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DYNAMICS OF ONE TWO-LEVEL-ATOM INTERACTING WITH A MULTIPLE CAVITY MODES

TAOUFIK SAID[*], ABDELHAQ CHOUIKH AND ZOUBIDA SAKHI

Laboratoire de Physique de la Matiere Condensee, Equipe Physique Quantique et Applications,
Faculte des Sciences Ben M'sik, Universite Hassan II,
Casablanca, B.P. 7955, Morroco

MOHAMED BENNAI[†]

Laboratoire de Physique de la Matiere Condensee, Equipe Physique Quantique et Applications,
Faculte des Sciences Ben M'sik, Universite Hassan II,
Casablanca, B.P. 7955, Morroco
and LPHE-Modelisation et Simulation, Faculte des Sciences, Universite Mohamed V,
Rabat, Morroco

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We discuss how to implement quantum logic gates by considering a two-level-atom driven by a strong microwave field and successively interacting with m+1 cavity modes. The scheme is insensitive to the initial state of the atom, and the operation time is independent of the number of cavity modes involved in the system operations. This scheme is used to realize two quantum logic gates (m-target-qubit controlled-global-phase gate and Multi-qubit phase shift gate) in a time much shorter than the photonic lifetime. We also studied the influence of decoherence on the fidelity. In general, our system is reasonably less sensitive to the photonic and atomic decay rates and therefore it can be experimentally realized.

Keywords: Quantum gate, cavity QED, MTCGPh gate, MPS gate.

1 Introduction and motivation

The Cavity quantum electrodynamics (QED) offers a good system to realize quantum information schemes [1]. Recent progress in cavity QED, in which microwave photons play the role of qubits of the one two-level-atom interacting with a multiple cavity modes, makes it stand out among the most promising candidates for implementing quantum information processing [2]. In the context of quantum information processing (QIP), it became feasible to investigate the interaction between cavity field and qubits to implement several quantum logic gates and generate entangled states[3, 4, 5, 6]. However, realistic QIP will most likely need a large number of qubits and placing all of them in a single cavity quickly runs into many fundamental and practical problems such as the increase of cavity decay rate and decrease of qubit-cavity coupling strength [7].

In this work we are interested in the implementation of quantum logic gates, which is a domain where many proposals have been presented using various physical systems [8, 9, 10, 11, 12, 13, 14, 15, 16, 17], because of its important role in quantum computation and quantum information processing.

[*]taoufik.said81@gmail.com, taoufik.said@univh2m.ma
[†]mohamed.bennai@univh2m.ma

At first, we achieve a m-target-qubit controlled-global-phase gate (MTCGPh gate) [18, 19], while in the second part of this study we achieve a Multi-qubit phase shift gate (MPS gate) [18]. The global-phase gate was originally called the phase-shift gate. It can be represented as matrix by

$$Ph(\theta) = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \tag{1}$$

and the controlled-global-phase gate by

$$CGPh = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-i\theta} & 0 \\ 0 & 0 & 0 & e^{-i\theta} \end{pmatrix} \tag{2}$$

likewise the name of phase shift gate arises because this gate shifts the phase of the $|1\rangle$ state relative to the $|0\rangle$ state. It can be represented as matrix by

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \tag{3}$$

We consider a qubit 1 simultaneously controlling m target qubits $(2, 3, ..., m+1)$, by having a sequence of off-resonant interactions with a two-level atom, and that atom will be an ancilla which will not be entangled with the final result of a gate (or step) operation. In order that we do not extract "information" from the system via the ancilla atom it is important for the sequence of atomic transition to start and end on the atomic same state. Then we show how to apply our general proposal to implement the proposed MTCGPh and MPS gates in cavity QED. Next, we study the fidelity, the possible experimental of these implementations and the influence of decoherence on the fidelity, we also calculate the implementation time and discuss the result.

2 Basic theory

Consider a two-level atom with states $|g\rangle$ and $|e\rangle$ and driven by a strong microwave field, successively interacting with $m+1$ cavity modes yielding cavity-enhanced resonances as shown in Figure 1 [20]. The relevant mode frequency of each cavity is coupled to the $|g\rangle \longleftrightarrow |e\rangle$ transition (Figure 1).

