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NON-MARKOVIAN QUANTUM INTERFERENCE IN MULTILEVEL QUANTUM SYSTEMS: EXACT MASTER EQUATION APPROACH

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We study the non-Markovian dynamics of multilevel quantum systems coupled with a bosonic dissipative environment. Based on the known exact quantum-state diffusion (QSD) equations, we propose a systematic approach to derive exact time-convolutionless master equations for multilevel quantum systems. Through a combination of analytical and numerical approaches, we extract the non-Markovian dynamics of quantum interference in different time scales. Also, we demonstrate the evolution of quantum interference in a four-level system controlled by an external electromagnetic field. Our findings are extended to few-body quantum networks, with a universal formalism established.

Keywords: Multilevel System, Quantum Interference, Non-Markovian Dynamics, Master Equation.
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1 Introduction

Any real physics system has to be investigated in the framework of an open quantum system, due to interplays with its environment. Open quantum system approaches have been extensively studied in quantum decoherence, atom emission and absorption, quantum dissipation, quantum transport, quantum interference, and quantum control [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 18]. However, most of the phenomena are studied in the Markov approximation [4], which assumes that the system of interest is weakly coupled to its environment and the influence of the environment on the system is a perturbation. The Markov approximation is

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no longer valid when the coupling is in the strong or deep strong regime. Therefore, a non-Markovian open quantum system theory which can exactly describe the temporal evolution of quantum features of a multilevel quantum system in both the short-time and long-time regime is crucial. However, analytical studies on the dynamics of multilevel quantum systems are technically challenging because of the memory effect in the non-Markovian evolution, which means all possible evolutionary trajectories of the system can affect the state of the system at later time instant [19, 20, 21, 22, 23, 24, 25, 26]. Additionally, the environment has infinite degrees of freedom, therefore tracing out the environment is not easy.

So far, many approaches, including the non-Markovian quantum-state diffusion equation (QSD) approach [30, 31, 32, 33, 34], the random matrix method [35] and the path integral approach [20, 36, 37], have been developed to study the non-Markovian dynamics of open quantum systems coupled to a bosonic environment[28, 29]. However, a generic non-Markovian master equation approach is still of fundamental importance for decoherence analysis and quantum dynamics simulations [38, 39, 40, 41], since only the master equation allows a direct investigation on the systems' reduced density matrix. Typically, a quantum system and its environment become entangled once they are coupled to each other. As a result, the states of the system are, in most cases, mixed states at a later time instant and can only be described by a reduced density matrix [1]. Furthermore, the environmental memory information is also included in the reduced density matrix. Analytical studies on this matrix help us better understand the evolution of quantum features, like quantum coherence and quantum interference patterns, and utilize the memory effect to engineer the environment.

Recently, we have successfully derived a set of exact non-Markovian master equations from the non-Markovian quantum-state diffusion equation [27]. In this paper, we establish a systematical method to derive the exact non-Markovian master equations for multilevel systems and investigate the temporal quantum features in multilevel systems numerically. Quantum interference, as one of the important features of the multilevel system arising from quantum information processing, quantum control, quantum coherence transfer, and quantum decoherence [3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18], is investigated in the non-Markovian regime. First of all, we work out the non-Markovian master equation for a four-level atomic system as an example in the presence of a dissipative environment. Secondly, we study the dynamics of quantum interference and the steady state in a four-level system controlled by an external laser field.

The paper is organized as follows. In Sec. II, we briefly review the principle ideas of the quantum-state diffusion equations (QSD) approach and show how to derive the generic master equations for multilevel systems formally. In Sec. III, we apply the non-Markovian master equations to investigate two examples followed by some brief discussions on the non-Markovian behavior of quantum interference. We conclude in Sec. IV.

