Quantum Information and Computation, Vol. 23, No. 1&2 (2023) 0067–0080

EXCHANGE THE MARKOVIANITY AND THE MEMORY OF MAGNETIC ENVIRONMENT INTERACTING LOCALLY WITH A SINGLE QUANTUM DOT

A. R. MOHAMMED T. M. EL-SHAHAT

Math. Dept., Faculty of Science, Al-Azhar University
Assiut 71524, Egypt.

N. METWALLY

Math. Dept., Faculty of Science, Aswan University
Aswan 81528, Egypt.
Department of Mathematics, College of Science, University of Bahrain
P. O. Box 32038, Kingdom of Bahrain.

Received June 22, 2022
Revised December 6, 2022

In this manuscript, we show that it is possible to change the environment Markovianity/memory into non-Markovianity/memoryless, and vice versa. This idea is clarified by considering a system of a single two level quantum dot interacts locally with a magnetic field. The Markovianity of the environment depends on whether the coupling between the two systems is time dependent/independent and whether the systems suffering from damping or not. The amount of the lost/gained information and its scrambling depends on the energy gap spacing between the levels of the quantum dot, where the Skew information and the out-of-time ordered are used as quantifiers for both phenomena. Thermally, one can freeze the environment properties to be memory/ memoryless, where our results show the amount of exchanging information and its scrambling are constant as the temperature increases.

Keywords: Quantum dot, Scrambling, Markovianity, memoryless.

1 Introduction

Quantum dots systems (QDs) are one of the most important candidates that may be used to build quantum computer [1, 2]. Moreover, the quantum dot systems are used as a solid state approach to implement teleportation [3]. Also, it is shown that QDs are considered as responsible qubit systems for encoding quantum information [4–9]. As multi-qubit systems, multi entangled -quantum dot systems are more powerful than single systems. Therefore various methodologies have been adopted to preserve the entanglement of the bipartite systems, such as the quantum zeno effect, or the turing paradox [10, 11], decoherence free subspace [12, 13], feed back control [14, 15], and dynamical decoupling [16, 17].

In reality, any quantum system interacts locally with its surrounding and consequently it may gain or lose information from this environment. In this context, this interaction is called Markovian if the information flows in one direction from the system to the environment. In contrast, it is called non-Markovian, if the information flow back into the system [18]. One of the most simplest method for calculating the degree of the Markovianity, is the trace distance between the initial state and the final state [19–22].

Due to the chaotic, the phenomenon of quantum information scrambling ($\mathcal{Q}_s$) may take place in any physical system. The scrambling degree of the information may be quantified by a function called, out-of-time ordered correlations (OTOCs) [23]. This function have been used to quantify the "fast scrambling" governed by universal Lyapunov exponents in the dynamics of black holes to describe the chaotic systems with holographic duals [24, 25]. In the context of the Markovian /non-Markovian interactions, the information scrambles in one or two directions, where information scrambles from the system into its environment, and vis versa [26–29]. The OTOCs is used to quantify the degree of information scrambling.

Therefore we are motivated to discuss the possibility of changing the Markovianty of an environment, and its memory/memorless efficiency. Moreover, the amount of scrambling information between the system and its environment is investigated. This idea is clarified by assuming a two-level single quantum dot (QD) interacts locally with magnetic field, where the interaction is considered as non-thermal and thermal. Moreover, the behavior of theses phenomena is studied within/witout damping. The manuscript is designed as following: in Sec.(2), we describe the system and its evolution for thermal/non-thermal QD system in the presence/absance of the damping. A mathematical forms are reviewed as; the measure of Markovianity, the skew information; which represent a quantifier of the gained/lost information, and the out time function (OTOCT) as measure of scrambling information, are introduced in Sec.(3). The numerical calculations and description of the behavior of all the quantifiers are introduced in Sec.(4). Finally, we summarize and conclude our results in Sec.(5).

2 The system and its evaluation

One of the proposals concerning new solid-state quantum computers (QC) is the possibility of using quantum dots (QDs). QDs is a semiconductor solid-state complex that behaves in many ways like an atom, which can be modelled as effective two-level atoms (TLAs). A TLA is able to describe some phenomena in quantum physics such as photon echoes, self-induced transparency, mode locking, and optical nutation [30]. The Hamiltonian which describe a single QD interacts with an electric field consists of two terms; the energy difference between the excited the ground states, and the interaction between the QD and the electric field. Mathematically, the total energy of the $QD - F$ system is given by, [31, 32]:

$$\mathcal{H} = \frac{\hbar\omega_0}{2}\sigma_z - \frac{\hbar\lambda(t)}{2}(\sigma^+ + \sigma^-), \tag{1}$$

where ω_0 is the energy difference between the ground and excited states, and $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$, is the well known Pauli operator, and $\lambda(t)$ describes the coupling between the field and the quantum dot, while $\sigma^+ = |1\rangle\langle 0|$ and $\sigma^- = |0\rangle\langle 1|$ are the raising and lowering operators.

1. *Non thermal interaction without damping*
   In this subsection we introduce the time evolution of the $QD - F$ system when the QD is in its ground state, namely $|\psi_{QD}(0)\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. At any $t > 0$, the time evolution of

the quantum dot is given by,

$$|\psi_{QD}(t)\rangle = U(t)\,|\psi_{QD}(0)\rangle\,,\ \mathcal{U}(t) = exp[-i\mathcal{H}t]. \tag{2}$$

The density operator of the quantum dot is written in the computational basis as,

$$\rho_{QD}(t) = \varrho_{11}\,|0\rangle\,\langle 0| + \varrho_{12}\,|0\rangle\,\langle 1| + \varrho_{21}\,|1\rangle\,\langle 0| + \varrho_{22}\,|1\rangle\,\langle 1|\,, \tag{3}$$

where,

$$\begin{aligned}
\varrho_{11} &= \frac{\phi(t)^2}{\mu(t)^2}\sin^2(\mu(t)),\\
\varrho_{12} &= \frac{-\phi(t)\sin(\mu(t))}{2\mu^2}(t\omega_0\sin(\mu(t)) + 2i\mu\cos(\mu(t))),\\
\varrho_{21} &= \varrho_{12}^*, \quad \varrho_{22} = \cos^2(\mu(t)) + \frac{t^2\omega_0^2}{4\mu^2}\sin^2(\mu(t)),
\end{aligned} \tag{4}$$

with, $\mu(t) = \sqrt{\frac{1}{4}\omega_0^2 t^2 + \phi^2(t)}$, and, $\phi(t) = \int_0^t \frac{\lambda(t)}{2}dt$.

