Quantum Information and Computation, Vol. 19, No. 1&2 (2019) 0014–0022

TELEPORTATION VIA THE ENTANGLED DERIVATIVE OF COHERENT STATE

ANAS OTHMAN
Department of Physics, Faculty of Science, Taibah University
P.O. Box 344, Al Madinah Al Munawwarah, Saudi Arabia
aothman@taibahu.edu.sa

Received July 4, 2018
Revised December 25, 2018

Recently, David Yevick and I published an article [Othman, A. & Yevick, D. Int J Theor Phys **57**, 2293 (2018)] about constructing a superposition of two nearly identical coherent states (near coherent state). We showed that this state becomes a superposition of a derivative state and a coherent state. Here, we use the definition of the derivative state to create the entangled derivative of coherent state (EDCS). We show that this state can be used to teleport qubits encoded in the near coherent states. The decoherence of EDCS is also studied. In addition, we propose an experimental scheme to produce the EDCS and to perform teleportation.

Keywords:

Communicated by: S Braunstein & R Laflamme

1 Introduction

Entanglement is one of the most essential resources in quantum information. Its main idea is that the different parts of a system, which could be particles, share an inseparable quantum state. Entanglement has been experimentally observed in different physical implementations such as optical [1] and ionic [2] systems. In addition, it is one of the main components of many applications such as dense coding [3], device-independent quantum cryptography [4], and teleportation [5].

Quantum teleportation was first proposed in 1993 [5] and has been applied in quantum cryptography and quantum communication. Many physical realizations using photons, atoms, electrons, and superconducting circuits have verified its concept [6, 7, 8, 9]. The main idea of quantum teleportation is to utilize the shared entanglement state and the classical channel between the sender (Bob) and the receiver (Alice) to send some quantum states.

Different shared entangled wavefunctions or EPR states have been used to teleport a variety of light states. For example, the two-mode squeezed state, which can have an arbitrary amount of ebits (entanglement measurement), can be used to teleport an arbitrary single light mode [10, 11]. In addition, the polarization-entangled state is employed to teleport polarized qubits [12, 13]. The conventional Bell states are also utilized to teleport qubits based on many physical implementations.

Another known optical class of EPR states, based on coherent states, is the entangled coherent state (ECS). It was proposed in 1992 [14], and was considered to teleport the Schrödinger cat state (SCS) [15]. The ECS has the following form:

$$|\alpha;\alpha\rangle^{\pm} = N_{\pm}(|\alpha\rangle_1|\alpha\rangle_2 \pm |-\alpha\rangle_1|-\alpha\rangle_2), \tag{1}$$

where $|\alpha\rangle_i$ is the coherent state of mode i, α is the coherent parameter, and $N_{\pm}$ is $1/\sqrt{2(1 \pm e^{-4|\alpha|^2})}$. Although this state depends on the parameter α, it has exactly one ebit for all values of α. This feature and other similar features of the ECS makes it one of the best options to teleport qubits. Many entanglement and teleportation properties of ECS are discussed in detail in [15, 16, 17, 18]. Moreover, it has been extended and designated to many practical applications (for an excellent review, see [19]).

Recently, David Yevick and I studied the properties of the near coherent state [20]. It is a superposition of two nearly identical coherent states. A special case of this superposition yields the derivative of a coherent state with respect to its absolute parameter, in short, a derivative state, which is denoted as $|d(\alpha)\rangle = d|\alpha\rangle/d|\alpha|$. We found that, in general, the near coherent state and the derivative state share many similarities between the coherent states and the SCS. Namely, they have statistical properties that are similar to coherent states, and they are mostly nonclassical, similar to SCS. In the same paper, an effective experimental method was also suggested to prepare these states.

Based on the importance of ECS and on the similarities between the derivative state and the coherent state, we are motivated to propose the entangled derivative of coherent state (EDCS), which has the following form:

$$|\Psi_{\alpha}^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[|d(\alpha)\rangle_1|\alpha\rangle_2 \pm i|\alpha\rangle_1|d(\alpha)\rangle_2\right]. \tag{2}$$

This state is identical to some types of ECS. For example, the ECS can be covered if the replacement $|d(\alpha)\rangle \rightarrow |-\alpha\rangle$ is made. Moreover, this state has a continuous parameter, α, similar to ECSs. In contrast, it contains a nonclassical component, which is the derivative state. On the other hand, an arbitrary near coherent state can be easily teleported using this EPR state, as we will see. Accordingly, this paper discusses the EDCS, its different properties, and its application to the teleportation of a near coherent state.