Figure 1 shows the diagrammatic sketch of the atom–cavity combined system where a single atom enters a multimode cavity and interacts with the photonic qubits present and the strong microwave field. We recall that the atom acts only as an ancilla to bring about the operation. The Hamiltonian for the whole system is given by (assuming $\hbar = 1$ for simplicity)[21, 22]

$$H = \sum_{j=1}^{m+1} \omega_{c,j} a_j^+ a_j + \frac{\omega_0}{2} S_z + \sum_{j=1}^{m+1} g_j (a_j S^+ + a_j^+ S^-) + \Omega(S^+ e^{-i\omega t} + S^- e^{i\omega t}) \tag{4}$$

where $m+1$ corresponds to the number of cavity modes, $S_z = (|g\rangle\langle g| - |e\rangle\langle e|)$, $S^+ = |e\rangle\langle g|, S^- = |g\rangle\langle e|$ with $|e\rangle(|g\rangle)$ is the excited state (ground state) of the atom; $a_j(a_j^+)$ is the photon annihilation (creation) operator of resonator j with frequency $\omega_{c,j}$, ω_0 is the atomic transition frequency between the two levels $|g\rangle$ and $|e\rangle$ of the atom and ω is the frequency of the classical field, g_j is the coupling constant between the resonator j and the $|g\rangle \longleftrightarrow |e\rangle$

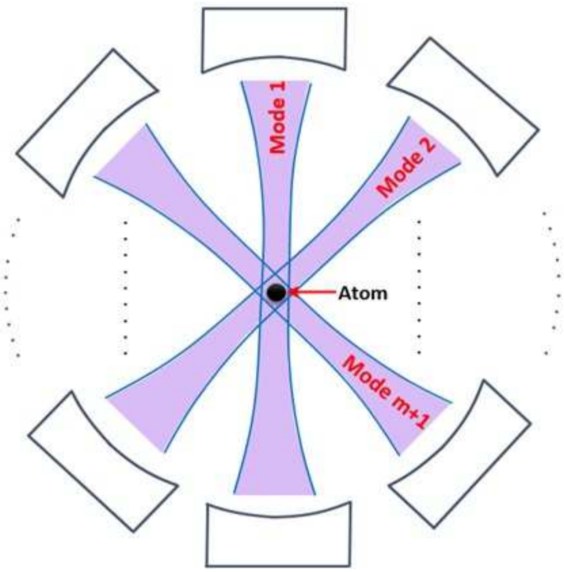

Fig. 1. Layout of the m modes cavity with $\omega_i = \omega$ $(i = 1, m)$. A single two-level atom enters a multi-mode cavity and interacts with the photonic qubits present. The atom acts only as an ancilla.

transition of the qubit and Ω the Rabi frequency of the classical field. In the interaction picture, the Hamiltonian H becomes [21, 22]

$$H_I = \sum_{j=1}^{m+1} g_j(a_j S^+ e^{i\Delta_{c,j}t} + a_j^+ S^- e^{-i\Delta_{c,j}t}) + \Omega(S^+ + S^-) \tag{5}$$

where $\Delta_{c,j} = \omega_0 - \omega_{c,j}$ is the detuning between the $|g\rangle \longleftrightarrow |e\rangle$ transition frequency ω_0 of the atom and the jth resonator frequency $\omega_{c,j}$.

Suppose that (i) $\Omega \gg \Delta_{c,j}$, (ii) the $|g\rangle \longleftrightarrow |e\rangle$ transition of the atom is dispersive interaction with the resonator j (i.e., $\Delta_{c,j} \gg g_j$) (Figure 1), and (iii) $\Delta_{c,j+1} - \Delta_{c,j-1}$ is on the same order of magnitude as the coupling constant g_j, such that the indirect interaction between any two resonator modes induced by the atom is negligible. Under these conditions, the Hamiltonian H_I reduces to [22, 23, 24]

$$H_{eff} = \frac{g^2}{\Delta}(m+1)|e\rangle\langle e| + \frac{g^2}{\Delta}X(|e\rangle\langle e| - |g\rangle\langle g|) + \Omega(|e\rangle\langle g| + |g\rangle\langle e|),(X = \sum_{j=1}^{m+1} a_j^+ a_j) \tag{6}$$

with $g_j = g$ and $\Delta \simeq \Delta_{c,j}$ $(j = 1, ..., m+1$ and $\Delta_{c,j} \gg g_j)$. The evolution operator for the Hamiltonian H_{eff} can be written as [25, 26]

$$U(t) = U_1.U_2.U_3.U_4.U_5 \tag{7}$$

with

$$U_1 \quad = \quad e^{-iA(t)|e\rangle\langle e|}$$

$$
\begin{array}{rcl}
U_2 &=& e^{-iB(t)X(|e\rangle\langle e|-|g\rangle\langle g|)} \\
U_3 &=& e^{-iC(t)\Omega(|e\rangle\langle g|+|g\rangle\langle e|)} \\
U_4 &=& e^{-iD(t)X(|e\rangle\langle g|-|g\rangle\langle e|)} \\
U_5 &=& e^{-iE(t)(|e\rangle\langle g|-|g\rangle\langle e|)}
\end{array} \tag{8}
$$