For the time independent coupling, namely $\lambda \neq \lambda(t)$, the time evolution of the initial state $\psi_{QD}(0)$ is given by $\tilde{\rho}_{QD}(t)$, which is similar to that given by (3), but with different coefficients, where

$$\begin{aligned}
\tilde{\varrho}_{11} &= \frac{\lambda^2}{\eta}\sin^2(\frac{1}{2}t\eta),\\
\tilde{\varrho}_{12} &= \frac{\lambda}{2\eta}(i\eta\sin(t\eta) + \omega_0(-\cos(t\eta)) + \omega_0),\\
\tilde{\varrho}_{21} &= \tilde{\varrho}_{12}^*, \quad \tilde{\varrho}_{22} = \frac{1}{2\eta}(\lambda^2(1+\cos(t\eta)) + 2\omega_0^2),\ \eta = \sqrt{\lambda^2 + \omega_0^2}.
\end{aligned} \tag{5}$$

2. *Non thermal interaction within damping*

In this case, the master equation which describes the dynamics of the quantum dot state interact locally with an intensity damping of an environment can be written as,

$$\frac{d\Re}{dt} = \frac{-i}{\hbar}[\mathcal{H}(t), \Re] - \frac{\gamma}{2}(\Re\sigma^+\sigma^- - 2\sigma^-\Re\sigma^+ + \sigma^+\sigma^-\Re). \tag{6}$$

The dynamics of this system is described by the following four differential equations,

$$\begin{aligned}
\frac{d\Re_{11}(t)}{dt} &= -\gamma\Re_{11} - \frac{i\lambda(t)}{2}(\Re_{12} - \Re_{21}),\\
\frac{d\Re_{12}(t)}{dt} &= (-\frac{\gamma}{2} - i\omega_0)\Re_{12} - \frac{i\lambda(t)}{2}(\Re_{11} - \Re_{22}),\\
\frac{d\Re_{21}(t)}{dt} &= (-\frac{\gamma}{2} + i\omega_0)\Re_{21} - \frac{i\lambda(t)}{2}(\Re_{22} - \Re_{11}),\\
\frac{d\Re_{22}(t)}{dt} &= \gamma\Re_{11} - \frac{i\lambda(t)}{2}(\Re_{21} - \Re_{12}).
\end{aligned} \tag{7}$$

The analytical solution of the system (7) is given by,

$$\begin{aligned} &\Re_{11}(t)=\frac{1}{2}e^{-\gamma t}(\cos(\phi)+1), \quad \Re_{12}(t)=-\frac{1}{2}i\sin(\phi)e^{-\frac{1}{2}t(\gamma+2i\omega_0)}, \\ &\Re_{22}(t)=1-\frac{1}{2}e^{-\gamma t}(\cos(\phi)+1), \quad \Re_{21}(t)=\Re_{12}^{*}(t),\ \phi(t)=\int_0^t \lambda(t)dt. \end{aligned} \tag{8}$$

The details of this solution are given in the appendix(A) [38, 39]. Now, the density operator of the thermal state $\Re_{QD}$ can be written as,

$$\Re_{QD}(t)=\Re_{11}\left|0\right\rangle\left\langle 0\right|+\Re_{12}\left|0\right\rangle\left\langle 1\right|+\Re_{21}\left|1\right\rangle\left\langle 0\right|+\Re_{22}\left|1\right\rangle\left\langle 1\right|. \tag{9}$$

For the time independent case, the density operator of the quantum dot system $\tilde{\Re}(t)$ is similar to (9), where the coefficients are given by,

$$\begin{aligned} &\tilde{\Re}_{11}(t)=e^{-\gamma t}\cos^2\left(\frac{\lambda t}{2}\right), \quad \tilde{\Re}_{12}(t)=-\frac{1}{2}ie^{-\frac{1}{2}t(\gamma+2i\omega_0)}\sin(\lambda t), \\ &\tilde{\Re}_{22}(t)=1-\frac{1}{2}e^{-\gamma t}(\cos(\lambda t)+1), \quad \tilde{\Re}_{21}(t)=\rho_{12}^{*}(t). \end{aligned} \tag{10}$$

3. *Thermal interaction:*
Let us assume that quantum dot is initially prepared in its ground state. By using the H (1), the final state of the thermal quantum dot is given by,

$$\rho_{QD}(T)=\frac{1}{Z}\sum(\exp(\frac{-E_i}{T}))|\psi_i\rangle\langle\psi_i|, \tag{11}$$

where, $Z=Tr(\rho_{QD}(T))$ is the partition function and E_i, $\left|\psi_i\right\rangle, i=1,2$ are the eigenvalues and the eigenvectors of the Hamiltonian (1) which are given by,

$$\begin{aligned} E_1&=-\frac{\beta}{2}, \quad E_2=\frac{\beta}{2}, \\ |\psi_1\rangle&=\frac{1}{\sqrt{(\beta-\delta)^2+1}}((\beta-\delta)|0\rangle+|1\rangle), \\ |\psi_2\rangle&=\frac{1}{\sqrt{(-\beta-\delta)^2+1}}((-\beta-\delta)|0\rangle+|1\rangle), \end{aligned} \tag{12}$$