This paper is organized as follows. In the next section, a brief review of some key results of the near coherent states are provided. In Sec.(III), we will study the EDCS, its preparation, and its decoherence. In Sec. (IV), the teleportation scheme based on EDCS is presented.

2 Near coherent states

The near coherent state is a superposition of two almost identical coherent states [20]

$$|\alpha,\Delta\theta\rangle = \lim_{|\Delta\alpha|\rightarrow 0} C_{\alpha}\left(|\alpha + |\Delta\alpha|e^{i\Delta\theta}\rangle - |\alpha\rangle\right), \tag{3}$$

where $|\Delta\alpha|$ is the source of the near coherent state, $\Delta\theta$ is its phase, and C_{α} is the normalization factor. Note that this definition is unique, which means that if the middle phase between the two superposed coherent states is different than $e^{i\pi} = -1$, the resultant state will be a single coherent state. We have shown that the near coherent state yields

$$|\alpha, \Delta\theta\rangle = \frac{e^{i\delta\theta}\frac{\partial|\alpha\rangle}{\partial|\alpha|} + i|\alpha|\sin(\delta\theta)|\alpha\rangle}{\sqrt{1+|\alpha|^2\sin(\delta\theta)^2}}, \tag{4}$$

where $\delta\theta = \Delta\theta - \theta$, and θ is the phase of the coherent parameter $\alpha = |\alpha|e^{i\theta}$. In this formula, the superposition of the two coherent states (the near coherent state) in eq.(3) becomes a superposition of a derivative state $\partial|\alpha\rangle/\partial|\alpha|$ and a coherent state. The derivative state can be expressed as

$$|d(\alpha)\rangle = \frac{\partial|\alpha\rangle}{\partial|\alpha|} = \frac{e^{-\frac{1}{2}|\alpha|^2}}{|\alpha|}\sum_{n=0}^{\infty}\frac{n\alpha^n}{\sqrt{n!}}|n\rangle - |\alpha||\alpha\rangle. \tag{5}$$

Note that when $\Delta\theta = \theta, \delta\theta = 0$, the near coherent state becomes solely $|d(\alpha)\rangle$. The derivative state is normalized and orthogonal to the coherent state, $\langle\alpha|d(\alpha)\rangle = 0$. Another important feature of the derivative state is that when $\alpha \to 0$, it becomes $|d(0)\rangle = |1\rangle$, which means that it always has at least one photon. This feature can be understood by noting that it can be rewritten as

$$|d(\alpha)\rangle = e^{i\theta}D(\alpha)|1\rangle, \tag{6}$$

where $D(\alpha)$ is the displacement operator. Therefore, the derivative state is the displaced one-photon state with an overall irrelevant phase. The displaced Fock states are studied under different contexts [21, 22, 23], but not in the context of derivative states.

An experimental scheme to generate the near coherent state is described in [20]. In that procedure, two coherent states $|\alpha\rangle$ and $|\beta\rangle$, a cross Kerr medium, beamsplitters, and detectors were used. The resultant state after the measurement of a certain detector is the near coherent state $|\gamma, \Delta\theta_{eff}\rangle$, where for arbitrary values of α and β, γ becomes

$$|\gamma| = \frac{1}{\sqrt{2}}\sqrt{|\alpha|^2 + |\beta|^2 + 2|\alpha||\beta|\cos(\theta - s)}, \tag{7}$$

and

$$\delta\theta_{eff} = \theta + \frac{3\pi}{2} - \tan^{-1}\left(\frac{|\alpha|\sin(\theta) + |\beta|\sin(s)}{|\alpha|\cos(\theta) + |\beta|\cos(s)}\right), \tag{8}$$