By solving the Schrodinger equation

$$
i\frac{dU(t)}{dt} = H_{eff}U(t), \tag{9}
$$

we obtain

$$
\begin{array}{rcl}
A'(t) &=& \frac{g^2}{\Delta}(m+1) \Rightarrow \quad A(t) = \frac{g^2}{\Delta}(m+1)t, \\
B'(t) &=& \frac{g^2}{\Delta} \quad \Rightarrow \quad B(t) = \frac{g^2}{\Delta}t, \\
C'(t) &=& 1 \quad \Rightarrow \quad C(t) = t, \\
D'(t)X &=& 0 \quad \Rightarrow \quad D(t) = a, \\
E'(t) &=& 0 \quad \Rightarrow \quad E(t) = b.
\end{array} \tag{10}
$$

The state for the system at time t is

$$
|\psi(t)\rangle = U(t)|\psi(0)\rangle with |\psi(0)\rangle = |n\rangle(\alpha|g\rangle + \beta|e\rangle) = |n_1\rangle|n_2\rangle....|n_{m+1}\rangle(\alpha|g\rangle + \beta|e\rangle) \tag{11}
$$

at time $t = 0$, we have $|\psi(0)\rangle = U(0)|\psi(0)\rangle$, and

$$
\begin{array}{rcl}
U(0) &=& U_4 \cdot U_5 \\
&=& e^{-iaX(|e\rangle\langle g|-|g\rangle\langle e|)} \cdot e^{-ia(|e\rangle\langle g|-|g\rangle\langle e|)} \\
&=& \cos(-i\,(aX+b))\,(|e\rangle\langle e| + |g\rangle\langle g|) + \sin(-i\,(aX+b))\,(|e\rangle\langle g| - |g\rangle\langle e|)
\end{array} \tag{12}
$$

then

$$
\begin{array}{rcl}
U(0)|\psi(0)\rangle &=& U_4.U_5|\psi(0)\rangle \\
&=& [\alpha\cos(-i\,(aX+b)) - \beta + \sin(-i\,(aX+b))]\,|n\rangle|g\rangle + [\beta\cos(-i\,(aX+b)) \\
&& +\alpha\sin(-i\,(aX+b))]\,|n\rangle|e\rangle \\
&=& |n\rangle\,(\alpha|g\rangle + \beta|e\rangle)
\end{array} \tag{13}
$$

wich gives us the system

$$
\left\{
\begin{array}{l}
\cos(-i\,(aM+b)) \cdot \alpha - \sin(-i\,(aM+b)) \cdot \beta = \alpha \\
\sin(-i\,(aM+b)) \cdot \alpha + \cos(-i\,(aM+b)) \cdot \beta = \beta
\end{array}
\right. \tag{14}
$$

with $M = \sum_{j=1}^{m+1} n_j$ and $a_j^+ a_j|n_j\rangle = n_j|n_j\rangle$, by taking $b = 0$ the system gives us

$$
\begin{array}{rcl}
D(t) &=& a = \frac{2ik\pi}{M}, \quad (k \in \mathbb{N}) \\
E(t) &=& b = 0.
\end{array} \tag{15}
$$

The evolution operator of our system can be written then as

$$U(t) = e^{-i\frac{g^2 t}{\Delta}(m+1)|e\rangle\langle e|} e^{-i\frac{g^2 t}{\Delta}X(|e\rangle\langle e|-|g\rangle\langle g|)} e^{-i\Omega t(|e\rangle\langle g|+|g\rangle\langle e|)} e^{-i\frac{2k\pi}{M}X(|e\rangle\langle g|-|g\rangle\langle e|)} \tag{16}$$

so that the state of the atom at the end of a step is the same as at the beginning, we will devellop the two last term in the equation "(16)" (i.e $e^{-i\Omega t(|e\rangle\langle g|+|g\rangle\langle e|)} e^{-i\frac{2k\pi}{M}X(|e\rangle\langle g|-|g\rangle\langle e|)}$) by finding à convenable evolution time τ. Then for $t = \tau = \frac{2k'\pi}{\Omega}$ we have

$$\begin{aligned} e^{-i\Omega\tau(|e\rangle\langle g|+|g\rangle\langle e|)} e^{-i\frac{2k\pi}{M}X|e\rangle\langle g|-|g\rangle\langle e|} &= [\cos(\Omega\tau)(|e\rangle\langle e| + |g\rangle\langle g|) - i\sin(\Omega\tau)(|e\rangle\langle g| + |g\rangle\langle e|)] \\ &\quad \times \left[\cos(\frac{2ik\pi}{M}X)(|e\rangle\langle e| + |g\rangle\langle g|) - \sin(\frac{2ik\pi}{M}X)(|e\rangle\langle g| - |g\rangle\langle e|)\right] \\ &= |e\rangle\langle e| + |g\rangle\langle g| \qquad \left(X|n\rangle = \left(\sum_{j=1}^{m+1} n_j\right)|n\rangle = M|n\rangle\right) \end{aligned} \tag{17}$$