with $\beta=\sqrt{\delta^2+1}$, $\delta=\frac{\omega_0}{\lambda}$. In the computational basis set, $\{\left|0\right\rangle, \left|1\right\rangle\}$, the state $\rho_T(QD)$ may be written as,

$$\rho_{QD}(T)=\kappa_{11}\left|0\right\rangle\left\langle 0\right|+\kappa_{12}\left|0\right\rangle\left\langle 1\right|+\kappa_{21}\left|1\right\rangle\left\langle 0\right|+\kappa_{22}\left|1\right\rangle\left\langle 1\right|, \tag{13}$$

where

$$\begin{aligned} \kappa_{11}&=\frac{e^{\frac{-\beta\eta}{2}}((\beta-\delta)e^{\beta\eta}+\beta+\delta)}{2\beta}, \quad \kappa_{12}=\rho_{21}=\frac{\sinh(\frac{\beta\eta}{2})}{\beta}, \\ \kappa_{22}&=\frac{\delta\sinh(\frac{\beta\eta}{2})}{\beta}+\cosh(\frac{\beta\eta}{2}),\ Z=2\cosh(\frac{\beta\eta}{2}),\ \eta=\frac{\lambda}{T}. \end{aligned}$$

Now, we have all the details to investigate the scrambling of information, the skew information, and behavior of the non-Markovianity.

3 Markovianity and non-Markovianity

It is well known that, the dynamics of open quantum systems is divided into two different classes Markovian (memory) and non-Markovian (memoryless). If the properties of the forward behavior of a quantum system are not linked to the past events, then this system is called Markovian, otherwise it is called non-Markovian [33]. However, for the Markovian dynamics, the transfers information from the system into its surrounding environment is irreversible. For Markovianity and non-Markovianity behavior of a single QD, we quantify the amount of information losses via some different measures. Breuer et.al [19] have introduced a measure of Markovianity based on the assumption that, the trace distance between any two states decreases monotonically during the evolution. However, non-Markovianity behavior is displayed when any increasing of the trace distance during the evolution is predicted. In this context, the Markovianity/non-Markovianity of a pure state is defined by the distance's behavior,

$$\mathcal{D} = \sqrt{1-\mathcal{F}}, \quad \mathcal{F} = |\langle\psi(0)|\psi(t)\rangle|, \tag{14}$$

where $\mathcal{F}$, is the fidelity between the initial state of the quantum dot $\psi_{QD}(0)$ and its state $\psi_{QD}(t),\ t > 0$ [34].

3.1 *Skew information*

The skew information $\mathcal{S}_f$ is a measure of the local quantum uncertainty, this physical quantity is invariant under the local unitary operator.

$$\mathcal{S}_f(\rho, K) = -\frac{1}{2}Tr[\sqrt{\rho}, K]^2, \tag{15}$$

where, K is a fixed conserved observable. From this definition,one can say that, the skew information quantifies the degree of non-commutativity of a state ρ and an observable K. In other words, it measures the information that contained in a state ρ with respect to a conserved observable. In particular, if $\rho = |\psi\rangle\langle\psi|$ is a pure state, then the skew information is the amount of information on the values of observable K [35]. Mathematically, it may be written as:

$$\mathcal{S}_f(|\psi\rangle, K) = \langle\psi|K^2|\psi\rangle - \langle\psi|K|\psi\rangle^2. \tag{16}$$

3.2 *Out-of-Time-Ordered-Correlator function*

It is well know that, for open system there is a possibility that quantum system interacts locally with its surrounding, and consequently the initially localized quantum information spreads from the system into its environment. The Out-of time-ordered correlators function (OTOCs) is used to quantify the degree of information scrambling [36]. Consider that, W and V denote Hermitian or unitary operators defined on the system's Hilbert space. The OTOC is defined as [35, 37]:

$$\mathcal{Q}_s = \frac{1}{2}(1 - F(t)), \qquad F(t) \text{ is a real}, \tag{17}$$

where,

$$F(t) = Tr[W(t)^{\dagger}.V^{\dagger}.W(t).V.\rho]. \tag{18}$$

Here $W(t) = U(t)^{\dagger} W U(t)$ is evolved in the Heisenberg picture with the unitary evolution operator $U(t) = e^{-i\hat{\mathcal{H}}t}$. In our numerical calculations, we assume that $W = V = \sigma_y$. The function $F(t)$ is used to diagnose the scrambling of quantum information and quantifies the possibility of recovering the information via local operations. The quantum fidelity for a pure state,

$$\mathcal{F} = |\langle\psi(0)|\psi(t)\rangle|. \tag{19}$$

4 Numerical results

In this section, we shall investigate the Markovianity of a single quantum dot that is initially prepared in a ground state. Moreover, scrambling of information from/to the quantum dot is discussed by considering the behavior of two measures; the information scrambling measure $\mathcal{Q}_s$ and the skew information $\mathcal{S}_f$.

1. Non thermal interaction

   Fig.(1) displays the behavior of the three quantifiers for a quantum dot interacts non-