where s is the phase of β, $\gamma = \frac{1}{\sqrt{2}}(\beta + \alpha)$, and $\Delta\theta_{eff} = \theta + 3\pi/2$. An arbitrary near coherent state can be generated in this way. For example, to produce derivative states, we let $\theta = -3\pi/2$; then, s has to be equal to $\tan^{-1}\left(\frac{|\alpha|/|\beta|}{\sqrt{1-|\alpha|^2/|\beta|^2}}\right)$. Consequently, the amplitude of the resultant derivative state is $|\gamma| = \sqrt{\frac{3|\alpha|^2+|\beta|^2}{2}}$, and its phase is $\arg(\gamma) = \tan^{-1}\left(\frac{2|\alpha|}{|\beta|\sqrt{1-|\alpha|^2/|\beta|^2}}\right)$. Note that in this particular selection of parameters, the derivative state will be produced for all $|\alpha|/|\beta| \leq 1$.

By using this procedure, we have shown that the required interaction time between the coherent states of the experiment and the Kerr medium could be very brief $\chi t \to 0$. And it is known that for certain interferometry experiments, similar to ours, which involve the Kerr medium, a brief interaction time is required [24, 25]. This suggests that the production of near coherent states can be effectively demonstrated with high fidelity, which makes them practical to apply in different areas.

The average number of photons in the near coherent state is

$$\langle n\rangle = |\alpha|^2 + M, \quad M = \frac{1+2|\alpha|^2\sin(\delta\theta)^2}{1+|\alpha|^2\sin(\delta\theta)^2}. \tag{9}$$

Note that M is bounded $1 \leq M \leq 2$ for all values of α, $\delta\theta$. The near coherent state has some nonclassical properties such as squeezing, anti-bunching, and negative Wigner function. All these features are discussed in the same article. The last feature that needs to be mentioned here is the inner product between the coherent states and the derivative states, which is given as

$$\langle\alpha|d(\beta)\rangle = e^{-\frac{1}{2}|\alpha|^2-\frac{1}{2}|\beta|^2+\beta\alpha^*}\left[\frac{\beta\alpha*}{|\beta|} - |\beta|\right]. \tag{10}$$

This relation confirms that if $\alpha = \beta$, then $\langle\alpha|d(\alpha)\rangle = 0$.

3 Entangled Derivative of Coherent State (EDCS)

We define the entangled derivative of coherent state (EDCS) as follows:

$$|\Psi_\alpha^\pm\rangle = \frac{1}{\sqrt{2}}\left[|d(\alpha)\rangle_1|\alpha\rangle_2 \pm i|\alpha\rangle_1|d(\alpha)\rangle_2\right]. \tag{11}$$

The above states are normalized and orthogonal to each other. They are orthogonal to each other because the derivative state and the coherent state are orthogonal to each other $\langle d(\alpha)|\alpha\rangle = 0$. This feature makes the EDCS a maximally entangled state. The alternative form of the EDCS can be written as

$$|\psi_\alpha^\pm\rangle = \frac{1}{\sqrt{2}}\left[|d(\alpha)\rangle_1|d(\alpha)\rangle_2 \pm |\alpha\rangle_1|\alpha\rangle_2\right]. \tag{12}$$

In general, this form and other similar forms of EDCSs can all be used for teleportation. Here, we focus on the formula of eq.(11) because it is easy to generate experimentally as we will see.

As we can write the coherent state as $|\alpha\rangle = D(\alpha)|0\rangle$ and the derivative state as in eq.(6), we can write

$$|\Psi_\alpha^\pm\rangle = \frac{D(\alpha)_1 D(\alpha)_2}{\sqrt{2}}\left[|1\rangle_1|0\rangle_2 + i|0\rangle_1|1\rangle_2\right], \tag{13}$$

where we have removed the irrelevant overall phase $e^{i\theta}$. In this formula, the EDCSs is almost identical to the Bell state, if the displacement operators are removed. Therefore, if we apply the inverse displacement operator, we will obtain the known Bell state

$$D(\alpha)_1^{-1}D(\alpha)_2^{-1}|\Psi_\alpha^\pm\rangle = \frac{1}{\sqrt{2}}\left[|1\rangle_1|0\rangle_2 \pm i|0\rangle_1|1\rangle_2\right]. \tag{14}$$

This mathematical similarity suggests that the EDCS features are between the ECS and the Bell states. This is the case at least when detecting the number of photons in the teleportation scheme, as we will see. However, it is worth stressing that although the EDCS from this formula appears as simple as a displaced Bell state, we should not forget that the EDCS originally comes from the near coherent state and yields to displace the Bell state and not the opposite. In addition, it appears that the contribution of the displacement operator is not trivial.