2 Exact Non-Markovian Master Equations

For a generic multilevel quantum system coupled to a zero-temperature bosonic environment, the total Hamiltonian is given by [15] (we set $\hbar = 1$)

$$H_{\rm tot} = H_{\rm s} + H_{\rm int} + H_{\rm b}, \tag{1}$$

$$H_{\rm int} = L\sum_k g_k b_k^\dagger + L^\dagger \sum_k g_k^* b_k, \tag{2}$$

$$H_{\mathrm{b}} = \sum_k \omega_k b_k^\dagger b_k, \tag{3}$$

where L is the Lindblad operator of the system, describing the coupling between the system and its environment. For a dissipative environment, the Lindblad operator L is a ladder operator in the form of $L = \sum_n |n\rangle\langle n+1|$, which satisfies the following condition

$$L^N = 0, \tag{4}$$

where the integer N is the total number of the energy levels in the multilevel system. Note that $b_k^\dagger(b_k)$ is the creation (annihilation) operator for the kth mode in the bosonic environment and g_k is the complex coupling constant between the system and the kth environmental mode, characterized by the correlation function

$$\alpha(t,s) = \sum_k |g_k|^2 e^{-i\omega_k(t-s)}, \tag{5}$$

when the bath is at zero temperature.

In the QSD approach, the dynamics of quantum systems is described by a stochastic Schrödinger equation. The total state Ψ_{tot} for the system and the environment can be expanded in the basis of Bargmann coherent states

$$\Psi_{\mathrm{tot}}(t) = \frac{1}{\pi}\int \mathrm{d}^2 z ||z\rangle|\psi_t(z^*)\rangle, \tag{6}$$

where ψ_t is the stochastic wave function for the multilevel system, defined as $\psi_t(z^*) = \langle z|\Psi_{tot}\rangle$. The formal QSD equation at zero temperature [30, 31] is

$$\partial_t\psi_t(z^*) = \left(-iH_{\mathrm{s}} + Lz_t^*\right)\psi_t(z^*) - L^\dagger\int_0^t \mathrm{d}s\alpha(t,s)\frac{\delta}{\delta z_s^*}\psi_t(z^*), \tag{7}$$

where $z_t^* = -i\sum_k g_k z_k^* e^{i\omega_k t}$ is a complex Gaussian stochastic process whose correlation function is $\mathcal{M}[z_t^* z_s] = \alpha(t,s)$. The symbol $\mathcal{M}[..] = \int \frac{\mathrm{d}^2 z}{\pi} e^{-|z|^2}..$ means taking the ensemble average over all of the possible stochastic trajectories $\psi_t(z^*)$. The last term in the QSD equation can be explicitly determined by introducing the so-called O operator

$$O(t,s)\psi_t(z^*) = \frac{\delta}{\delta z_s^*}\psi_t(z^*). \tag{8}$$

It is easy to show the initial condition $(s=t)$ of the O operator is the Lindblad operator L. In order to clearly show the structure of the O operator in the terms of noise, we expand the O operator in the form of

$$O(t,s) = O_0 + \int_0^t ds_1 z_{s_1}^* O_1(t,s,s_1) + \int\int_0^t ds_1 ds_2 z_{s_1}^* z_{s_2}^* O_2(t,s,s_1,s_2) + ..., \tag{9}$$

where the terms $O_j(j=0,1,2,3...)$ are the operators involving the jth order of the noise. To satisfy the consistency condition of the O operator

$$\frac{\partial}{\partial t}\frac{\delta}{\delta z_s^*}\psi_t = \frac{\delta}{\delta z_s^*}\frac{\partial}{\partial t}\psi_t, \tag{10}$$

the O operator can be formally determined by its own evolution equation

$$\partial_t O = \left[-iH_{\rm s} + Lz_t^* - L^\dagger \bar{O},\, O\right] - L^\dagger \frac{\delta \bar{O}}{\delta z_s^*}, \tag{11}$$

where $\bar{O}(t,z^*) = \int_0^t \mathrm{d}s\alpha(t,s)O(t,s,z^*)$. Together with the QSD equation (7), the reduced density matrix ρ_t is recovered from the ensemble average over all trajectories. Then the formal master equation can be written as

$$\partial_t \rho_t = -i\left[H_{\rm s},\, \rho_t\right] + L\mathcal{M}[z_t^* P_t] - L^\dagger \mathcal{M}[\bar{O}P_t] + \mathcal{M}[z_t P_t]L^\dagger - \mathcal{M}[P_t \bar{O}^\dagger]L, \tag{12}$$