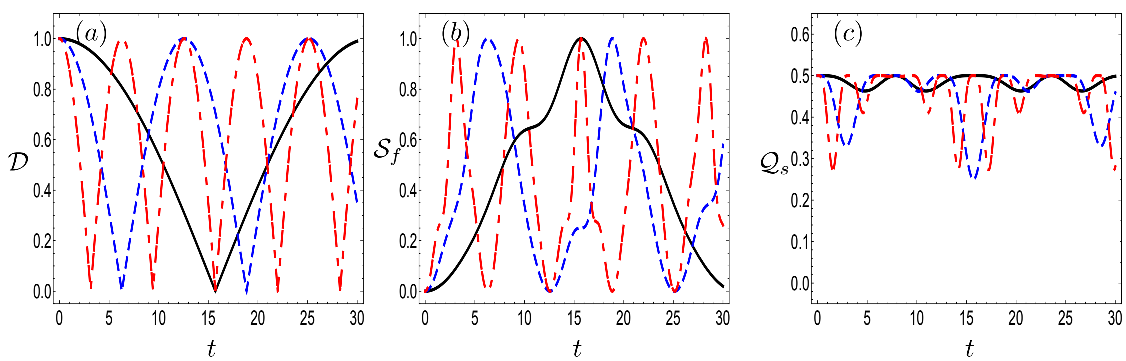

Fig. 1. For the non-thermal interaction and time independent coupling with $\omega_0 = 0.2$.(a) The Markovianity, $\mathcal{D}$, (b) the skew information $\mathcal{S}_f$, and (c) the quantum information scrambling $\mathcal{Q}_s$. The solid, dot and dash-dot curves represent the behavior of the three quantifiers at $\lambda = 0.2, 0.5$ and 1, respectively.

   thermally with a local magnetic field at different values of the interaction coupling constant. In this investigation, it is assumed that the energy gap between the excited and the ground state of the quantum dot, $\omega_0 = 0.2$, and several values of the coupling constant are considered. From Fig.(1a), it is clear that, as soon as the interaction is switched on, the Markovian behavior is depicted, where it decreases gradually at small values of the coupling constant. The periodic behavior of Markovianity is displayed as one increases the coupling constant. The QD losses its Markovianity fast at large values of the coupling constant. The behavior of the skew information $\mathcal{S}_f$ is displayed in Fig.(1b), where it increases as the interaction time increases. However, the increasing rate depends on the coupling constant between the QD and the magnetic field. As soon as the skew information reaches its maximum bound, the QD turns into a completely mixed state and the Markovianity is maximum, namely, the QD loses a large amount of its local coded information. The behavior of quantum scrambling of information from/into the quantum dot is exhibited in Fig.(1c), where it decreases as the time increases. This behavior shows that, $F(t)$ increases and consequently the possibility of Markovianity increases. However, at small values of the coupling, the behavior of $\mathcal{Q}_s$ predicates the non-Markovianity.

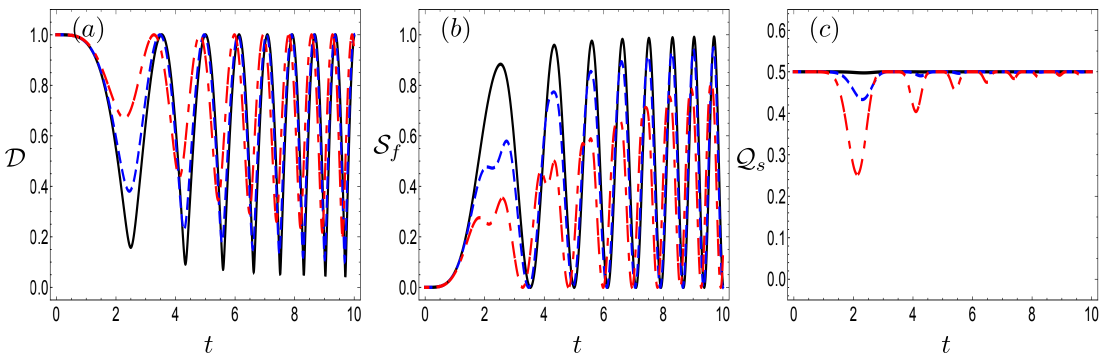

Fig. 2. The same as Fig.(1), but $\lambda(t) = t$.

The effect of time dependent coupling on the three quantifiers is discussed in Fig.(2), where we set $\lambda(t) = t$ and different values of the energy gap parameter ω_0. It is clear that, the oscillatory behavior of Markovianity, skew information are displayed clearly, where their amplitudes increase as the interaction time increases. However, as it is shown from Fig.(2a), at large values of ω_0, the displayed degree of Markovianity, is smaller than that shown at large values of ω_0. Fig.(2b) shows that, the skew information, $\mathcal{S}_f$ increases gradually as t increases, where the upper bounds are depicted at large values of ω_0. This means that, the a mount of the lost information increases when the gap between the excited and ground states is small. The degree of scrambling is displayed in Fig.(2c), where the constant behavior is predicted as the interaction time increases. However, the oscillation behavior is displaced at a small values of the interaction time, only at large values of ω_0.

From Figs.(1) and (2), it us clear that the three quantifiers depend on the interaction's strength, whether it is time dependent/independent. The unstable behavior of the Markovianity is displayed in the presences of time dependent coupling, where the predicted number of the oscillations are larger than those displayed in the presences of time independent coupling.

2. Non-thermal interaction with damping

In Fig.(3), we investigate the behavior the Markovianity of the quantum dot system in the presence of damping. The predicted behavior shows a different features, where the non-Markovianity increases gradually as the time increases. As it is displayed from Fig.(3a), the increasing rate depends on the energy gap between the ground and the exited state of the quantum dot. The oscillation behavior of the Markovianity is displayed at small values of ω_0. However, as the interaction time increases, these oscillations are frozen and the non-Markovian behavior of the quantum dot is displayed, namely the