From eq.(14), the discrimination of the two modes $(1, 2)$ can be performed by first applying the inverse of the displacement operator and then detecting where the photon is. If the photon appears in mode (1), we can be certain that the other mode is a vacuum, and vice versa.

The average number of photons in the EDCS is

$$\langle n \rangle = 2|\alpha|^2 + 1. \tag{15}$$

It becomes 1 when $\alpha = 0$, and therefore it has, as the derivative state, at least one photon. The average number of photons can grow unlimitedly as the ECS. The amount of entanglement in the EDCS, as the ECS, is exactly 1 ebit. We can see this by noting that the trace of the reduced density matrix equals 1.

Next, in order to prepare the EDCS, we assume that there is a source of derivative coherent states whose values are $d(|\sqrt{2}\alpha\rangle)$. Such a source can be prepared using our method, which was briefly described earlier. Then, we assume that this state is incident to a beam splitter, so

$$|d(\sqrt{2}\alpha)\rangle_1|0\rangle_2 \rightarrow \frac{1}{\sqrt{2}}\left[|i\alpha\rangle_1|d(\alpha)\rangle_2 + i|d(i\alpha)\rangle_1|\alpha\rangle_2\right]. \tag{16}$$

After letting mode (1) pass through a phase shifter of phase equal to $\Theta = 3\pi/2$, the state becomes

$$\rightarrow \frac{1}{\sqrt{2}}\left[|\alpha\rangle_1|d(\alpha)\rangle_2 + i|d(\alpha)\rangle_1|\alpha\rangle_2\right]. \tag{17}$$

This state is the EDCS of eq.(11). It can be seen that this method is accessible since it requires only the essential optical tools.

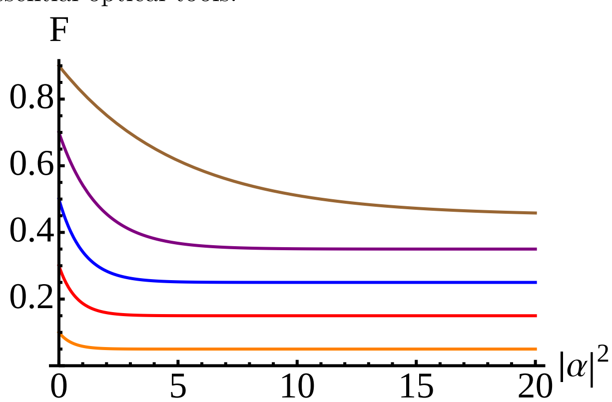

Fig. 1. Fidelity of eq.(23) against the coherent parameter $|\alpha|$ for different values of η. The values are $\eta = 0.9, 0.7, 0.5, 0.3, 0.1$, from the top to bottom, where the uppermost curve is for $\eta = 0.9$.

Let us now study the decoherence of the EDCS. Here, we use a simple but effective model to introduce the decoherence. We assume that the decoherence is modelled by a beamsplitter whose coefficient is η. The coherent state will decohere as

$$|\alpha\rangle_i|0\rangle_E \rightarrow |S_1\rangle_i|S_2\rangle_E, \tag{18}$$

where $S_1 = \sqrt{\eta}\alpha$, $S_2 = \sqrt{1-\eta}\alpha$, E refers to the environment causing the decoherence, and η is the decoherence parameter. The derivative state will be decohered as

$$|d(\alpha)\rangle_i|0\rangle_E \rightarrow \sqrt{\eta}|d(S_1)\rangle_i|iS_2\rangle_E + i\sqrt{1-\eta}|S_1\rangle_i|d(iS_2)\rangle_E. \tag{19}$$