where $\rho_t = \mathcal{M}[|\psi_t(z^*)\rangle\langle\psi_t(z)|]$. We denote the stochastic reduced density matrix as $P_t = |\psi_t(z^*)\rangle\langle\psi_t(z)|$. Applying the Novikov theorem, the ensemble average can be calculated $\mathcal{M}[z_t^* P_t] = \int_0^t \mathrm{d}s\mathcal{M}[z_t^* z_s]\mathcal{M}[\frac{\delta}{\delta z_s} P_t]$. Since the trajectory $|\psi_t(z^*)\rangle$ is independent from the noise z_t, we have the result that $\mathcal{M}[\frac{\delta}{\delta z_s} P_t] = \mathcal{M}[P_t O^\dagger]$. Then the above formal master equation can be further simplified as

$$\partial_t \rho_t = -i\left[H_{\rm s},\, \rho_t\right] + \left[L,\, \mathcal{M}[P_t \bar{O}^\dagger]\right] - \left[L^\dagger,\, \mathcal{M}[\bar{O}P_t]\right]. \tag{13}$$

If the environment is Markov, then the correlation function is a Dirac delta function $\alpha(t,s) = \delta(t,s)$. Then we can derive that $\mathcal{M}[z_t^* P_t] = \mathcal{M}[P_t L] = \rho_t L$, and Eq.(13) matches the standard Lindblad form master equation [2]. However, only in several trivial models, the O operator is noise-free. For most studied models, we expand the O operator in terms of noise, so the term $\mathcal{M}[P_t \bar{O}^\dagger]$ turns out to be pretty complicated. By repeatedly using the Novikov theorem, the average over the term involving the nth power of noise is

$$\mathcal{M}[z_{s_1} z_{s_3} ... z_{s_{2n-1}} P_t] = \int_0^t \mathrm{d}s_2 \alpha_{1,2} \mathcal{M}\left[\left(z_{s_3} ... z_{s_{2n-1}}\right) \frac{\delta P_t}{\delta z_{s_2}^*}\right] \tag{14}$$

$$= \int_0^t \mathrm{d}s_2, .., \mathrm{d}s_{2n} \left(\prod_{j=1}^n \alpha_{j,j+1}\right) \mathcal{M}\left[\left(\prod_{j=1}^n \frac{\delta}{\delta z_{s_{2j}}^*}\right) P_t\right], \tag{15}$$

where $\alpha_{i,j} = \alpha(s_i, s_j)$. With the inference of the Novikov theorem, one can observe that the term $\mathcal{M}[P_t \bar{O}^\dagger]$ is a sum of series of ascending power of the O operator and its hermit operator $O^\dagger$. To resolve this difficulty, we recall the identity function of the O operator in Eq. (11), the right hand side contains up to $(N-2)$th power of noise, while in the left hand side, the commutator term $[L^\dagger \bar{O}_j,\, O_k]$ creates up to $(j+k)$th power of noise. Mathematically, the two sides of Eq. (11) match with each other, therefore the term $\bar{O}_j O_k$ must go to zero when $j + k > N - 2$,

$$O_j O_k = 0, \qquad (j + k > N - 2). \tag{16}$$

Such a particular selection rule is named the "forbidden conditions" [27]. Then it's easy to show that the $\mathcal{M}[z_{s_1} z_{s_3} ... z_{s_{2n-1}} P_t]$ must be a sum of finite terms and the number of terms is model dependant. If we denote $R(t) = \mathcal{M}[P_t \bar{O}^\dagger]$, a compact form of the exact master equation is obtained

$$\partial_t \rho_t = -i\left[H_{\rm s},\, \rho_t\right] + \left[L,\, R(t)\right] + \left[L,\, R(t)\right]^\dagger. \tag{17}$$

3 Temporal quantum interference based on the exact master equations

3.1 A four-level atom model

The first model we consider as an example is a four-level atom model described by the total Hamiltonian in the same form as Eq.(1), with

$$H_{\rm s} = \sum_{m=1}^{4} \omega_m |m\rangle\langle m|, L = \sum_{m=1}^{3} \kappa_m |m\rangle\langle m+1|, \tag{18}$$

where the operator $|m\rangle\langle m+1|$ is the ladder operator. It is obvious that $L^4 = 0$ in a four-level system. As discussed in the Sec. II, the O operator contains up to the second order of noise and is in the form of

$$O(t,s) = O_0(t,s) + \int_0^t ds_1 z^*_{s_1} O_1(t,s,s_1) + \int\int_0^t ds_1 ds_2 z^*_{s_1} z^*_{s_2} O_2(t,s,s_1,s_2). \tag{19}$$