For the following we will take $k' = 1$ ($\tau = \frac{2\pi}{\Omega}$). The evolution operator $U(t)$ becomes

$$U(t) = e^{-i\frac{g^2\tau}{\Delta}(m+1)|e\rangle\langle e|} \cdot e^{-i\frac{g^2\tau}{\Delta}X(|e\rangle\langle e|-|g\rangle\langle g|)} \cdot (|e\rangle\langle e| + |g\rangle\langle g|) \tag{18}$$

By developing the exponentials in the expression of $U(t)$, we finally found

$$U(\tau) = e^{-i\frac{g^2\tau}{\Delta}(X+m+1)}|e\rangle\langle e| + e^{-i\frac{g^2\tau}{\Delta}X}|g\rangle\langle g| \tag{19}$$

According to the form of the evolution operator $U(\tau)$, we have the choice to take either $|g\rangle$ or $|e\rangle$ as the initial state of the atom. We will take state $|e\rangle$ as the initial state of the atom, so we keep the first part of the expression of U

$$\begin{aligned} U(\tau) &= e^{-i\frac{g^2\tau}{\Delta}(X+m+1)}|e\rangle\langle e| \\ &= U_{ph}(\tau)|e\rangle\langle e| \end{aligned} \tag{20}$$

where

$$U_{ph}(\tau) = e^{-i\alpha(m+1)} \prod_{j=1}^{m+1} e^{-i\alpha a_j^+ a_j} with\ \alpha = \frac{g^2\tau}{\Delta} \tag{21}$$

2.1 Implementation of the M-Controlled-Global-Phase gate

As shown in Introduction section, let us consider a two-level atom with states $|g\rangle$ and $|e\rangle$ and driven by a strong microwave field, successively interacting with a high Q cavity containing $(m+1)$ modes. By taking the term δ such that $\delta = \frac{2\pi g^2}{\Omega m\varphi}$ ($\delta > 0$), we consider two special cases: positive detuning $\Delta = \delta$ ($\Delta > 0$) , as well as negative detuning $\Delta = -\delta$ ($\Delta < 0$). φ is a parameter which define the phase of the gate which depend on the choice of δ, Ω and g. The results from the unitary evolution, obtained for these two special cases, will be employed for the controlled-global-phase gate implementation. For that, we will also need to a third step to accomplish. The necessary operations and the unitary evolutions after each step of gate operations, are listed below:

Step(i): Adjust the transition frequency for atom such that the cavity modes $(1, 2, ..., m+1)$ are coupled to atom with a positive detuning $\Delta = \delta$ $(\Delta > 0)$. In this case, the evolution operator for the qubit system for an interaction time $\tau_1 = 2\pi/\Omega$, becomes

$$U_{ph}(\tau_1) = e^{-i\alpha_1(m+1)} \prod_{j=1}^{m+1} e^{-i\alpha_1 a_j^+ a_j} \; with \; \alpha_1 = \frac{g^2\tau_1}{\delta} = m\varphi \tag{22}$$

Step(ii): Adjust the transition frequency for atom such that the cavity modes $(1, 2, ..., m+1)$ are coupled to atom with a negative detuning $\Delta = -\delta$ $(\Delta < 0)$. In this case, the evolution operator for the qubit system for an interaction time $\tau_2 = 2\pi/\Omega$, becomes

$$U_{ph}(\tau_2) = e^{-i\alpha_2(m+1)} \prod_{j=2}^{m+1} e^{-i\alpha_2 a_j^+ a_j} with \; \alpha_2 = -\frac{g^2\tau_2}{\delta} = -m\varphi \tag{23}$$

Step(iii): Adjust the transition frequency for atom, such that the cavity modes $(2, 3, ..., m+1)$ are coupled to atom with a detuning $\Delta' > 0$ with $\Delta' = \frac{g^2}{\Omega}$ and $\tau_3 = \tau_2$ then $\alpha_3 = 2\pi$ and set the parameters of cavity such that atom is decoupled from the cavity mode 1 in third region

$$U_{ph}(\tau_3) = e^{-i\alpha_3(m+1)} \prod_{j=2}^{m+1} e^{-i\alpha_3 a_j^+ a_j} \; with \; \alpha_3 = \frac{g^2\tau_2}{\Delta'} = 2\pi \tag{24}$$

