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EXTENSION OF REMOTELY CREATABLE REGION VIA LOCAL UNITARY TRANSFORMATION ON RECEIVER SIDE

G.A. BOCHKIN and A.I. ZENCHUK
Institute of Problems of Chemical Physics, RAS
Chernogolovka, Moscow reg., 142432, Russia
bochkin.g@yandex.ru, zenchuk@itp.ac.ru

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We show that the length of the effective remote state creation via the homogeneous spin-1/2 chain can be increased more than three times using the local unitary transformation of the so-called extended receiver (i.e., receiver joined with the nearest node(s)). This transformation is most effective in the models with all-node interactions. We consider an example of communication lines with the two-qubit sender, one-qubit receiver and two-qubit extended receiver.

Keywords:

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1 Introduction

The problem of remote state creation [1, 2, 3, 4, 5] is an alternative to the state teleportation [6, 10, 11, 7, 8, 9] and the state transfer problem [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]. All of them are aimed at the proper way of the information transfer [37, 38, 39] from the sender to the receiver. In most experiments the information carriers are photons [2, 3, 4, 9, 10, 11]. However, the spin chain as a transmission line between the sender and receiver is popular in numerical simulations, see [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27].

In the recent paper [36] we propose the detailed description of the remote state creation in long homogeneous spin-1/2 chains as the map (control parameters) $\to$ (creatable parameters). Here, we call the arbitrary parameters of the sender's initial state the control parameters, while the creatable parameters are the parameters of the receiver's state (which are eigenvalue-eigenvector parameters in that paper). As a characteristic of the state creation effectivity, the creatable interval of the largest eigenvalue was proposed. The critical length $N_c = 34$ was found such that any eigenvalue can be created, i.e., the largest eigenvalue can take any value from the interval $\frac{1}{2} \le \lambda_{max} \le 1$.

It was shown [36] that the creatable region of the receiver's state space (i.e., the subregion of the receiver's state space which can be remotely created by varying the control parameters) shrinks to $\lambda_{max} = 1$ with an increase in the length of the homogeneous spin chain. This fact restricts the applicability of homogeneous spin-1/2 chains in communication lines. A formal straightforward way to avoid the above shrinking is using the chains with specially valued

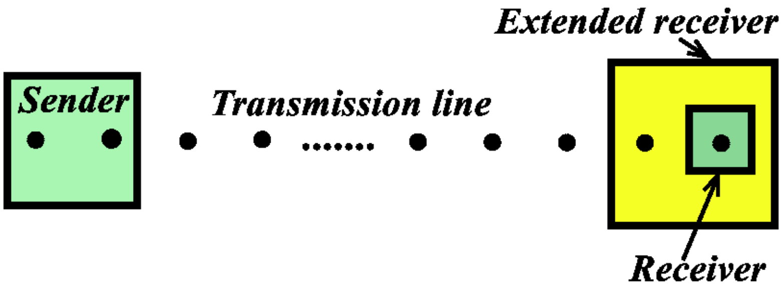

Fig. 1. The communication line with the extended receiver. Using the optimized local unitary transformation of the extended receiver we increase the critical length N_c and thus extend the creatable region in comparison with the model without the above local transformation of the extended receiver.

coupling constants providing the either perfect state transfer (PST) [13, 15] or high probability state transfer (HPST) [16, 17, 22, 23, 24]. However, these chains require additional efforts for their practical realization, unlike the homogeneous chain with all equal nearest-neighbor coupling constants. Therefore, the problem of improving the characteristics of communication lines based on homogeneous spin chains is of principal meaning. This conclusion motivates our further study of the remote state creation via a homogeneous spin chain.

In this regard, we have found an additional simple way to compensate the above mentioned shrinking of the creatable region in long communication lines based on a homogeneous spin chain via a local unitary transformation of the receiver side. However, we shall remark that a unitary transformation applied to the receiver itself can not change the eigenvalues (which are part of the creatable parameters) of the receiver state. Nevertheless, it is remarkable that the receiver's eigenvalue can be changed by a local unitary transformation applied to the so-called extended receiver involving the receiver as a subsystem, Fig. 1. Further numerical simulations with the one-node receiver (justified by the theoretical arguments) show that this procedure is most effective in chains governed by the Hamiltonian with all-node interactions rather than with nearest-neighbor ones. As a result, we manage to significantly extend the creatable region and increase the mentioned above critical length from $N_c = 34$ to $N_c = 109$ in homogeneous chains (i.e., more than three times). We emphasize that the above mentioned optimizing unitary transformation is uniquely (up to non-significant phase transformation) defined for a given communication line and does not depend on the particular state we need to create. Therefore, the particular created state is completely defined by the values of control parameters and there is a one-to-one map (control parameters) $\to$ (creatable parameters). Thus, the only requirement for the tool at the receiver side is the implementation of that unique optimizing unitary transformation.

All in all, we consider the communication line based on the homogeneous spin chain with all-node interactions consisting of the following parts, Fig. 1.

1. The two node sender with an arbitrary pure state whose parameters are referred to as the control parameters (the first and the second nodes of the spin chain).