Substituting this solution into Eq.(11), we have

$$\partial_t O_0(t,s) = [-iH_{\rm s},\, O_0] - \left[L^\dagger \bar{O}_0,\, O_0\right] - L^\dagger \bar{O}_1(t,s), \tag{20}$$

$$\partial_t O_1(t,s,s_1) = [-iH_{\rm s},\, O_1] - \left[L^\dagger \bar{O}_0,\, O_1\right] - \left[L^\dagger \bar{O}_1,\, O_0\right] - 2L^\dagger \bar{O}_2(t,s,s_1), \tag{21}$$

$$\partial_t O_2(t,s,s_1,s_2) = [-iH_{\rm s},\, O_2] - \left[L^\dagger \bar{O}_0,\, O_2\right] - \left[L^\dagger \bar{O}_1,\, O_1\right] - \left[L^\dagger \bar{O}_2,\, O_0\right]. \tag{22}$$

Meanwhile, we have the boundary conditions for each term of O operator:

$$[L,\, O_0] = O_1(t,s,t), \tag{23}$$

$$[L,\, O_1] = 2O_2(t,s,t,s_1). \tag{24}$$

Next, the ensemble average term $R(t)$ in the formal master equation (17) is

$$R(t) = \rho_t \bar{O}_0^\dagger + \int_0^t ds_1 \alpha_{1,2} \mathcal{M}[z_{s_1} P_t] \bar{O}_1^\dagger + \int_0^t ds_1 \alpha_{1,2} \mathcal{M}[z_{s_1} z_{s_2} P_t] \bar{O}_2^\dagger. \tag{25}$$

By repeatedly applying the Novikov theorem (Eq.15), the two terms $\mathcal{M}[z_{s_1} P_t]$ and $\mathcal{M}[z_{s_1} z_{s_2} P_t]\bar{O}_2^\dagger$ can be extensively written as a sum of an infinite long series. Simplified by the "forbidden conditions" (16), we have

$$\mathcal{M}[z_{s_1} P_t]\bar{O}_1^\dagger = \int_0^t ds_2 \alpha_{1,2} O_0(t,s_2) \rho_t \bar{O}_1^\dagger + \int\int\int_0^t ds_2 s_3 s_4 \alpha_{1,2}\alpha_{3,4} O_1(t,s_2,s_3) \rho_t O_0^\dagger(t,s_4) \bar{O}_1^\dagger, \tag{26}$$

and

$$\mathcal{M}[z_{s_1} z_{s_2} P_t]\bar{O}_2^\dagger = \int\int_0^t ds_3 s_4 \alpha_{1,3}\alpha_{2,4} O_0(t,s_3) O_0(t,s_4) \rho_t \bar{O}_2^\dagger + \int\int_0^t ds_3 s_4 \alpha_{1,3}\alpha_{2,4} O_1(t,s_3,s_4) \rho_t \bar{O}_2^\dagger. \tag{27}$$

Substituting these terms into Eq. (25), we obtain that

$$R(t) = \rho_t \bar{O}_0^\dagger + \int\int_0^t ds_1 ds_2 \alpha_{1,2} O_0(t,s_2) \rho_t \bar{O}_1^\dagger(t,s_1) \tag{28}$$

$$+\int\int\int\int_0^t \mathrm{d}\bar{s}\alpha_{1,2}\alpha_{3,4}O_1(t,s_2,s_3)\rho_t O_0^\dagger(t,s_4)\bar{O}_1^\dagger(t,s_1) \quad (29)$$

$$+\int\int\int\int_0^t \mathrm{d}\bar{s}\alpha_{1,3}\alpha_{2,4}O_0(t,s_3)O_0(t,s_4)\rho_t\bar{O}_2^\dagger(t,s_1,s_2) \quad (30)$$

$$+\int\int\int\int_0^t \mathrm{d}\bar{s}\alpha_{1,3}\alpha_{2,4}O_1(t,s_3,s_4)\rho_t\bar{O}_2^\dagger(t,s_1,s_2), \quad (31)$$

where $\mathrm{d}\bar{\mathrm{s}}$ denotes $\mathrm{ds}_1\mathrm{ds}_2\mathrm{ds}_3\mathrm{ds}_4$ in the integral. The exact non-Markovian master equation for four-level system is explicitly determined.