The total evolution operator can be then written as

$$U_{ph}(\tau_1 + \tau_2 + \tau_3) = U_{ph}(\tau_1).U_{ph}(\tau_2).U_{ph}(\tau_3) \tag{25}$$

and since $\alpha_1 = -\alpha_2$, then

$$U_{ph}(\tau_1).U_{ph}(\tau_2) = e^{-i\alpha_1 a_1^+ a_1} \tag{26}$$

We then obtain

$$U_{ph}(\tau_1 + \tau_2 + \tau_3) = e^{-i\alpha_1 a_1^+ a_1}.e^{-i\alpha_3(m+1)} \prod_{j=2}^{m+1} e^{-i\alpha_3 a_j^+ a_j} \tag{27}$$

By using $\alpha_1 = m\varphi$ and $\alpha_3 = 2\pi$ we found

$$U_{ph}(\tau) = \prod_{j=2}^{m+1} U_p(1, j) \tag{28}$$

with

$$U_p(1, j) = e^{-i(\varphi a_1^+ a_1 + 2\pi a_j^+ a_j)} \tag{29}$$

It can be easily shown that for the qubit pair $(1, j)$, we have

$$\begin{aligned} U_P(1,j)|0_1\rangle|0_j\rangle &= |0_1\rangle|0_j\rangle \\ U_P(1,j)|0_1\rangle|1_j\rangle &= |0_1\rangle|1_j\rangle \\ U_P(1,j)|1_1\rangle|0_j\rangle &= e^{-i\varphi}|1_1\rangle|0_j\rangle \\ U_P(1,j)|1_1\rangle|1_j\rangle &= e^{-i\varphi}|1_1\rangle|1_j\rangle \end{aligned} \tag{30}$$

which shows that a Controlled-Global-Phase gate described by $U_p(1,j) = e^{-i(\varphi a_1^+ a_1 + 2\pi a_j^+ a_j)}$ is achieved for the qubit pair $(1, j)$. Equation "(28)" demonstrates that a m-Controlled-Global-Phase gates are simultaneously performed on the qubit pairs $(1, 2)$, $(1, 3)$,..., and $(1, m+1)$, respectively. Note that each qubit pair contains the same control qubit (qubit 1) and a different target qubit (either qubit 2, 3, ..., or $m+1$). Hence, an MCGPh gate with m target qubits (2, 3, ..., $m+1$) and one control qubit (qubit 1) is obtained after the above three-step process.

2.2 Implementation of the Multi-qubit phase shift gate

In this subsection, we present an approach to implement the Multi-qubit phase shift gate (MPS gate). We will show that it can be realized just by one step, which may cause a great reduction of the complexity for some quantum circuits.

Following the method introduced in the beginning of this section, we now discuss how to implement the MPS gate with $(m+1)$ charge qubits (1, 2, ..., $m+1$), coupled to one atom. Then we consider the evolution operator in the equation "(21)", which gives us

$$U_{ph}(\tau) = \prod_{j=1}^{m+1} e^{-i\alpha a_j^+ a_j} with\ \alpha = \frac{g^2\tau}{\Delta} \tag{31}$$

where an overall phase factor $e^{-i\alpha(m+1)}$ is omitted. The total evolution operator can then be written as

$$U_{ph}(\tau) = \prod_{j=1}^{m+1} U_p(j) \tag{32}$$

with

$$U_p(j) = e^{-i\alpha a_j^+ a_j} = e^{i\varphi a_j^+ a_j} \tag{33}$$

where $\varphi = -\alpha = -\frac{g^2\tau}{\Delta} = -\frac{2\pi g^2}{\Delta\Omega}$

It can be easily shown that for the qubit j, we have

$$\begin{aligned} U_P(j)|0_j\rangle &= |0_j\rangle \\ U_P(j)|1_j\rangle &= e^{i\varphi}|1_j\rangle \end{aligned} \tag{34}$$

which shows that a phase shift gate described by $U_p(j) = e^{i\varphi a_j^+ a_j)}$ is achieved for the qubit (j). Equation "(32)" demonstrates that a m-phase shift gates are simultaneously performed on the qubit 1, 2, 3,..., and $(m+1)$, respectively. Hence, an MPS gate with $(m+1)$ qubits (1, 2, ..., $m+1$) is obtained after only one step process.