2. The one-qubit receiver whose state-parameters are referred to as the creatable parameters (the last node of the chain).

3. The two-node extended receiver consisting of the two last nodes of the chain (involving the receiver itself).

4. The transmission line connecting the sender with the extended receiver.

Thus, the modification of the remote state creation algorithm of ref.[36] by including the extended receiver together with the local unitary transformation allows us to perform the significant improvement of the characteristics of the communication line based on a homogeneous spin-1/2 chain without involving the special engineering of coupling constants.

The paper is organized as follows. In Sec.2, we specify the interaction Hamiltonian together with the initial condition used for the remote state creation. Sec.3 is devoted to the optimization of the local unitary transformation of the extended receiver with the purpose to obtain the largest creatable region. Numerical simulations confirming the theoretical predictions are presented in Sec.4. General conclusions and discussion of results are given in Sec.4.

2 XY Hamiltonian and initial state of communication line

Our model of communication line is based on the homogeneous spin chain with the one-spin excitation whose dynamics is governed by the XY-Hamiltonian

$$H = \sum_{\substack{i,j=1\\ j>i}}^{N} D_{ij}(I_{i,x}I_{j,x} + I_{i,y}I_{j,y}), \quad D_{ij} = \frac{\gamma^2\hbar}{r_{ij}^3}, \tag{1}$$

where γ is the gyromagnetic ratio, r_{ij} is the distance between the ith and the jth spins, $I_{i,\alpha}$ ($\alpha = x, y, z$) is the projection operator of the ith spin on the α axis, D_{ij} is the dipole-dipole coupling constant between the ith and the jth nodes. Below we use the dimensionless time (formally setting $D_{12} = 1$). Obviously, this Hamiltonian commutes with the z-projection of the total angular momentum I_z, so that the evolution of the one-spin excitation can be described in the $N+1$-dimensional basis (instead of the general 2^N dimensional one)

$$|i\rangle, \quad i = 0, \dots, N, \tag{2}$$

where $|i\rangle$, $i > 0$, denotes the state with the ith excited spin, $|0\rangle$ corresponds to the ground state of the spin chain with zero (by convention) eigenvalue.

The general form of the initial state of the N-node chain with the one-excitation initial state of the two-qubit sender reads

$$|\psi_0\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle, \quad \sum_{i=0}^{2} |a_i|^2 = 1, \tag{3}$$

where the real parameter a_0 and the complex parameters a_1, a_2 are given as:

$$a_0 = \sin\frac{\alpha_1\pi}{2}, \quad a_1 = \cos\frac{\alpha_1\pi}{2}\cos\frac{\alpha_2\pi}{2}e^{2i\pi\varphi_1}, \quad a_2 = \cos\frac{\alpha_1\pi}{2}\sin\frac{\alpha_2\pi}{2}e^{2i\pi\varphi_2}, \tag{4}$$

$$0 \le \alpha_i \le 1, \quad 0 \le \varphi_i \le 1, \quad i = 1, 2. \tag{5}$$

Note that formula (3) means that both extended receiver and transmission line are in the ground state initially.

3 Optimal local transformation of the extended receiver

In this section we derive the optimal local unitary transformation of the extended receiver which maximizes the creatable region. For this purpose we first find the general formula for the state of the extended receiver, Sec.3.1. Then we diagonalize this state using the appropriate unitary transformation V and show that both non-zero eigenvalues depend on the probability of the excitation transfer to the extended receiver, Sec.3.2. After that we maximize this excitation transfer probability optimizing the control parameters, Sec.3.3. The unitary transformation V corresponding to the optimized control parameters is the needed unitary transformation of the extended receiver, Sec.3.4. This is the transformation which provides the transfer of both nonzero eigenvalues of the extended receiver to the one-node receiver. After optimization of V over the time t (Sec.3.5.1), we obtain the algorithm of remote state creation in Sec.3.6. It is remarkable that the optimization of the transformation V can be done using the singular value decomposition of some special matrix P whose elements are defined in terms of transition amplitudes (23) which simplifies numerical simulations, Sec.3.5.

3.1 General state of extended receiver

As mentioned above, the state of the extended receiver is described by the density matrix reduced over all the nodes except the two last ones. Written in the basis

$$|0\rangle,\ \ |N-1\rangle,\ \ |N\rangle,\ \ |(N-1)N\rangle, \tag{6}$$

this state reads

$$\rho_{R_{ext}} \equiv \mathrm{Tr}_{1,2,\dots,N-2}\rho = \left(\begin{array}{cccc} 1-|f_{N-1}|^2-|f_N|^2 & f_0 f^*_{N-1} & f_0 f^*_N & 0\\ f^*_0 f_{N-1} & |f_{N-1}|^2 & f_{N-1} f^*_N & 0\\ f^*_0 f_N & f^*_{N-1} f_N & |f_N|^2 & 0\\ 0 & 0 & 0 & 0\end{array}\right). \tag{7}$$