After tracing over the environment modes, the density matrix of $|\Psi^{\pm}_{\alpha}\rangle$ becomes

$$\begin{array}{rcl}\rho & = & \frac{\eta}{2}|A\rangle\langle A| + (1-\eta)|B\rangle\langle B| + \frac{\eta}{2}|C\rangle\langle C| \\ & \pm & \frac{1}{2}\sqrt{\eta(1-\eta)}(k|A\rangle\langle B| + k^{*}|B\rangle\langle A|) \\ & \pm & \frac{i\eta l}{2}(|A\rangle\langle C| - |C\rangle\langle A|) \mp \frac{1}{2}\sqrt{\eta(1-\eta)}\times \\ & & (k^{*}|B\rangle\langle C| + k|C\rangle\langle B|),\end{array} \tag{20}$$

where $|A\rangle = |S_1\rangle_1|d(S_1)\rangle_2$, $|B\rangle = |S_1\rangle_1|S_1\rangle_2$, $|C\rangle = |d(S_1)\rangle_1|S_1\rangle_2$, $k = t_1 t_2^{*}$, and $l = |t_1|^2$. The coefficients t_1 and t_2 are given by

$$t_1 = \langle S_2|iS_2\rangle = e^{-|\alpha|^2(1-\eta)(1-i)}, \tag{21}$$

$$t_2 = \langle S_2|d(iS_2)\rangle = -|\alpha|\sqrt{1-\eta}(1-i)e^{-|\alpha|^2(1-\eta)(1-i)}, \tag{22}$$

where the inner multiplication of the derivative state with the coherent state for arbitrary amplitudes is taken from eq.(10). In addition, the states $|A\rangle$, $|B\rangle$, and $|C\rangle$ are all orthogonal to each other. The maximum EDCS of a given value of η can be written as $|\Psi^{\pm}_{S_1}\rangle = \frac{1}{\sqrt{2}}(|C\rangle \pm i|A\rangle)$. The projection of this state with the decoherence density matrix of eq.(20) is given by

$$\langle\Psi^{\pm}_{S_1}|\rho|\Psi^{\pm}_{S_1}\rangle = \frac{\eta}{2}(1+l) = \frac{\eta}{2}\left(1 + e^{-2|\alpha|^2(1-\eta)}\right). \tag{23}$$

This defines the fidelity between the dissipated state and the EDCS. In fig.(1), we have plotted eq.(23) for different values of η.

Equation (23) indicates that the fidelity will eventually reach $\eta/2$ when $|\alpha| \to \infty$, as shown in fig.(1). The width of F when it equals $3\eta/4$ (the width of the half-height in this case) is $\approx \frac{0.589}{\sqrt{1-\eta}}$. In general, the half-height of the decoherence fidelity of the ECS [15] is similar to our range, although ours is larger when η is close to unity. Therefore, the EDCS is less decohered than the ECS when the environment is close to ideal ($\eta \to 1$). Another difference between our fidelity and ECS fidelity is that the ECS fidelity drops to $1/2$ at $|\alpha| \to \infty$, while ours always tries to reach $\eta/2$. In the next section, we will discuss how to use the EDCS for teleportation.

4 Teleportation scheme

Here, we propose a teleportation scheme using EDCS. This scheme may not be the best, but it is adequate to illustrate the concept. Here, we wish to teleport a qubit encoded in a near coherent state. Accordingly, suppose that Alice wants to teleport to Bob, the following state:

$$|f_1, f_2\rangle = f_1|d(\alpha)\rangle + f_2|\alpha\rangle, \quad |f_1|^2 + |f_2|^2 = 1, \tag{24}$$

where f_1 and f_2 are the amplitudes of the derivative and coherent states, respectively. This kind of qubits can be prepared using the definition of the near coherent states in eq.(4). The EDCS state is prepared as explained in the previous section, that is, by splitting the derivative state and then applying the phase shifter as shown in fig. (2). The state in Alice's possession is

$$\begin{array}{rcl}|in\rangle & = & \frac{|f_1,f_2\rangle_c}{\sqrt{2}}D(\alpha)_a D(\alpha)_b[|0\rangle_a|1\rangle_b + i|0\rangle_b|1\rangle_a], \\ & = & \frac{|f_1,f_2\rangle_c}{\sqrt{2}}D(\alpha)_a D(\alpha)_b[a_b^{\dagger} + i a_a^{\dagger}]|0\rangle_a|0\rangle_b, \\ & = & \frac{D(\alpha)_a D(\alpha)_b D(\alpha)_c}{\sqrt{2}}[a_b^{\dagger} + i a_a^{\dagger}][f_1 a_c^{\dagger} + f_2]|0\rangle_a|0\rangle_b|0\rangle_c\end{array} \tag{25}$$