For simplicity, we choose the Ornstein-Uhlenbeck type correlation function, $\alpha(t,s) = \frac{\gamma}{2}e^{-\gamma|t-s|}$, which covers both the Markov limit ($\gamma \to \infty$) and the non-Markovian (γ is small) regime. However, it needs to point out that, our generic master equations are valid for any type of correlation function.

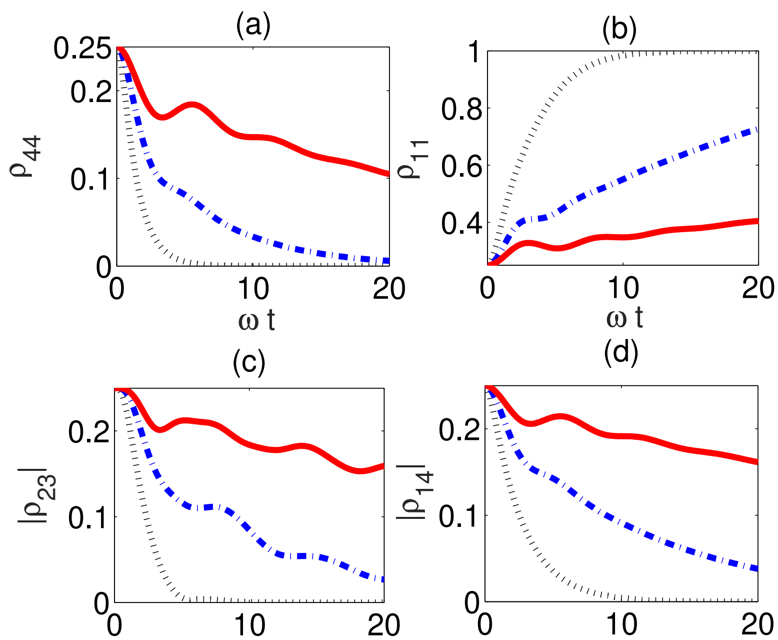

Fig. 1. (Color online) Non-Markovian evolution of four-level atom. The initial state is set as $|\psi\rangle_0 = |1\rangle + |2\rangle + |3\rangle + |4\rangle)/2$. We show the population of highest level (a) and ground level (b), also the coherence of $|\rho_{23}|$ (c) and $|\rho_{14}|$ (d). The solid (red) curve is for $\gamma = 0.2$, dash-dotted (blue) is for $\gamma = 0.5$ and dotted (black) is for $\gamma = 2$.

As shown in Fig. 1, the time evolution of the population of each level and the quantum coherence between levels is displayed. Our results clearly reveal that the decay speed of the population of each level is mediated by the environment via its non-Markovian correlation function. Long-lasting behavior of both the population and the quantum coherence comparing to the Markov limit, and the fluctuation in the short time regime, indicate the influence of the environment via its memory effect. This tool supplies a easy access to study the multilevel systems in the presence of the dissipative environments with arbitrary decaying factors. For other types of correlation functions, we can also use this approach to study the impacts on the evolution of the multilevel systems induced by other controllable parameters of the corresponding environments.

3.2 A four-level system under control

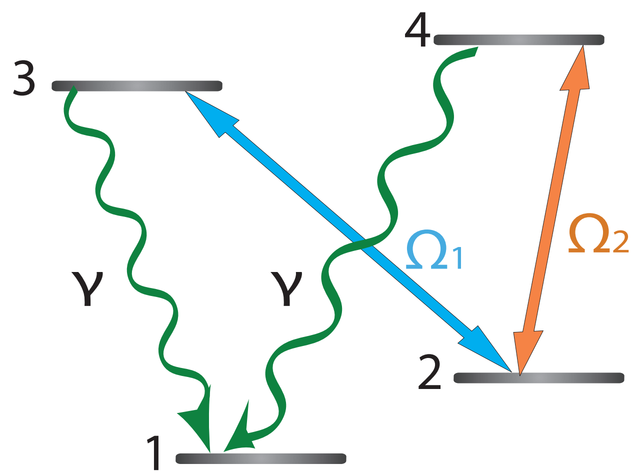

Fig. 2. Schematic diagram of quantum interference model, which shows the two upper level ($|4\rangle$, $|3\rangle$) coupled to the intermediate level ($|2\rangle$) with Rabi frequency Ω_1 and Ω_2, and decaying to ground state ($|1\rangle$).