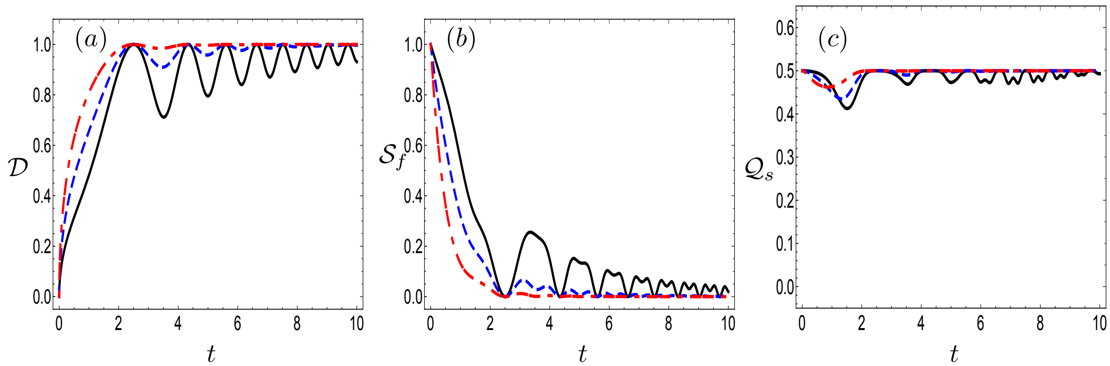

Fig. 3. The same as Fig.(2), but in the presence of damping where we set $\gamma = 0.2$.

forward state of the quantum dot doesn't depend on its back state. Fig.(3b), shows that the skew information decreases suddenly as soon as the interaction is switched on. However, the freezing behavior is predicted as one increases the interaction time or increasing the gap energy ω_0. The degree of scrambling is shown in Fig.(3c), where the amount of scrambling is very small due to the large difference between the temporary state of the QD and its initial state. Moreover, the stable behavior is depicted clearly at large values of the interaction time.

From Figs.(2) and (3), one may conclude that the damping has the ability to change the Markovian dynamics into non- Markovian ones. Accordingly, the memory and memoryless properties of the environment may be exchanged.

In Fig.(4), it is assumed that the coupling between the QD system and the magnetic field is give by $\lambda(t) = e^t$, and different values of the energy gap spacing ω_0 are considered. The general behaviors of the three quantifiers are similar to those are shown in Fig.(3). From Fig.(4), it is clear that the non-Markovianity increases gradually as the interaction time increases. The oscillatory behavior with a large amplitude is displayed at small values of the energy spacing. However, the amplitudes of these oscillations decrease as the interaction time increases. The quantum dot system reaches its non-Markovianity at large values of the energy gap spacing between its ground and excited states. These results are confirmed from the behavior of the skew information that displayed in Fig.(4b), where $\mathcal{S}_f$ decreases as t increases, and the oscillatory behavior is displayed only at small values of ω_0. This means that, the memory environment turns into memoryless one, where the possibility of exchanging the information is very small. The behavior of scrambling information that is displayed in Fig.(4c) is similar to that displayed in Fig.(3c). However, the information scrambling function oscillates faster.

From Figs.(3) and (4), it is clear that the time dependent coupling between the quantum dot system and its environment may be used to change memory environment into memoryless one. In this case, one can decreases the possibility of exchange the information between the QD system and its environment. Moreover, as one increases the interaction the possibility of protecting the local information is improved.

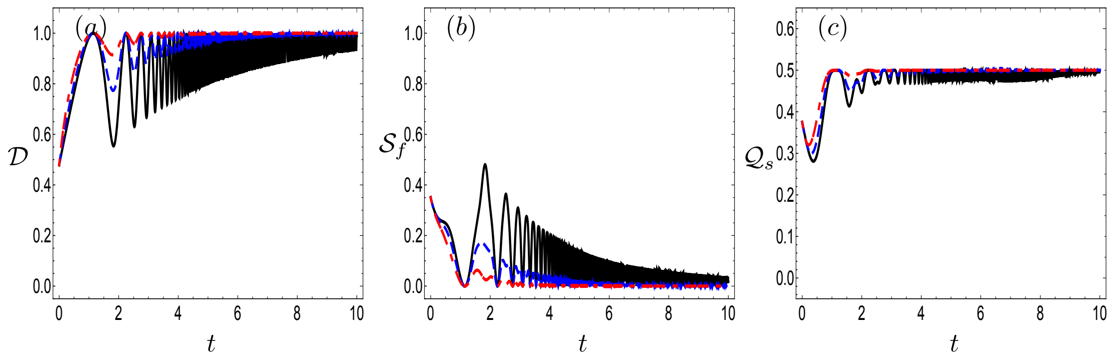

Fig. 4. The same as Fig.(3), but $\lambda(t) = e^t$.

3. Thermal interaction

   Now, it is important to shed the light on the behavior of the three quantifiers on the thermal interaction. The effect of the coupling constant and the energy spacing ω_0 on the Markovianity, skew information, and the quantum scrambling of information as function on the temperature is displayed in Fig.(5). From Fig.(5a), the distance $\mathcal{D}$ decreases as soon as the interaction is switched on, namely the Markovianity increases. It is clear that, as one increases the temperature, the degree of Markovianity is frozen. As it is shown from Fig.(5b), the Skew information exhibits a different behavior, where it decreases in a small interval of temperature $T \in [0,\ 2]$, namely the amount of the lost information is small, and depends on the initial coupling constant. However, as the temperature increases, the exchange rate of information between the QD system and its environment is stable, and consequently, one can say that the environment is partially non-Markovian.

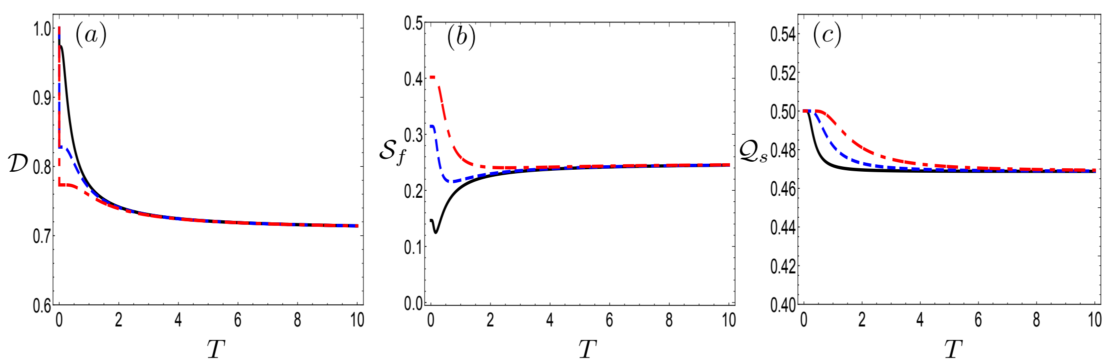

Fig. 5. For the thermal interaction with $\omega_0 = 0.4$ and $\lambda = 0.2, 1, 2$(a) The Markovianity, $\mathcal{D}$, (b) the skew information $\mathcal{S}_f$, and (c) the quantum information scrambling $\mathcal{Q}_s$. The solid, dot and dash-dot curves represent the behavior of the three quantifiers at $\lambda = 0.2, 1$ and 2, respectively.

