the process due to the traveling of distance a. Hence the total phase factor is added up to 360-degree for the new orientation of $\bar{S}_1$ vector after time, τ. Similarly, for the 180-degree phase $\bar{S}_2$ at $l-1$ is brought to interact with zero degree $\bar{S}_2$ vector at l, it will acquire additional 180-degree phase and hence the result is 360-degree phase factor for the new vector $\bar{S}_2$ and that process has occurred two times already during the time interval τ. Similarly, for $\bar{S}_3$ and $\bar{S}_4$ vectors with three times and four times occurrences during the same interval. The net result is that the four new eigen-vectors all orientated in the zero-degree direction to arrive at the new S_4 computation state automatically in the time interval, τ. This picture explains one of the 16 rules, rule #7 in the Rule I set shown in Fig. 3 and is represented by the pattern transformation notation in part (c). There are four complete rule sets (Rule I, II, III and IV) listed in Fig. 3, including the mix conjugation relations. The phase-computing here are conservative and a new set of phase relations on the entire chain emerges at every time interval, τ. Hence it is a mimic of Newtons first law in discrete time. The evolution results in a periodic occurrence of any initial configuration (the information content) with a Poincare cycle, the period of the time crystal that lies between the value of $2N\tau$ and $4N\tau$. We also note that the existence of phase-relations implies that an entangled atomic chain is also a four-channel distributed superconductor between two ends.