The quantum interference in the multilevel systems leads to many interesting effects, for example quantum absorption, reduction and cancellation [4]. Here in this section, we consider a four-level system under a pumping laser, as shown in Fig. 2. The Hamiltonian of this model in the system-environment frame is

$$H_{\text{tot}} = H_{\text{s}} + H_{\text{int}} + H_{\text{b}}, \tag{32}$$

$$H_{\text{s}} = \sum_{m=1}^{4} \omega_m |m\rangle\langle m| + (\Omega_1 e^{-i\mu t}|3\rangle\langle 2| + h.c.) + (\Omega_2 e^{-i\mu t}|4\rangle\langle 2| + h.c.), \tag{33}$$

$$H_{\text{int}} = \sum_k \sum_{j=3,4} g_k |j\rangle\langle 1| b_k + h.c., \tag{34}$$

$$H_{\text{b}} = \sum_k \omega_k b_k^\dagger b_k, \tag{35}$$

where the $\Omega_1(\Omega_2)$ is the Rabi frequency of the driving field pumping between the upper two levels $|4\rangle(|3\rangle)$ and $|2\rangle$. There are two decaying channels for the spontaneous emission. The dissipative coupling constant between the system and kth mode in the environment is marked as g_k. Thus, the Lindblad operator for this model is

$$L = |1\rangle\langle 3| + \kappa|1\rangle\langle 4|,$$

where κ is the ratio of decaying constants between these two channels. It is easy to prove that this model satisfies the "forbidden condition" $L^3 = 0$. Because the dissipative channels are independent of the driving field, we can apply the known O operator (Eq. 19) and the exact master equation (Eq.17) directly to numerically simulate the quantum interference evolution under the external pumping laser field.

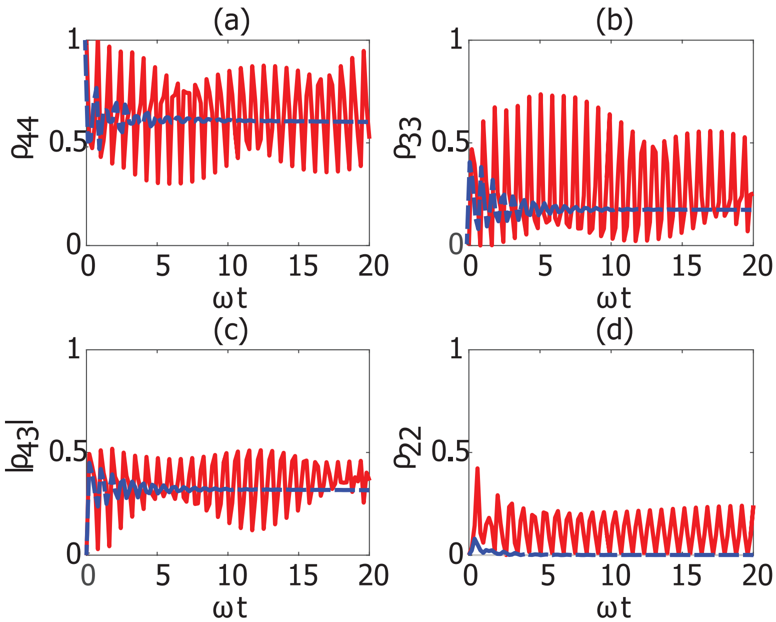

Fig. 3. (Color online) Time evolution of population of $|4\rangle$, $|3\rangle$, $|3\rangle$ and coherence $|\rho_{43}|$ between $|4\rangle$ and $|3\rangle$. Initial state is set as $|4\rangle$. The constants are set as: $\omega_4 - \omega_3 = \omega$, $\Omega_1 = 5\omega$, $\Omega_2 = 2\Omega_1$, $\mu = 2\omega$, $\kappa = 2$. Take comparison for two γ values. (1) $\gamma = 0.5$, solid (red), (2) $\gamma = 10$, dashed (blue).