In (6), $|(N-1)N\rangle$ means the state with two last excited nodes of the chain, the trace is taken over the nodes $1,\dots,N-2$, the star means the complex conjugate value and f_i, $i = 0,\ N,\ N-1$, is the projections of the state on the basis vector $|i\rangle$, i.e.,

$$\begin{aligned} f_i &= \langle i|e^{-iHt}|\Psi_0\rangle = R_i e^{2\pi i \Phi_i},\ \ i=0,\dots,N, \\ & \quad 0\le \Phi_i \le 1,\quad R_i\ge 0. \end{aligned} \tag{8}$$

Remember the natural constraint

$$|f_0|^2+|f_N|^2+|f_{N-1}|^2\le 1 \ \ \Rightarrow\ \ R_0^2+R_N^2+R_{N-1}^2\le 1, \tag{9}$$

where the equality corresponds to the pure state transfer to the nodes of the extended receiver because in this case $f_i\equiv 0$ $(0<i<N-1)$.

Since the initial state is a linear function of the control parameters a_i, the projections f_i are also linear functions of these parameters:

$$f_N(t) = \langle N|e^{-iHt}|\Psi_0\rangle = \sum_{j=1}^2 a_j \langle N|e^{-iHt}|j\rangle = \sum_{j=1}^2 a_j p_{Nj}(t), \tag{10}$$

$$f_0(t) = \langle 0|e^{-iHt}|\Psi_0\rangle = a_0\equiv R_0, \tag{11}$$

where p_{kj} are transition amplitudes:

$$p_{kj}(t) = \langle k|e^{-iHt}|j\rangle = r_{kj}(t)e^{2\pi i\chi_{kj}(t)}, \quad k,j>0, \qquad (12)$$
$$r_{kj} \ge 0, \quad 0 \le \chi_{kj} \le 1.$$

In eq. (11), we use the fact that the ground state has zero energy. We emphasize that the transition amplitudes represent the inherent characteristics of the communication line and do not depend on the control parameters a_i of the sender's initial state.

3.2 Eigenvalues of extended receiver

The construction of optimal local transformation of the extended receiver is based on maximization of the creatable interval of the largest eigenvalue of the density matrix $\rho_{R_{ext}}$ (7) corresponding to the extended receiver. The eigenvalues of $\rho_{R_{ext}}$ read as follows:

$$\tilde\lambda_\pm = \frac{1}{2}\left(1 \pm \sqrt{(1-2R^2)^2 + 4R^2R_0^2}\right), \qquad (13)$$

where we introduce the probability of the excitation transfer to the nodes of the extended receiver

$$R^2 \equiv |f_{N-1}|^2 + |f_N|^2 = R_N^2 + R_{N-1}^2. \qquad (14)$$

The largest eigenvalue $\tilde\lambda_+$ as a function of R and R_0 varies inside of some interval

$$\tilde\lambda_0 \le \tilde\lambda_+ \le 1. \qquad (15)$$

Thus, to obtain the largest variation interval we need to minimize $\tilde\lambda_0$ as a function of R and R_0. It is simple to show that the minimum $\tilde\lambda_0^{min}$ corresponds to R_0 =0. For this purpose we use the following substitution prompted by constraint (9):

$$R_N = \tilde R_N\sqrt{1-R_0^2}, \quad R_{N-1} = \tilde R_{N-1}\sqrt{1-R_0^2}, \quad \tilde R^2 = \tilde R_N^2 + \tilde R_{N-1}^2. \qquad (16)$$

In terms of the new notations, the largest eigenvalue reads

$$\tilde\lambda_+ = \frac{1}{2}\left(1 + \sqrt{1-4(1-R_0^2)^2\tilde R^2(1-\tilde R^2)}\right) \qquad (17)$$

Calculating the derivative of $\tilde\lambda_+$ with respect to $\tilde R$ we find the extremum at $\tilde R^2 = \frac{1}{2}$:

$$\tilde\lambda_+^{min} = \frac{1}{2}\left(1 + R_0\sqrt{2-R_0^2}\right) \qquad (18)$$

which is minimal at $R_0 = a_0 = 0$:

$$\tilde\lambda_+^{min}|_{R_0=0} = \frac{1}{2}. \qquad (19)$$

Note that R is a continuous function of the control parameters a_i, $i=1,2$, and $R=0$ at $a_1=a_2=0$. Consequently, if R reaches some value R^{opt}, then with varying a_i, we can obtain any value of R inside of the interval

$$0 \le R \le R^{opt}. \qquad (20)$$

The largest variation interval $0 \le R \le 1$ corresponds to the communication line allowing the perfect state transfer of the excitation to the extended receiver. In this case the variation interval of $\tilde\lambda_+$ is also maximal, $\frac{1}{2} \le \tilde\lambda_+ \le 1$. However, in general, this variation interval is following:

$$\begin{aligned} &1-(R^{opt})^2 \le \tilde\lambda_+ \le 1, \;\; 0 \le (R^{opt})^2 \le \frac{1}{2}, \\ &\frac{1}{2} \le \tilde\lambda_+ \le 1, \;\; (R^{opt})^2 \ge \frac{1}{2}. \end{aligned} \tag{21}$$

Eq.(21) shows that the interval of creatable $\tilde\lambda_+$ is completely defined by the probability of the excitation transfer to the extended receiver. Therefore, the maximization of this quantity deserves special consideration.