3 Possible experimental implementation

Now, we briefly discuss experimental feasibility of the current scheme. We consider a microwave cavity-QED experiment in [27, 28, 29], highly excited Rydberg atoms (typically 85Rb) have been used to interact with a superconducting cavity ($Q = 8 \times 10^8$), the coupling constant is $\frac{g}{2\pi} = 50KHz$ [30, 31, 32], the cavity mode frequency is $\frac{\omega_c}{2\pi} = 51,2 \times 10^3 MHz$ [27] and the rabi frequency is $\frac{v_{rabi}}{2\pi} = 440MHz$. The photon lifetime inside the cavity is in order $\tau_{ph} = 130ms$. The lifetime τ for the both MCGPh and MPS gates must be shorter than the photonic lifetime. From the conditions of implementation of these gates, we have $\tau = 2\pi/\Omega$ (for one step). The interaction time becomes: $\tau = 0,36ns$, which is much shorter than τ_{ph}.

In this numerical simulation, we take $\varphi = \pm\pi$. So according to the implementation condition (see equation $\varphi = -\frac{2\pi g^2}{\Delta\Omega}$), we have $\Delta = \frac{2g^2}{\Omega}$. If a variation on the coupling strength g occur in a region of the cavity, we can adjust the value of $3c6$ by varying the value of 2126 to realize the same quantum logic gates.

We plot in Figure 2 the fidelity versus the qubit number to show how our scheme works with the multi-qubit case. We show in Figure 2 that the fidelity is close to the unity for the qubit number ranging from 4 to 6.

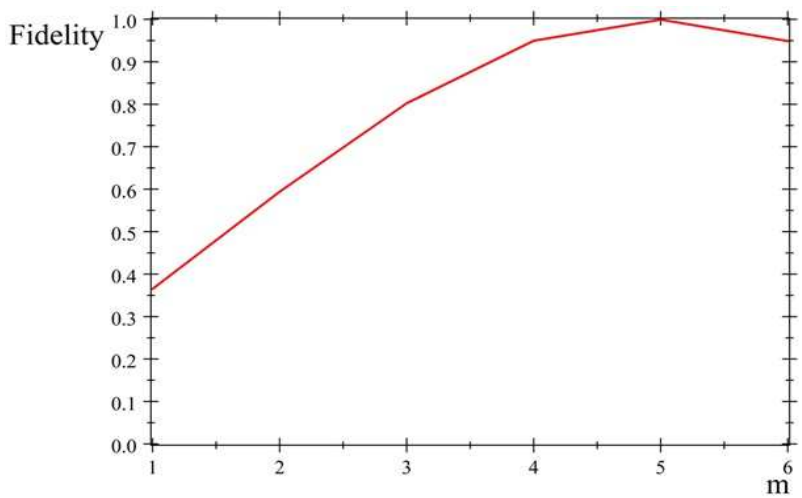

Fig. 2. The fidelity by numerical simulation versus the qubit number m with considering the initial state $|g\rangle|1_1\rangle|1_2\rangle|1_3\rangle|1_4\rangle$. The relevant parameters needed for the simulation can be found in the text.

In Figure 3, we have also plotted the probability of the state $|g\rangle|1_1\rangle|1_2\rangle|1_3\rangle|1_4\rangle$ as a function of gt (for $\varphi = \pi$) and we find that the simulation results are in excellent agreement with the calculated results. If m (qubit number) is even then we have a change of sign otherwise the sign is unchanged (since we have four qubit then a four phase gate with $\varphi = \pi$ that each change sign of state).

Also, the fidelity in this model is the probability of the system to be in the state $|g\rangle|1_1\rangle|1_2\rangle|1_3\rangle|1_4\rangle$, which we take as initial state for example, i.e. [33, 34]

$$F\left(|\psi(t)\rangle\langle\psi(t)|, |g, 1_1, 1_2, 1_3, 1_4\rangle\langle g, 1_1, 1_2, 1_3, 1_4|\right) = |\langle g, 1_1, 1_2, 1_3, 1_4|\psi(t)\rangle|^2 \qquad (35)$$

where, in our case both the wave-function $|\psi(t)\rangle$ describing the system in a time t and the

