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ERRATUM TO " COHERENCE MEASURES AND OPTIMAL CONVERSION FOR COHERENT STATES" [QUANTUM INFORMATION AND COPUTATION, Vol. 15(2015), 1307-1316]

SHUANPING DU ZHAOFANG BAI
School of Mathematical Sciences, Xiamen University
Xiamen 361000, China
dushuanping@xmu.edu.cn baizhaofang@xmu.edu.cn

XIAOFEI QI
Department of Mathematics, Shanxi University
Taiyuan 030006, China
xiaofeiqisxu@aliyun.com

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In our previous paper "Coherence measures and optimal conversion for coherent states" [Quantum Information and Computation, Vol. 15(2015), 1307-1316], there is an error in the proof of Theorem 1. Here, we correct the error.

In our previous paper [1], we discuss a general strategy to construct coherence measures. One can build an important class of coherence measures which cover the relative entropy measure for pure states, the l_1-norm measure for pure states and the α-entropy measure. One of the main results is the following

Theorem 1. Any function C_f satisfying (2)-(6) is a coherence measure, i.e.,

$$\text{Eqs. } (2)-(6) \Rightarrow C1, C2b, C3. \tag{1}$$

In the proof of Theorem 1, a key step is to check that the defined C_f has the property C2b. However there is an error in the Eq. (12). That is, the cross term of (12) does not vanish. The aim of this note is to correct the error.

For the convenience of readers, here we recall the contents of Eqs. $(2)-(6), C1, C2b, C3$. And more details can be found in [1].

Let $\Omega = \{\mathbf{x} = (x_1, x_2, \cdots, x_d)^t | \sum_{i=1}^d x_i = 1, x_i \geq 0\}$, given any nonnegative function $f : \Omega \mapsto \mathcal{R}^+$ such that it is

- $$f(P_\pi(1, 0, \cdots, 0)^t) = 0, \tag{2}$$

  for every permutation π,

- invariant under any permutation transformation P_π, i.e.

$$f(P_\pi \mathbf{x}) = f(\mathbf{x}) \text{ for every } \mathbf{x} \in \Omega, \tag{3}$$

- concave, i.e.

$$f(\lambda \mathbf{x} + (1-\lambda)\mathbf{y}) \geq \lambda f(\mathbf{x}) + (1-\lambda) f(\mathbf{y}) \tag{4}$$

  for any $\lambda \in [0,1]$ and $\mathbf{x}, \mathbf{y} \in \Omega$,

a coherence measure can be derived by defining it for pure states (normalized vectors $|\psi\rangle = (\psi_1, \psi_2, \cdots, \psi_d)^t$ in the fixed basis $\{|i\rangle\}_{i=1}^d$) as

$$C_f(|\psi\rangle\langle\psi|) = f((|\psi_1|^2, |\psi_2|^2, \cdots, |\psi_d|^2)^t), \tag{5}$$

and by extending it over the whole set of density matrices as

$$C_f(\rho) = \min_{p_j, \rho_j} \sum_j p_j C_f(\rho_j), \tag{6}$$

where the minimization is to be performed over all the pure-state ensembles of ρ, i.e., $\rho = \sum_j p_j \rho_j$.

Based on Baumgratz et al.'s suggestion [2], any proper measure of coherence $\mathcal{C}$ must satisfy the following axiomatic postulates.

(C1) The coherence measure vanishes on the set of incoherent states, $\mathcal{C}(\rho) = 0$ for all $\rho \in \mathcal{I}$;

(C2a) Monotonicity under incoherent operation Φ, $\mathcal{C}(\Phi(\rho)) \leq \mathcal{C}(\rho)$,

or (C2b) Monotonicity under selective measurements on average: $\sum_n p_n \mathcal{C}(\rho_n) \leq \mathcal{C}(\rho)$, where $p_n = tr(K_n \rho K_n^\dagger)$, $\rho_n = \frac{1}{p_n} K_n \rho K_n^\dagger$, for all $\{K_n\}$ with $\sum_n K_n^\dagger K_n = I$ and $K_n \rho K_n^\dagger / Tr(K_n \rho K_n^\dagger) \in \mathcal{I}$ for all $\rho \in \mathcal{I}$;

(C3) Non-increasing under mixing of quantum states (convexity),

$$\mathcal{C}(\sum_n p_n \rho_n) \leq \sum_n p_n \mathcal{C}(\rho_n)$$

for any ensemble $\{p_n, \rho_n\}$.