3.3 Control parameters maximizing R

The probability of the excitation transfer R^2 (14) is equal to the squared norm of the two-component vector $f = (f_{N-1} \;\; f_N)^T$ (the superscript T means transposition),

$$f = Pa, \quad a = \left(\begin{array}{c} a_1 \\ a_2 \end{array}\right), \tag{22}$$

$$P = \left(\begin{array}{cc} p_{(N-1)1} & p_{(N-1)2} \\ p_{N1} & p_{N2} \end{array}\right) \tag{23}$$

Thus, the maximum $(R^{opt})^2$ of the probability R^2 as a function of the control parameters can be found as

$$(R^{opt})^2 = \max_{|a_1|^2+|a_2|^2=1} (|f_N|^2 + |f_{N-1}|^2). \tag{24}$$

To proceed further, we write R^2 in the following form

$$R^2 \equiv f^+ f = a^+ P^+ P a. \tag{25}$$

and diagonalize the matrix P^+P:

$$P^+P = U^+\Lambda_0^2 U, \;\; \Lambda_0^2 = \mathrm{diag}(\lambda_-^2, \lambda_+^2), \;\; 0 \le \lambda_- \le \lambda_+, \tag{26}$$

where

$$\begin{aligned} \lambda_\pm^2 \;\; = \;\; & \frac{1}{2}\Big(r^2_{(N-1)1} + r^2_{(N-1)2} + r^2_{N1} + r^2_{N2} \pm \sqrt{Q}\Big), \\ & Q = (r^2_{(N-1)1} + r^2_{(N-1)2} + r^2_{N1} + r^2_{N2})^2 - \\ & 4\Big(r^2_{(N-1)2} r^2_{N1} + r^2_{(N-1)1} r^2_{N2} - \\ & 2\cos(2(\chi_{(N-1)1} - \chi_{(N-1)2} - \chi_{N1} + \chi_{N2})) r_{(N-1)1} r_{(N-1)2} r_{N1} r_{N2}\Big). \end{aligned} \tag{27}$$

So, by virtue of (26), eq.(25) reads

$$f^+ f = b^+ \Lambda_0^2 b, \;\; b = Ua. \tag{28}$$

The mutual position of the eigenvalues in the matrix Λ_0^2 is taken for convenience and will be used in Sec.3.5. Now, by virtue of eq.(28), we rewrite eq.(24) as follows:

$$(R^{opt})^2 = \max_{|b_1|^2+|b_2|^2=1} (\lambda_-^2|b_1|^2 + \lambda_+^2|b_2|^2), \quad b = (b_1, \ \ b_2)^T. \tag{29}$$

Obviously, the maximal value is achieved when $b_2 = 1$ and $b_1 = 0$:

$$(R^{opt})^2 = \lambda_+^2. \tag{30}$$

The appropriate expression for the vector of control parameters a^{opt} follows from the relation between a and b given in the second of eqs.(28):

$$Ua^{opt} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \Rightarrow a^{opt} = U^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{31}$$

Formula (31) gives us the sender's initial state leading to the maximal value $(R^{opt})^2$ of the probability R^2 at a given time instant.

Note that $|f_{N-1}| = 0$ at the extremum point of $|f_N|$ in the case of nearest neighbor approximation [36]. As a result, $R^{max} \equiv \max\limits_{|a_1|^2+|a_2|^2=1} |f_N|$ and there is no contribution from the $(N-1)$th node of the chain. That is why the local transformation of the two-node extended receiver is not effective in the case of nearest neighbor approximation.

3.3.1 Explicit form of U

We can also write the explicit form of U (and, consequently, the explicit form of a^{opt}) in terms of the probability amplitudes p_{ij}. This can be done using the definition (26) written as

$$UP^+P = \Lambda_0^2 U. \tag{32}$$

Let us represent the matrix U in terms of the parameters α_i^{opt} and φ_i^{opt}:

$$U = \begin{pmatrix} \cos\frac{\alpha^{opt}\pi}{2} & -\sin\frac{\alpha^{opt}\pi}{2} e^{-2i\varphi^{opt}} \\ \sin\frac{\alpha^{opt}\pi}{2} e^{2i\varphi^{opt}} & \cos\frac{\alpha^{opt}\pi}{2} \end{pmatrix}. \tag{33}$$

Substituting matrix (33) into eq.(32) we can solve it for the parameters φ^{opt} and α^{opt}:

$$\tan(\frac{\alpha^{opt}\pi}{2}) = \tag{34}$$
$$\frac{r_{N1}^2 + r_{(N-1)1}^2 - \lambda_-}{\cos(2\pi(\phi_{12} - \chi_{(N-1)1} + \chi_{(N-1)2}))r_{(N-1)1}r_{(N-1)2} + \cos(2\pi(\phi_{12} - \chi_{N1} + \chi_{N2}))r_{N1}r_{N2}},$$
$$\tan(2\varphi^{opt}\pi) = \frac{\sin(2\pi(\chi_{(N-1)1} - \chi_{(N-1)2}))r_{(N-1)1}r_{(N-1)2} + \sin(2\pi(\chi_{N1} - \chi_{N2}))r_{N1}r_{N2}}{\cos(2\pi(\chi_{(N-1)1} - \chi_{(N-1)2}))r_{(N-1)1}r_{(N-1)2} + \cos(2\pi(\chi_{N1} - \chi_{N2}))r_{N1}r_{N2}}.$$

Then formula (31) with U given by (33,34) gives us the expressions for the control parameters maximizing the probability R^2 of the excitation transfer to the nodes of the extended receiver.