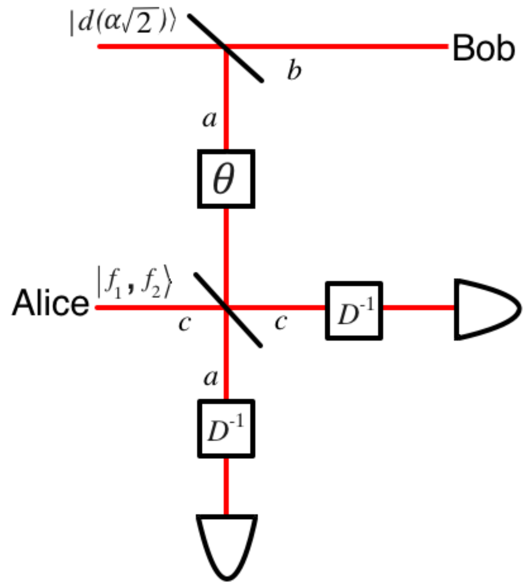

Fig. 2. Teleportation scheme using EDCS.

Then, Alice mixes her EDCS with the near coherent qubit using the beam splitter; therefore, the outcome state is

$$\begin{aligned}|in\rangle &\rightarrow \frac{D(\alpha)_b D(\alpha e^{i\pi/4})_a D(\alpha e^{i\pi/4})_c}{\sqrt{2}}[a_b^\dagger + \frac{i}{\sqrt{2}}(a_a^\dagger + ia_c^\dagger)]\times \\ &= [\frac{f_1}{\sqrt{2}}(a_c^\dagger + ia_a^\dagger) + f_2]|0\rangle_a|0\rangle_b|0\rangle_c\end{aligned} \tag{26}$$

Then, after performing the inverse of the displacement operator $D^{-1}(\alpha e^{i\pi/4})$ on mode a and c, we obtain

$$\begin{aligned}\rightarrow\ & \frac{f_2|d(\alpha)\rangle_b}{\sqrt{2}}|00\rangle + \frac{f_1|d(\alpha)\rangle_b}{2}(|01\rangle + i|10\rangle)\ + \\ & i\frac{f_2|\alpha\rangle_b}{2}(|10\rangle + i|01\rangle) - \frac{f_1|\alpha\rangle_b}{2}(|20\rangle + |02\rangle),\end{aligned} \tag{27}$$

where we abbreviate $|x\rangle_a|y\rangle_c \rightarrow |xy\rangle$. After rearranging the terms, we get

$$\begin{aligned}\rightarrow\ & \frac{f_2|d(\alpha)\rangle_b}{\sqrt{2}}|00\rangle + \frac{1}{2}(f_1|d(\alpha)\rangle_b - f_2|\alpha\rangle_b)|01\rangle\ + \\ & +\frac{i}{2}(f_1|d(\alpha)\rangle_b + f_2|\alpha\rangle_b)|10\rangle - \frac{f_1|\alpha\rangle_b}{2}(|20\rangle + |02\rangle).\end{aligned} \tag{28}$$

Now, if the detectors measure the state $|10\rangle$, the teleported state to Bob is $|f_1, f_2\rangle$, whereas if the detectors measure $|01\rangle$, the teleported state is $|f_1, -f_2\rangle$. However, if the detectors register $|00\rangle$ or $|20\rangle, |20\rangle$, the teleported state is the coherent state $|\alpha\rangle$ or the derivative state $|d(\alpha)\rangle$, respectively. Therefore, as in most teleportation methods, a classical channel is required between Alice and Bob. Alice should send Bob the detectors measurements, so that Bob knows what to perform in his state.