5 conclusion

In this manuscript, we consider a system consists of two level single quantum dot interacts locally with a magnetic field. It is assumed that, the coupling strength may be time dependent or independent. Different types of interactions are considered; non-thermal/theramal interaction, within and without damping. We discussed three different phenomenon; the Markovianity of the QD system, the skew information, and the information scrambling from/into the quantum dot.

The effect of coupling constant on the three phenomena is investigated, where the degree of the Markovianity increases as the coupling constant increases. The oscillatory behavior and the number of oscillations of the Markovianity of the QD system are displaced at large values of the coupling constant. The sudden changes between Markovianity and non Markovianity are depicted periodically, where the large periodic time is shown at small values of the coupling constant. This means that, the physical property of the environment can be changed suddenly from memory into memoryless and vis versa. However, this property is discussed when we consider that the coupling between the two systems is linearly time dependent. Our results show that, the changing speed from Markovianity into non Markovianity increases, where the number of oscillations increases. However, the amplitudes of these oscillations depend on the energy spacing gap between the ground and excited state of the QD system.

The memory and memoryless phenomena of the environment are discussed in the presences of damping interaction, where we consider either the coupling between the two systems time dependent or independent. It is shown that, the Markovianity increases as the interaction time increase, where the increasing rate depends on the value of the coupling constant. The results reveals a different behavior of the QD Markovianity, when the coupling is time dependent, where the non-Markovianity increases as the interaction time increases. However, the oscillatory behavior depends on the energy spacing gap between the ground and excited states of the QD system, as well as the type of the coupling function, where the faster oscillation is displayed if the coupling constant is given as an exponential function.

The Markovianity of the QD system is discussed when we treat the problem thermally. It is shown that, in a small range of temperature the Markovianity increases and reaches into saturating position, where the environment reduces to be partially non-Markovianity as one increases the temperature.

The rate of exchanging information between the QD system and its environment is quantified by means of the skew information. Our results show that, as the Markovianity increases the amount of the skew information increases, namely, environment is of a memory type. The predicted behavior indicates that, the environment turns from memory into memoryless and vis versa periodically. The coupling between the QD system and its environment plays a similar role to that shown for Markovianity, where the exchanging information rate decreases if we consider the coupling is a time dependent.

Similarly, the phenomena of the information scrambling is investigated for different cases. It is shown that, the energy spacing gap between the ground and excited states and the type of coupling between the quantum dot and its environment may be used as control parameters of increasing/ decreasing the coded information from/into the QD system. It is clear that, the scrambling degree depends on whether the environment is memory or memoryless. However, one can observe that, there is a possibility of freezing/fix the scrambling process. Moreover, in the thermal case, the amount of scrambling information are smaller than those predicted for the non-thermal case. The stability of scrambling is depicted as one increases the interaction temperature.

In conclusion, if one consider that the coupling between the QD system and its environment as time dependent function, one can change the Markovianity property of the environment. This means that, the environment can be switched between memory and memoryless types. The used coupling and the energy spacing gap between the levels of the QD system may be considered as a control parameter to increase/decreases or freezing(fixing) the loses of the amount of information.

Acknowledgements

We would like to thank the refree for his/her report which help us to improve our manuscript.

References

1. Loss, D., & DiVincenzo, D. P. (1998). Quantum computation with quantum dots. Physical Review A, 57(1), 120.
2. Friesen, M., Rugheimer, P., Savage, D. E., Lagally, M. G., van der Weide, D. W., Joynt, R., & Eriksson, M. A. (2003). Practical design and simulation of silicon-based quantum-dot qubits. Physical Review B, 67(12), 121301.
3. J. H. Reina and N. F. Johnson, Phys Rev. A 63 012303 (2001).
4. Barenco, A., Deutsch, D., Ekert, A., & Jozsa, R. (1995). Conditional quantum dynamics and logic gates. Physical Review Letters, 74(20), 4083.-
5. Sanders, G. D., Kim, K. W., & Holton, W. C. (1999). Optically driven quantum-dot quantum computer. Physical Review A, 60(5), 4146.-
6. Sherwin, M. S., Imamoglu, A., & Montroy, T. (1999). Quantum computation with quantum dots and terahertz cavity quantum electrodynamics. Physical Review A, 60(5), 3508.-
7. Li, X. Q., & Yan, Y. (2002). Quantum computation with coupled quantum dots embedded in optical microcavities. Physical Review B, 65(20), 205301.-
8. Brandi, H. S., Latge, A., & Oliveira, L. E. (2001). Light-induced shifts in the electronic and shallow-donor states in GaAs-(Ga, Al) As quantum dots. Physical Review B, 64(23), 233315.-
9. Brandi, H. S., Latge, A., & Oliveira, L. E. (2003). Rabi oscillations, coherent properties, and model qubits in two-level donor systems under terahertz radiation. Physical Review B, 68(23), 233206.-
10. Dicke, R. H. (1954). Coherence in spontaneous radiation processes. Physical review, 93(1), 99.-
11. Wootters, W. K. (1998). Entanglement of formation of an arbitrary state of two qubits. Physical Review Letters, 80(10), 2245.-
12. Lehmberg, R. H. (1970). Radiation from an N-atom system. I. General formalism. Physical Review A, 2(3), 883.-
13. Ficek, Z., Tanaś, R., & Kielich, S. (1987). Quantum beats and superradiant effects in the spontaneous emission from two nonidentical atoms. Physica A: Statistical Mechanics and its Applications, 146(3), 452-482.-
14. Rau, A. R. P., Ali, M., & Alber, G. (2008). Hastening, delaying, or averting sudden death of quantum entanglement. EPL (Europhysics Letters), 82(4), 40002.-
15. Hussain, M. I., & Ikram, M. (2012). Entanglement engineering of a close bipartite atomic system in a dissipative environment. Journal of Physics B: Atomic, Molecular and Optical Physics, 45(11), 115503.-
16. Qian, X. F., Eberly, J. H. (2012). Initial conditions and entanglement sudden death. Physics Letters A, 376(45), 2931-2934.-
17. Qian, X. F. (2014). Pre-management of disentanglement. Journal of Physics A: Mathematical and Theoretical, 47(26), 265304.-
18. Zhi He, J. Zou, Lin Li, and B. Shao, "Effective method of calculating the non-Markovianity N for