3.4 Optimized local transformation of extended receiver

In Sec.3.3, we find the values of the control parameters for construction of R^{opt}. Namely, $a_0 = 0$ (or $\alpha_1 = 0$), φ_1 is arbitrary, while α_2 and φ_2 are determined by expressions (34). Now we write the explicit form of the local transformation diagonalizing the state of the extended receiver obtained for the above control parameters. Before the diagonalization, density matrix (7) reads (we mark the appropriate quantities with the superscript *opt*):

$$\rho^{opt}_{R_{ext}} \equiv \mathrm{Tr}_{1,2,\dots,N-2}\rho = \begin{pmatrix} 1-(R^{opt})^2 & 0 & 0 & 0 \\ 0 & |f^{opt}_{N-1}|^2 & f^{opt}_{N-1}(f^{opt}_N)^* & 0 \\ 0 & (f^{opt}_{N-1})^* f^{opt}_N & |f^{opt}_N|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{35}$$

It is remarkable that the central nonzero 2×2 block of the density matrix $\rho^{opt}_{R_{ext}}$ can be factorized as

$$\begin{pmatrix} |f^{opt}_{N-1}|^2 & f^{opt}_{N-1}(f^{opt}_N)^* \\ (f^{opt}_{N-1})^* f^{opt}_N & |f^{opt}_N|^2 \end{pmatrix} = ff^+. \tag{36}$$

It is clear that this block can be diagonalized by the matrix V_0 of the following form:

$$V_0 = \frac{1}{R^{opt}} \begin{pmatrix} f^{opt}_N & -f^{opt}_{N-1} \\ (f^{opt}_{N-1})^* & (f^{opt}_N)^* \end{pmatrix} \tag{37}$$

with the eigenvalue matrix

$$\Lambda_b = \mathrm{diag}(0, (R^{opt})^2), \tag{38}$$

so that we can write

$$\rho^{opt}_{R_{ext}} = V^+ \Lambda V, \tag{39}$$

where

$$V = \mathrm{diag}(1, V_0, 1), \tag{40}$$

$$\Lambda = \mathrm{diag}(1-(R^{opt})^2, \Lambda_b, 0). \tag{41}$$

Consequently, applying the unitary transformation V (40) to $\rho^{opt}_{R_{ext}}$, we obtain the diagonal density matrix

$$\tilde\rho^{opt}_{R_{ext}} = V\rho^{opt}_{R_{ext}} V^+ = \Lambda. \tag{42}$$

The transformation (40) with V_0 from (37) is the needed local unitary transformation of the extended receiver.

3.4.1 The optimized state of one-qubit receiver

To obtain the optimized state of the receiver, we reduce the density matrix (42) to the state of the last node using the basis (6). Owing to the mutual positions of the eigenvalues in the diagonal matrix (41), this state reads:

$$\rho^{opt}_R = \mathrm{diag}(1-(R^{opt})^2, (R^{opt})^2). \tag{43}$$

Thus both non-zero eigenvalues are transferred from the extended receiver to the receiver itself.

3.5 Singular value decomposition of P in terms of matrices V_0^+, U and Λ_0

It is remarkable that the matrices V_0, U and Λ_0 can be given another meaning. In fact, the central 2×2 block of eq.(42) by virtue of eqs.(37,38,40,41) yields

$$V_0 f f^+ V_0^+ = (R^{opt})^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{44}$$

On the other hand, eq.(25) by virtue of eq.(26) can be written in the form

$$f^+ f = a^+ U^+ \Lambda_0 \tilde{V} \tilde{V}^+ \Lambda_0 U a, \tag{45}$$

where $\tilde{V}$ is some unitary matrix. Now we can formally split eq. (45) into equation for f

$$f = \tilde{V}^+ \Lambda_0 U a \;\Rightarrow \tag{46}$$

$$\tilde{V} f = \Lambda_0 U a \stackrel{(31)}{=} \begin{pmatrix} 0 \\ \lambda_+ \end{pmatrix} \tag{47}$$

and its Hermitian conjugate. Multiplying eq.(47) by its Hermitian conjugation from the right we obtain

$$\tilde{V} f f^+ \tilde{V}^+ = \lambda_+^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{48}$$

Comparison of eqs. (44) and (48) prompts us to identify

$$\tilde{V} = V_0, \;\; R_{opt}^2 = \lambda_+^2. \tag{49}$$

Comparing eq.(47) with eq.(22) for f by virtue of eqs.(49) we conclude that

$$P = V_0^+ \Lambda_0 U, \tag{50}$$

i.e., the matrices V_0^+, U and Λ constructed in Secs. 3.3 and 3.4 represent the singular value decomposition of the matrix P. This fact allows us to simplify the algorithm of the numerical construction and time-optimization of the probability $(R^{opt})^2$ together with the unitary transformations V and U. This algorithm reads as follows.