The success measurements in our teleportation protocol, as the ECS and Bell states, are an odd number of photons. This makes our state a special case of what is proven to be true in many teleportation protocols [26]. Although from this side, our state seems to be similar to ECS, there is a big difference between them. Namely, in our state, the required odd number for all α is a photon, while that in ECS is unlimited. This fact gives a potential advantage to our state, in which the required measurements as the Bell states is a photon. At the same time, the average number of photons of the carrier state EDCS grows unlimitedly as the ECS.

The probability of the success measurements that leads to the teleportation of the qubit is 1/2, which is the probability of the detectors to measure the states $|01\rangle$ or $|10\rangle$. There are many ways to increase this probability, but they will not be studied in this article.

5 Conclusion

In conclusion, we proposed an EPR state based on the derivative state, which is EDCS. We showed that this state carries exactly one ebit of entanglement and provided a scheme to prepare it experimentally. Based on this state, we suggested a teleportation configuration to teleport the near coherent state.

This work provides a new method to apply the teleportation. There are many similarities between our state and the ECS. This suggests that the near coherent state teleportation might be as efficient as ECS, which is used to teleport Schrödinger cat states and other states. More theoretical and experimental studies are needed to validate this work.

Acknowledgments

We acknowledge the financial support from Taibah University.

References

1. A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. **47**, 460 (1981).
2. C. A.Sackett, D. Kielpinski, B. E. King, C. Langer, V. Meyer, C. J. Myatt, M. Rowe, Q. A. Turchette, W. M. Itano, D. J. Wineland, C. Monroe (2000), Nature **404**, 256 (2000).
3. C. H. Bennett, S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
4. A. Acn, N. Brunner, N. Gisin, S. Massar, S. Pironio, V. Scarani, Phys. Rev. Lett. **98**, 230501 (2007).
5. C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
6. S. Stenholm and P. J. Bardroff, Phys. Rev. A **58**, 4373 (1998).
7. D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature 390, 575 (1997); D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
8. A. Furusawa, J. L. Srensen, S. L. Braunstein, C. A. Fuchs, H. J. Kim- ble, and E. S. Polzik, Science **282**, 706 (1998).
9. L. Vaidman, Phys. Rev. A. **49**, 1473 (1994).
10. D. F. Walls and G. J. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994).
11. A. Furasawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble and E. S. Polzik, Science, **282**, 706, (1998).
12. J. Fulconis, O. Alibart, J. L. O?Brien, W. J. Wadsworth, and J. G. Rarity, Phys. Rev. Lett. **99**, 120501, (2007).
13. M. Fiorentino and R. G. Beausoleil, Opt. Express **16**, 20149 (2008).
14. B. C. Sanders, Phys. Rev. A **45**, 6811 (1992).
15. S. J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001).
16. O. Hirota, S. J. Enk, K. Nakamura, M. Sohma and K. Kato, "Entangled Nonorthogonal States and Their Decoherence Properties", arXiv:quant-ph/0101096v1 (2001).
17. H. Jeong, M. S. Kim, and Jinhyoung Lee, Phys. Rev. A **64**, 052308 (2001).
18. H. Jeong and M. S. Kim, Quantum Information and Computation **2**, 208 (2002).

19. B. C. Sanders, J. Phys. A: Math. and Theor. **45**, 244002 (2012).
20. A. Othman, D. Yevick, Int. J. Theor. Phys. **57**, 2293 (2018).
21. P. Král, J. Mod. Opt. **37**, 889 (1990).
22. F. A. M. de Oliveira, M. S. Kim, P. L. Knight, and V. Buek, Phys. Rev. A **41**, 2645 (1990).
23. M. M. Nietoab, Phys. Lett. A **229**, 135 (1997).
24. H. Jeong, M. S. Kim, T. C. Ralph, and B. S. Ham. Phys. Rev. A, **70**, 061801 (2004).
25. S. Glancy and H. M. de Vasconcelos, J. Opt. Soc. Amer. B, **25**, 712 (2008).
26. S. J. van Enk, Phys. Rev. A **67**, 022318 (2003).