single-channel open systems", Phys. Rev. A 83, 012108 (2011).
19. H.-P. Breuer, E.-M Laine, and J. Piilo, "Measure for the Degree of Non-Markovian Behavior of Quantum Processes in Open Systems", Phys. Rev. Lett. 103, 210401 (2009).
20. Roberts, D. A., & Swingle, B. (2016). Lieb-Robinson bound and the butterfly effect in quantum field theories. Physical review letters, 117(9), 091602.-
21. Breuer, H. P., & Vacchini, B. (2008). Quantum semi-Markov processes. Physical review letters, 101(14), 140402.-
22. Breuer, H. P. (2004). Genuine quantum trajectories for non-Markovian processes. Physical Review A, 70(1), 012106.
23. Sachdev, S., & Ye, J. (1993). Gapless spin-fluid ground state in a random quantum Heisenberg magnet. Physical review letters, 70(21), 3339.
24. Shenker, S. H., & Stanford, D. (2014). Black holes and the butterfly effect. Journal of High Energy Physics, 2014(3), 1-25.-
25. Banerjee, S., & Altman, E. (2017). Solvable model for a dynamical quantum phase transition from fast to slow scrambling. Physical Review B, 95(13), 134302.
26. Hayden, P., & Preskill, J. (2007). Black holes as mirrors: quantum information in random subsystems. Journal of high energy physics, 2007(09), 120.-
27. Sekino, Y., & Susskind, L. (2008). Fast scramblers. Journal of High Energy Physics, 2008(10), 065.-
28. Lashkari, N., Stanford, D., Hastings, M., Osborne, T., & Hayden, P. (2013). Towards the fast scrambling conjecture. Journal of High Energy Physics, 2013(4), 1-33.-
29. Landsman, K. A., Figgatt, C., Schuster, T., Linke, N. M., Yoshida, B., Yao, N. Y., & Monroe, C. (2019). Verified quantum information scrambling. Nature, 567(7746), 61-65.-
30. Allen, L., & Eberly, J. H. (1987). Optical resonance and two-level atoms (Vol. 28). Courier Corporation.-
31. Cartar, W. K. (2017). Theory and modelling of light-matter interactions in photonic crystal cavity systems coupled to quantum dot ensembles (Doctoral dissertation, Queen's University (Canada)).-
32. Riedijk, S. (2018). Rabi oscillations in a cavity-quantum dot system.– -
33. Sharma, K. K., and Gerdt, V. P. (2021). Quantum information scrambling and entanglement in bipartite quantum states. Quantum Information Processing, 20(6), 1-16.
34. Pitchford, A. (2019). Control of Many-body Quantum Systems (Doctoral dissertation, Aberystwyth University).
35. Chen, Z. (2005). Wigner-Yanase skew information as tests for quantum entanglement. Physical Review A, 71(5), 052302.
36. Alonso J.R. G, Halpern,N. Y. and Dresse J.(2009). Out-of-Time-Ordered-Correlator Quasiprobabilities Robustly Witness Scrambling. Phys. Rev. Lett. 122, 040404.
37. Khemani, V., Vishwanath, A., & Huse, D. A. (2018). Operator spreading and the emergence of dissipative hydrodynamics under unitary evolution with conservation laws. Physical Review X, 8(3), 031057.
38. W. H. Louisell, "Quantum statistical properties of radiation",by John Wiley& Sons, (1973).
39. M. Y. Abd-Rabbou, Salman Khan & M. Shamirzaie," Quantum fisher information and quantum coherence of an entangled bipartite state interacting with a common classical environment in accelerating frames", Quantum Inf Process 21, 218 (2022).

-

Appendix A: Solving the system of Eqs.(8)

In this appendix we review the details of solving the system of Eqs.7. In this context, one can write the system in a matrix form as [38, 39],

$$\begin{pmatrix} \dot{\mathcal{R}}_{11} \\ \dot{\mathcal{R}}_{12} \\ \dot{\mathcal{R}}_{21} \\ \dot{\mathcal{R}}_{22} \end{pmatrix} = \begin{pmatrix} -\gamma & -\frac{i\lambda(t)}{2} & \frac{i\lambda(t)}{2} & 0 \\ -\frac{i\lambda(t)}{2} & -\frac{\gamma}{2} - i\omega_0 & 0 & \frac{i\lambda(t)}{2} \\ \frac{i\lambda(t)}{2} & 0 & -\frac{\gamma}{2} + i\omega_0 & -\frac{i\lambda(t)}{2} \\ \gamma & \frac{i\lambda(t)}{2} & -\frac{i\lambda(t)}{2} & 0 \end{pmatrix} \begin{pmatrix} \mathcal{R}_{11} \\ \mathcal{R}_{12} \\ \mathcal{R}_{21} \\ \mathcal{R}_{22} \end{pmatrix}. \tag{A.1}$$