When we verify C2b, there is an error in calculation in the Eq. (12) [1]. Now we give an emendation. Indeed, one can only obtain

$$\begin{cases} \psi_1^2 + \sum_n \delta_{1,f_n(2)} |k_2^{(n)}|^2 \psi_2^2 \\ \quad + \sum_n \delta_{1,f_n(3)} |k_3^{(n)}|^2 \psi_3^2 \\ \quad + \sum_n \delta_{1,f_n(2)} \delta_{1,f_n(3)} \overline{k_2^{(n)}} k_3^{(n)} \psi_2 \psi_3 \\ \quad + \sum_n \delta_{1,f_n(2)} \delta_{1,f_n(3)} \overline{k_3^{(n)}} k_2^{(n)} \psi_3 \psi_2 \\ \ \ = \sum_n |\phi_1^{(n)}|^2, \\ \sum_n \delta_{2,f_n(2)} |k_2^{(n)}|^2 |\psi_2^2 + \sum_n \delta_{2,f_n(3)} |k_3^{(n)}|^2 \psi_3^2 \\ \quad + \sum_n \delta_{2,f_n(2)} \delta_{2,f_n(3)} \overline{k_2^{(n)}} k_3^{(n)} \psi_2 \psi_3 \\ \quad + \sum_n \delta_{2,f_n(2)} \delta_{2,f_n(3)} \overline{k_3^{(n)}} k_2^{(n)} \psi_3 \psi_2 \\ \ \ = \sum_n |\phi_2^{(n)}|^2, \\ \sum_n \delta_{3,f_n(3)} |k_3^{(n)}|^2 \psi_3^2 = \sum_n |\phi_3^{(n)}|^2. \end{cases} \tag{7}$$

Note that in the first equation of (7),

$$\begin{aligned}
&\sum_n \delta_{1,f_n(2)}|k_2^{(n)}|^2\psi_2^2 \\
&\quad + \sum_n \delta_{1,f_n(3)}|k_3^{(n)}|^2\psi_3^2 \\
&\quad + \sum_n \delta_{1,f_n(2)}\delta_{1,f_n(3)}\overline{k_2^{(n)}}k_3^{(n)}\psi_2\psi_3 \\
&\quad + \sum_n \delta_{1,f_n(2)}\delta_{1,f_n(3)}\overline{k_3^{(n)}}k_2^{(n)}\psi_3\psi_2 \\
&= \sum_n |\delta_{1,f_n(2)}k_2^{(n)}\psi_2 + \delta_{1,f_n(3)}k_3^{(n)}\psi_3|^2.
\end{aligned} \tag{8}$$

This tells us that

$$\psi_1^2 \leq \sum_n |\phi_1^{(n)}|^2.$$

Adding the first and the second equation of (7) together, we have

$$\psi_1^2 + \psi_2^2 \leq \sum_n |\phi_1^{(n)}|^2 + \sum_n |\phi_2^{(n)}|^2.$$

From the definition of majorization, one can check that

$$\begin{aligned}
& ((|\psi_1|^2, |\psi_2|^2, |\psi_3|^2)^t \\
\prec \quad & (\sum_n |\phi_1^{(n)}|^2, \sum_n |\phi_2^{(n)}|^2, \sum_n |\phi_3^{(n)}|^2)^t.
\end{aligned} \tag{9}$$

That is, the Eq.(13) of [1].

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