In Figs. 3 and 4, the time evolution of the population of level 2, level 3 and level 4 and the quantum interference of the upper two levels are displayed. By choosing different decay rate γ in the Ornstein-Uhlenbeck correlation function $\frac{\gamma}{2}e^{-\gamma|t-s|}$, we demonstrate the significant difference between the Markov and the non-Markovian regimes. With our exact master equation, numerical simulations show that the populations of the upper two levels swap in a quick frequency, as well as the modulated dynamics due to the detuning between the system and the driving field, when $\gamma = 0.5$ (the red solid line). At the same time, the average population of the highest level $|4\rangle$ fluctuates between the range from 0.4 to 1, while that quickly decays monotonically to zero in the Markov regime, $\gamma = 10$ (blue dashed line). The second conclusion is that the populations of the upper two levels inverse only in a restrict parameter space. In Fig. 4, we study the unbiased model, setting $\kappa = 1$, while the detuning for the upper two levels are bias, setting $\Gamma_2 = \Gamma_1$. The populations of level 4 and level 3 of the steady state are close to each other. Although the conditions of population inverse was mentioned in [4], the discussion was based on the spectral density analysis and Markov assumption. Our exact master equation approach allows an exact way to study the steady state of the system under arbitrary environmental spectrum.

More than a tool to simulate the non-Markovian dynamics of the multilevel system, our master equation can also be applied to study the influence on the steady state in arbitrary

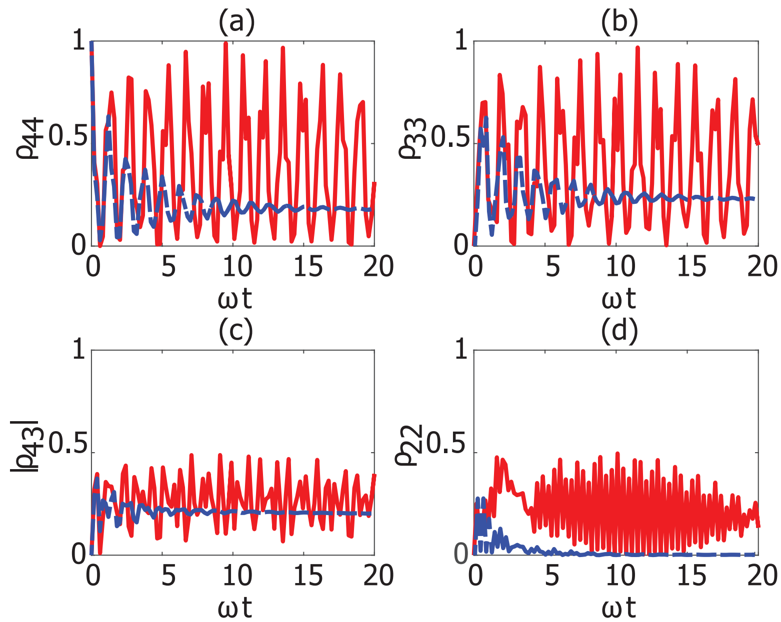

Fig. 4. (Color online) Time evolution of population of $|1\rangle$, $|2\rangle$, $|3\rangle$ and coherence $|\rho_{43}|$ between $|4\rangle$ and $|3\rangle$. Initial state is set as $|4\rangle$. The constants are set as: $\omega_4 - \omega_3 = \omega$, $\Omega_1 = \Omega_2 = 5\omega$, $\mu = 2\omega$, $\kappa = 1$. Take comparison for two γ values. (1) $\gamma = 0.5$, solid (red), (2) $\gamma = 10$, dashed (blue).