The solution of this system takes the form,

$$\mathcal{R}(t) = e^{\int \mathcal{A}(t)dt}\mathcal{R}(0), \tag{A.2}$$

where,

$$\mathcal{A}(t) = \begin{pmatrix} -\gamma & -\frac{i\lambda(t)}{2} & \frac{i\lambda(t)}{2} & 0 \\ -\frac{i\lambda(t)}{2} & -\frac{\gamma}{2} - i\omega_0 & 0 & \frac{i\lambda(t)}{2} \\ \frac{i\lambda(t)}{2} & 0 & -\frac{\gamma}{2} + i\omega_0 & -\frac{i\lambda(t)}{2} \\ \gamma & \frac{i\lambda(t)}{2} & -\frac{i\lambda(t)}{2} & 0 \end{pmatrix}, \text{and} \quad \Re(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{A.3}$$

$$\begin{aligned} \int \mathcal{A}(t)dt &= \begin{pmatrix} -\gamma t & 0 & 0 & 0 \\ 0 & -\frac{\gamma t}{2} - it\omega_0 & 0 & 0 \\ 0 & 0 & -\frac{\gamma t}{2} + it\omega_0 & 0 \\ \gamma t & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\frac{i\phi}{2} & \frac{i\phi}{2} & 0 \\ -\frac{i\phi}{2} & 0 & 0 & \frac{i\phi}{2} \\ \frac{i\phi}{2} & 0 & 0 & -\frac{i\phi}{2} \\ 0 & \frac{i\phi}{2} & -\frac{i\phi}{2} & 0 \end{pmatrix} \\ &= \mathcal{A}_1 + \mathcal{A}_2, \end{aligned} \tag{A.4}$$

where $\phi = \int \lambda(t)dt$. Now, we can write the solution explicitly as,

$$\mathcal{R}(t) = \mathcal{B}_1\mathcal{B}_2\mathcal{R}(0), \ \mathcal{B}_1 = e^{\mathcal{A}_1} = V_1J_1V_1^{-1}, \mathcal{B}_2 = e^{\mathcal{A}_2} = V_2J_2V_2^{-1} \tag{A.5}$$

where V_1 are the eigenvectors of $\mathcal{A}_1$, and the eigenvalues of the matrix $\mathcal{A}_1$ are given by the set $\{0, -\gamma t, \frac{1}{2}t(-\gamma - 4i\omega_0), -\frac{1}{2}t(\gamma - 4i\omega_0)\}$. Similarly the eigenvectors of the matrix $\mathcal{A}_2$ are described by the matrix V_2, while the set of its eigenvalues is $\{-i\phi, i\phi, 0, 0\}$. By using these the eigenvalues of the matrices $\mathcal{A}_1$ and $\mathcal{A}_2$, one can evaluate the matrices J_1 and J_2, where $J_1 = e^{E_1}I$, $J_2 = e^{E_2}I$.

$$\begin{aligned} V_1 &= \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \quad J_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-\gamma t} & 0 & 0 \\ 0 & 0 & e^{\frac{1}{2}t(-\gamma-2i\omega_0)} & 0 \\ 0 & 0 & 0 & e^{\frac{1}{2}t(-\gamma+2i\omega_0)} \end{pmatrix}, \\ V_2 &= \begin{pmatrix} -1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} e^{-i\phi} & 0 & 0 & 0 \\ 0 & e^{i\phi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{A.6}$$

After some straightforward calculations, the explicit forms of $\mathcal{B}_1$ and $\mathcal{B}_2$, are given by,

$$\mathcal{B}_1 = \begin{pmatrix} e^{-\gamma t} & 0 & 0 & 0 \\ 0 & e^{\frac{-t}{2}(\gamma+2i\omega_0)} & 0 & 0 \\ 0 & 0 & e^{\frac{-t}{2}(\gamma-2i\omega_0)} & 0 \\ 1-e^{-\gamma t} & 0 & 0 & 1 \end{pmatrix}, \quad \mathcal{B}_2 = \begin{pmatrix} \cos^2\left(\frac{\phi}{2}\right) & -\frac{i}{2}\sin(\phi) & \frac{i}{2}\sin(\phi) & \sin^2\left(\frac{\phi}{2}\right) \\ \frac{-i}{2}\sin(\phi) & \cos^2\left(\frac{\phi}{2}\right) & \sin^2\left(\frac{\phi}{2}\right) & \frac{i}{2}\sin(\phi) \\ \frac{i}{2}\sin(\phi) & \sin^2\left(\frac{\phi}{2}\right) & \cos^2\left(\frac{\phi}{2}\right) & \frac{-i}{2}\sin(\phi) \\ \sin^2\left(\frac{\phi}{2}\right) & \frac{i}{2}\sin(\phi) & \frac{-i}{2}\sin(\phi) & \cos^2\left(\frac{\phi}{2}\right) \end{pmatrix}, \tag{A.7}$$

where V_1^{-1} and V_2^{-1}, are the inverse matrices of V_1 and V_2, respectively. Now, by equations A.5-A.7, the final solution of the system 7 is given by,

$$\mathcal{R}(t) = \begin{pmatrix} \frac{1}{2}e^{-\gamma t}(\cos(\phi)+1) \\ -\frac{1}{2}i\sin(\phi)e^{-\frac{1}{2}t(\gamma+2i\omega_0)} \\ \frac{1}{2}i\sin(\phi)e^{-\frac{1}{2}t(\gamma-2i\omega_0)} \\ 1-\frac{1}{2}e^{-\gamma t}(\cos(\phi)+1) \end{pmatrix}. \tag{A.8}$$