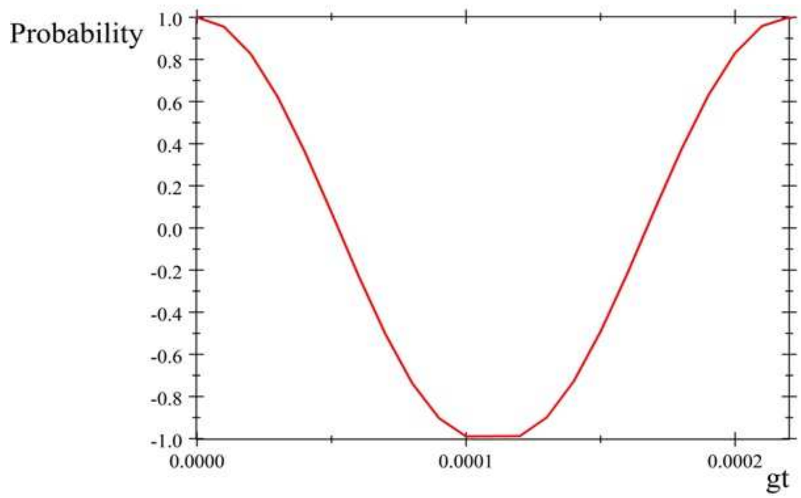

Fig. 3. The plot of probability of the state $|g\rangle|1_1\rangle|1_2\rangle|1_3\rangle|1_4\rangle$ in the case of four qubits. Other relevant parameters needed for the simulation can be found in the text.

target $|g, 1_1, 1_2, 1_3, 1_4\rangle$ is both pure states. We illustrate, then, the plot of fidelity as a function of $\frac{\Delta}{g}$ in Figure 4 (green line). We find that high fidelity is obtained for $\frac{\Delta}{g} \leqslant 100$.

In the following, we perform an analysis the fidelity of the MPS gate by considering influence of photon loss and atomic spontaneous emission, with resort to the conditional Hamiltonian.

We notice that for $\frac{\Delta}{g} \leqslant 120$ we have good fidelity, as well as in this region our model is less sensitive to the photonic and atomic decay rates. While for values of $\frac{\Delta}{g}$ greater than 120 we see a slight decrease in fidelity.

4 Effects of decoherence

To implement any quantum gate operation, it is necessary to measure its robustness taking into account decoherence. In real systems, the atom-field interaction is not completely controlled. Since quantum computers always require an interaction between the quantum operations and the outside world [35, 36], practical quantum gates have suffered from the decoherence process and then leads to loss of the quantum information stored in qubits.

Some improvements recently introduced in modern cavities have reduced the impact of previous dampings.Semiconductor quantum dot experiments in nanocavities, for example, exhibit that (at low temperatures) decoherence processes other than relaxation decay rates can be neglected (e.g. the radiative lifetime $\tau = 2ns$ and other dissipative processes $\sim 30ns$) [37, 38].

Also, it is reported that for the interaction between highly excited Rydberg atoms and high-Q cavity in either microwave or optical regimes [39, 40, 41]. we assume all processes other than cavity dissipation and spontaneous emission are negligible. These experimental schemes and other setups show that the atomic and photonic relaxation can be the only dominant loss mechanism in the present cavity-QED techniques.

In the following, we measure the variation of the population and the fidelity when the dissipative processes, namely via photonic and atomic decays, are to be considered. We only give the detailed analysis of the influence of decoherence processes when the initial state is

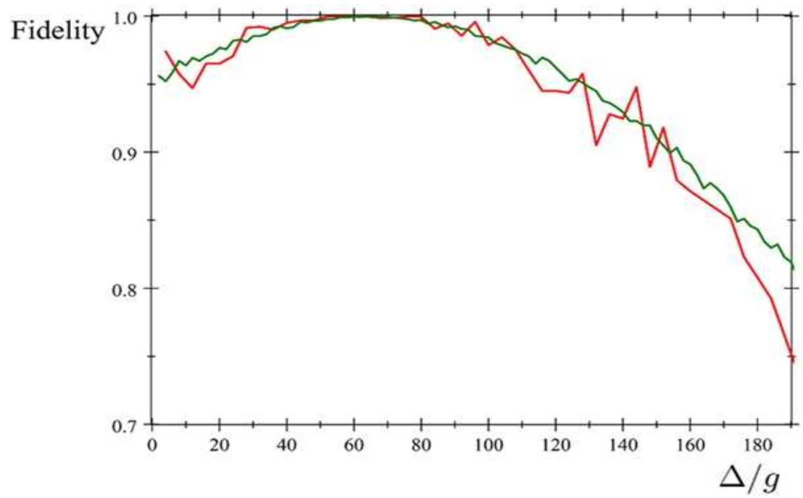

Fig. 4. Fidelity as a function of $\frac{\Delta}{g}$. The parameters are defined in the text. The green and red lines, respectively, represent the fidelity in the absence and in the presence of decay rates.