1. Calculate the matrix P as a function of the time t for the given Hamiltonian governing the spin dynamics and calculate the largest singular value λ_+ of P as a function of the time t.

2. Find the time instant t_0 maximizing the largest singular value of the matrix P. This maximal singular value gives the maximized probability:
$$(R^{(max)})^2 \equiv (R^{(opt)})^2|_{t=t_0} = (\lambda_+)^2|_{t=t_0}. \tag{51}$$

3. Construct the singular decomposition of P at the time instant $t = t_0$ obtaining the matrices U^{max} (composed of the optimal control parameters) and V^{max} (the optimized unitary transformation of the extended receiver):
$$V^{max} = V|_{t=t_0}, \quad U^{max} = U|_{t=t_0}. \tag{52}$$

Especially important in the above algorithm is the time-optimization of the probability $(R^{opt})^2$ in no.2 which is given the special consideration in the next subsection.

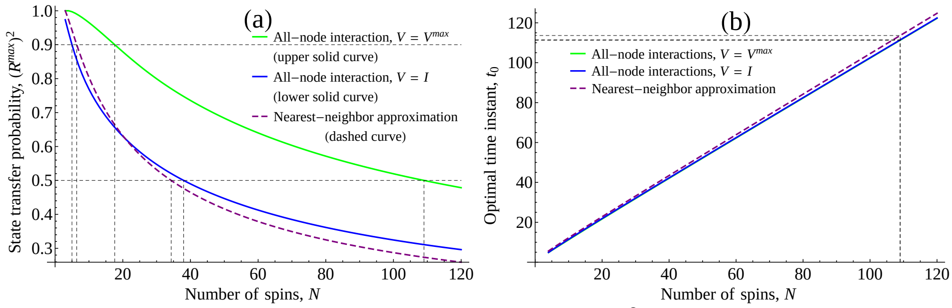

Fig. 2. The maximal excitation transfer probability $(R^{max})^2$ (Fig.(a)) and the corresponding time instant t_0 (Fig.(b)) as functions of the chain length N in different models based on the homogeneous spin-1/2 chain with XY-Hamiltonian: nearest neighbor approximation (the dash-line), all-node interactions without the local transformation of the extended receiver (the lower solid line in Fig.(a)), all-node interactions involving the optimal local transformation of the extended receiver (the upper solid line in Fig.(a)). Both solid lines almost coincide in Fig.(b), although the first one is a little below the second one. In all cases, t_0 is essentially a linear function of N. The upper and lower horizontal dot-lines in Fig. (a) indicate, respectively, the lower limit of the high-probability state transfer ($(R^{max})^2 = 0.9$) and the minimal value of $(R^{max})^2 = \frac{1}{2}$ providing the creation of any eigenvalue ($\frac{1}{2} \le \lambda_{max} \le 1$) in the receiver's state. The values $(R^{max})^2$ and associated values t_0 corresponding to the critical length $N_c = 109$ are shown for all three models.

3.5.1 Time-maximization of probability $(R^{opt})^2$

In this subsection we give some remarks regarding the maximization of the probability $(R^{opt})^2$ as a function of time. The probability $(R^{opt})^2$ is an oscillating function of the time t with the well defined first maximum [36]. The value of this maximum $(R^{max})^2$ together with the corresponding time instant t_0 as functions of the chain length N are shown in Fig.2. For comparison, $(R^{max})^2$ as function of N for the state creation without the local transformation of the extended receiver is shown for both nearest-neighbor approximation (the dash-line, $(R^{max})^2 = |f_N|^2$ in this case) and all-node interaction (the lower solid line). We see that the high probability state transfer ($(R^{max})^2 \ge 0.9$) is possible if $N \le 6$, $N \le 4$ and $N \le 17$ in the models, respectively, with nearest-neighbor interactions, with all-node interactions without the optimized unitary transformation V and involving this transformation.

Note that the transfer probability $(R^{max})^2$ is a smooth function of the time in the neighborhood of the maximum t_0 and small deviation of the time instant from t_0 does not significantly reduce the value of $(R^{max})^2$. Thus, for the chain of $N = 109$ with all node interactions we have $t_0 = 111.288$ and the probability $(R^{max})^2$ takes more then 99% of its maximal value over the time interval $110.856 < t < 111.712$. Similarly, for the chain of $N = 34$ with nearest neighbor interactions we have $t_0 = 37.279$ and $(R^{max})^2$ takes more then 99% of its maximal value over the time interval $36.7992 < t < 37.7769$. In other words, we have a high accuracy over the time interval ~ 1 (dimensionless units) in both cases.