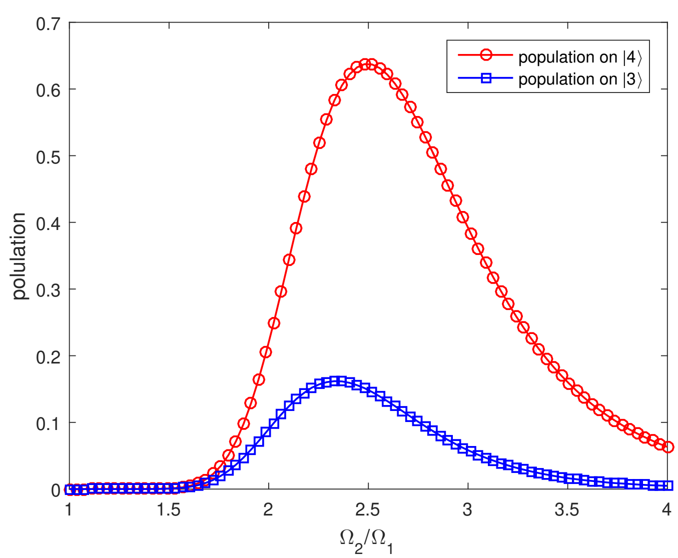

Fig. 5. (Color online) The population of $|4\rangle$ (red-circle), $|3\rangle$ (blue-square) at long time limit depending on the ratio of Ω_2/Ω_1. Initial state is set as $|4\rangle$. The constants are set as: $\mu = 2\omega$, $\kappa = 2$.

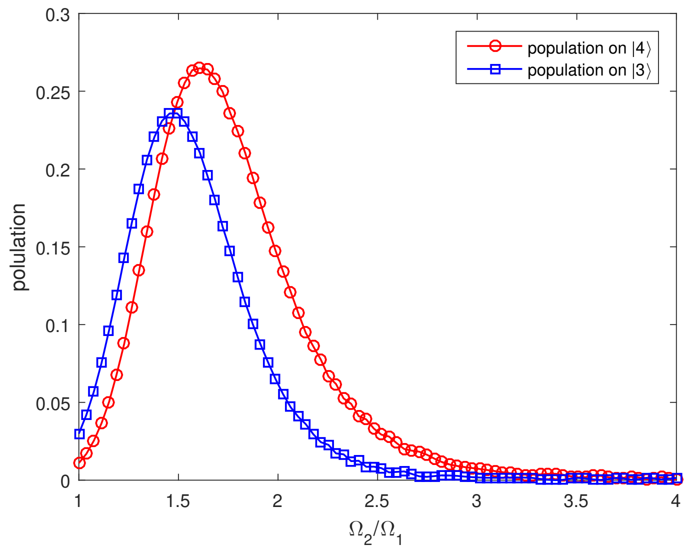

Fig. 6. (Color online) The population of $|4\rangle$ (red-circle), $|3\rangle$ (blue-square) at long time limit depending on the ratio of Ω_2/Ω_1. Initial state is set as $|4\rangle$. The constants are set as: $\mu = 2\omega$, $\kappa = 1$

parameter spaces. We numerically demonstrate the relationship between the steady state populations of the upper two levels as a function of the parameter κ and the ratio factor Ω_2/Ω_1. In Fig. 5, the population inverse happens in a particular range, especially when the ratio $\Omega_2/\Omega_1 = 2.5$ and the difference of the populations of level 4 and level 3 achieves the maximum, around 0.65. While in Fig. 6, although the population inverse can be observed in a narrow range comparing with the last setup, when the ratio factor $1.5 < \Omega_2/\Omega_1 < 3.5$, the steady state populations of level 4 and level 3 are close to each other, around 0.25. With our master equation approach, we can study the dynamics of the multilevel systems under an arbitrary quantum control, either it is a continuous laser or a sequence of discrete pumps.

4 Conclusion

In summary, we demonstrate the non-Markovian dynamics of multilevel systems induced by a dissipative bosonic environment with the aid of an exact master equation. Two examples are demonstrated to show how the master equation approaches work and how the memory of the environment impacts the population of each level and the quantum interference in the multi-level system. The master equation approach plays a critical role in exploring non-Markovian open system dynamics, even when quantum control is on. One important feature illustrated by the exact master equations in each case is that quantum interference can be investigated in both long-time and short-time scales. With the exactly resolved reduced density matrix, we can extract all information of the multilevel system and access the analytical relationship between the quantum features and the parameters of the environment. The results also show us how to engineer the environment to modulate the central system for particular desires. Additionally, the exact master equation sheds light on deriving the analytical solution of an open quantum system and approaching the non-equilibrium dynamics of a quantum system.

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