$|g, 1_1, 1_2, 1_3, 1_4\rangle$. For this purpose, we recall Liouville's equation (or general master equation) that can be written, in the density matrix framework, as follows

$$\frac{\partial}{\partial t}\rho = -i\left[H_I, \rho\right] + L_\rho \tag{36}$$

Where ρ is the density operator of the atom- field system and L_ρ is known as Liouville's operator which describes the dissipative mechanisms in the system. At zero temperature, the Liouvillian L_ρ has the so-called Lindblad form and can be expressed as [35, 42]

$$L_\rho = \sum_i \frac{1}{2}\eta_i\left(2L_i\rho L_i^\dagger - L_i^\dagger L_i\rho - \rho L_i^\dagger L_i\right) \tag{37}$$

Where η_i represents the loss of population. In our case, we consider two dominant channels for decoherence mechanisms, namely the spontaneous emission γ and the cavity field rate κ. The operators L_i and $L_i^\dagger$ are the corresponding system operators. More explicitly, in the presence of the atomic decay L_i and $L_i^\dagger$ can be replaced by the atomic operators σ and σ_+, and in the case of the cavity decay they are represented by the field operators a and $a^\dagger$.

We actually indicate that we can use the wave-function approach [43] instead of density matrix approach in equation equation "(36)" and therefore an analytical solution can be deduced. To this end, we rewrite the previous Liouville's equation as

$$\frac{\partial}{\partial t}\rho = -i\left(H'\rho - \rho H'^\dagger\right) + J_\rho \tag{38}$$

Where $H' = H_I - \frac{i}{2}\sum_i \eta_i L_i^\dagger L_i$ and J_ρ is the quantum-jump superoperator[42] given by $J_\rho = \sum_i \eta_i L_i \rho L_i^\dagger$. H_I is the original Hamiltonian of the system in the absence of any decay.

Since decays in our system result in an irreversible loss of population, we can propagate the wave-function $|\Psi(t)\rangle$ instead of the density matrix ρ, with the Schrödinger equation using the non-Hermitian Hamiltonian H' ($\frac{\partial}{\partial t}|\Psi(t)\rangle = -iH'|\Psi(t)\rangle$). We now use the wave-function treatment to study the influence of the atomic and photonic relaxations over our system described in the previous section.

Given the initial state to be $|g, 1_1, 1_2, 1_3, 1_4\rangle$, then the effectif non-Hermitian Hamiltonian [42, 43] describing the system when decays to be considered can be redefined from equation“(6)” As follows

$$H^{'}_{eff} = H_{eff} - \frac{i\gamma_e}{2}|e\rangle\langle e| - \frac{i\gamma_g}{2}|g\rangle\langle g| - \frac{i}{2}\sum_{j=1}^{m+1}\kappa_j a_j^{+} a_j \tag{39}$$

where γ_e (γ_g) and κ_j denote the atomic spontaneous emission rate and the cavity decay rate, respectively.

After the effective interaction time τ under the non-Hermitian Hamiltonian in $H^{'}_{eff}$, we can plot the gate fidelity by considering influence of photon loss and atomic spontaneous emission in Figure 4 (red line) (on the assumption that $g_i = g$, $\gamma_e = \gamma_g = \gamma$ and $\kappa_j = \kappa$).

5 Conclusion

In Summary, We have presented a method to implement a m-target-qubit controlled-global-phase gate (MTCGPh gate), while in the second part of this study we have achieved a Multi-qubit phase shift gate (MPS gate). We also have discussed the influence of the atomic spontaneous emission and the decay of the cavity modes on the gate fidelity. In general, the system is reasonably less sensitive to the photonic and atomic decay rates and therefore it can be experimentally realized. Else our scheme is quite general, which can be applied to other types of physical qubit systems with two levels, such as quantum dots and NV centers coupled to cavity modes.

We note that by considering the strong coupling, the evolution of the atom and cavity field is fully coherent inside the cavity ($g(t) = 2\Omega_{Rabi}$ for Jaynes Cummings model [44]). Also, to simplify the analysis and to focus on the intrinsic high-performance limitations for our scheme, we assume that all processes other than cavity dissipation and spontaneous emission are negligible.

As shown below, our proposal has the following advantages: (i) The m two-qubit global-phase involved in the MTCGPh gate can be performed simultaneously; (ii) The operation time required for the gate implementation is independent of the number $m+1$ of qubits; (iii) This proposal is insensitive to the initial state of the atom, and thus no preparation for the initial state of the atom is needed; (iv) No measurement on the atom is needed and thus the operation is simplified; and (v) The proposal requires only three steps of operations for the MTCGPh gate and one step for the MPS gate.

In addition the direct implementation of multiqubit controlled-global-phase gate would be more efficient than implementation built from a series of one- and two-qubit rotations, and this efficiency would become even more significant with an increasing number of the qubits. Obviously, a smaller number of gate steps keeps the scheme easier to implement from the experimental point of view.

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