Another parameter indicated in Fig.2 is the critical length N_c (corresponding to $R_c^2 = \frac{1}{2}$) such that any eigenvalue can be created in the receiver if $N \le N_c$. We see that $N_c = 34$, 37 and 109 in the models, respectively, with nearest-neighbor interactions, with all-node interactions without the optimized unitary transformation V and involving this transformation.

3.6 The algorithm of remote state creation. Analysis of creatable region

As the main result of this section we formulate the complete algorithm of the remote state creation.

1. Construct the optimized unitary transformations U^{max} and V^{max} and find the optimal time instant t_0 using the algorithm in Sec.3.5.

2. Create the initial state (3) of the whole chain (i.e., the one-excitation pure state of the sender and the ground state of the rest of the chain).

3. Apply the unitary transformation U^{max} (52) to the sender.

4. Switch on the evolution of the spin chain.

5. Apply the local unitary transformation V^{max} to the extended receiver at the time instant t_0.

6. Determine the state of the receiver at the time instant $t = t_0$ as the trace of the whole density matrix over the all nodes except the receiver's node. The resulting density matrix reads as follows (we use the basis $|0\rangle$, $|N\rangle$):

$$\rho_R = \mathrm{Tr}_{N-1}\left[V^{max}\rho_{R_{ext}}(V^{max})^+\right] = \left(\begin{array}{cc} 1-|z|^2 & f_0 z \\ f_0 z^* & |z|^2 \end{array}\right), \tag{53}$$

$$z = \frac{1}{R^{max}}(f_N^* f_N^{max} + f_{N-1}^* f_{N-1}^{max}) = R_z e^{2i\Phi_z\pi}. \tag{54}$$

This matrix coincides with ρ_R^{opt} (43) if we use optimized initial state (31) with $f_0 = 0$ at the step no.2. The function z in eq.(53) is nothing but the transition amplitude to the last node (compare with ref. [36]) after the evolution followed by the local optimal transformation V^{max} (don't mix z with f_N!). We see that the probability of the excitation transfer to the last node $|z|^2$ reaches its maximal value $|z_{max}|^2 = (R^{max})^2 = |f_{N-1}^{max}|^2 + |f_N^{max}|^2$ for the optimal initial state (31). The latter statement is the consequence of the optimizing transformation V^{max}. Without this transformation, we would have just $|z_{max}|^2 = |f_N^{max}|^2$. Thus, again, the probability of the excitation transfer to the receiver of the communication line is the parameter responsible for the area of the creatable region in the state-space of the receiver [36]. We emphasize that our model allows us to increase the length of the high probability ($\geq 90\%$) state transfer through the homogeneous spin chain with all node interactions from $N = 4$ to $N = 17$, see Fig.2.

Analyzing the creatable region we follow ref.[36] and use the eigenvalue-eigenvector parametrization of the receiver state:

$$\rho^B = U^B \Lambda^B (U^B)^+, \tag{55}$$

where Λ^B is the diagonal matrix of eigenvalues and U^B is the matrix of eigenvectors:

$$\Lambda^B = \mathrm{diag}(\lambda, 1-\lambda), \tag{56}$$

$$U^B = \left(\begin{array}{cc} \cos\frac{\beta_1\pi}{2} & -e^{-2i\beta_2\pi}\sin\frac{\beta_1\pi}{2} \\ e^{2i\beta_2\pi}\sin\frac{\beta_1\pi}{2} & \cos\frac{\beta_1\pi}{2} \end{array}\right). \tag{57}$$

In the ideal case, varying λ and β_i $(i = 1, 2)$ inside of the intervals

$$\frac{1}{2} \le \lambda \le 1, \tag{58}$$

$$0 \le \beta_i \le 1, \quad i = 1, 2, \tag{59}$$

we can create the whole state-space of the receiver. However, the parameters λ and β_i are not arbitrary because they depend on the control parameters via the functions R_0, R_z and Φ_z in accordance with the formulas [36]:

$$\lambda = \frac{1}{2}\left(1 + \sqrt{(1-2R_z^2)^2 + 4R_z^2 R_0^2}\right), \tag{60}$$

$$\cos\beta_1\pi = \frac{1-2R_z^2}{\sqrt{(1-2R_z^2)^2 + 4R_z^2 R_0^2}}, \quad \Rightarrow \tag{61}$$

$$\beta_1\pi = \arccos\frac{1-2R_z^2}{\sqrt{(1-2R_z^2)^2 + 4R_z^2 R_0^2}}, \tag{62}$$

$$\beta_2 = \Phi_z. \tag{63}$$

As a result, the variation intervals of the creatable parameters λ and β_1 become restricted, so that the creatable region does not cover the whole state space of the receiver. On the contrary, any value of β_2 can be constructed by the proper choice of the phases φ_i, $i = 1, 2$ in the initial state (3) [36]. This conclusion follows from the explicit expression for z in (54). Therefore, below we consider the simplified map

$$(\alpha_1, \alpha_2) \to (\lambda, \beta_1). \tag{64}$$

4 Numerical simulations

Now we apply the algorithm proposed in Sec.3.6 to the numerical study of map (64) in the case of spin chain having the critical length $N_c = 109$. In Fig.3, we collect the results of such simulations for the different models shown in Fig.2: the model with all-node interactions involving the optimized local unitary transformation of the extended receiver ($V = V^{max}$), Fig.3a; the model with all-node interactions without the optimized local transformation of the extended receiver (V equals the identity matrix I), Fig.3b; the model with nearest neighbor interactions, Fig.3c. We see that using the all-node interaction without optimized local transformation we can only slightly extend the creatable region (compare Figs. 3b and 3c), while the optimized transformation V^{max} allows us to significantly extend it, see Fig.3a. Results of our numerical simulations confirm the theoretical predictions of Sec.3 regarding the extension of the creatable region.

5 Conclusions and discussion

Among the spin-1/2 chains, the homogeneous ones are most simply reproducible because they do not require the special engineering of the coupling constants among the nodes. This is an important advantage of these chains in comparison with the chains engineered for the either PST [13, 15] or HPST [16, 17, 22, 23, 24], which require special efforts for the proper adjustment of the either all coupling constants (PST) or boundary coupling constants (HPST).

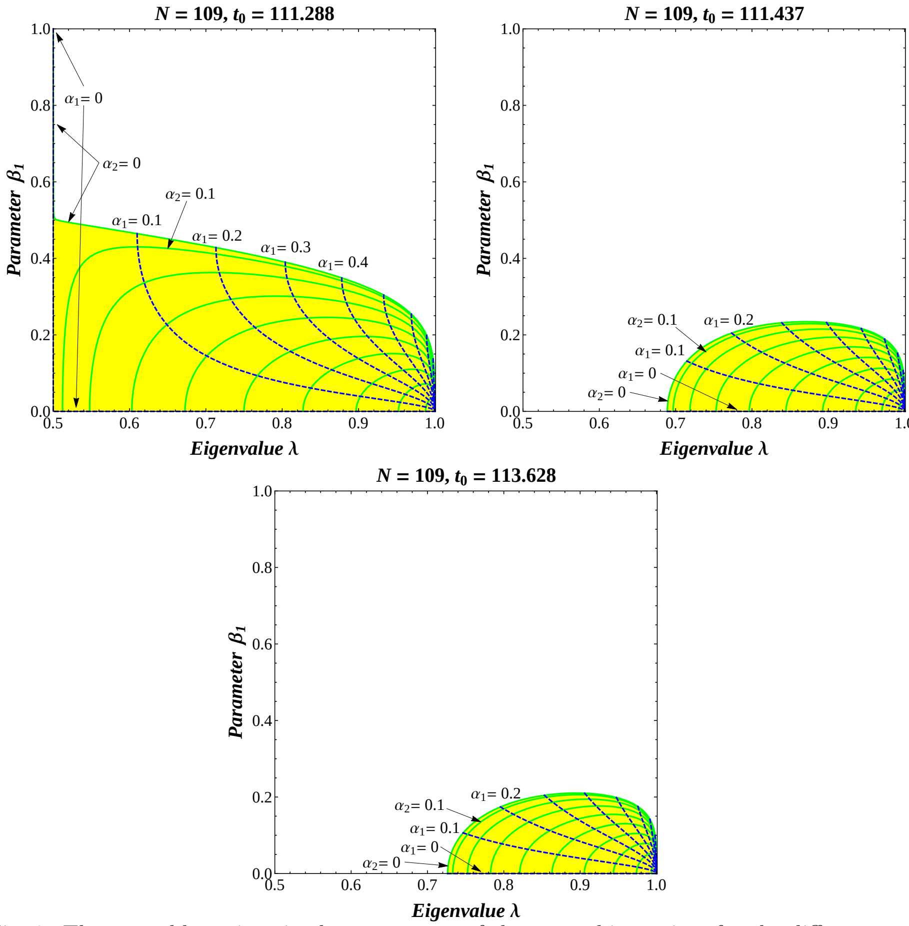

Fig. 3. The creatable regions in the state-space of the one-qubit receiver for the different models based on the homogeneous spin-1/2 chain and XY-Hamiltonian. (a) The model with all-node interactions and the optimal local transformation V^{max} of the extended receiver; (b) The model with all-node interactions without the local transformation V^{max} of the extended receiver; (c) The model with nearest neighbor interactions. Solid- and dash-lines correspond to $\alpha_1 = const$ and $\alpha_2 = const$ respectively; the interval between the neighboring lines is 0.1 in both families of curves.

However, unlike the above non-homogeneous chains, the long homogeneous chains (been used in communication lines) yield poor characteristics of both state transfer [12] and remote state creation [36] yielding, respectively, a significant decrease of the state-transfer fidelity and an area of creatable region with an increase in chain length. Therefore, the method improving these characteristics of homogeneous spin chains without changing the coupling constants is a problem of a special interest.

In this paper we show that an effective method of increasing both distance of the high probability state transfer and creatable region is the specially constructed local unitary transformation applied at the receiver side of the chain. This local transformation must involve not only the nodes of the receiver itself, but also some nodes from the close neighborhood. In our case of the one-node receiver we involve only the one additional node. This node and the one-node receiver form the so-called two-node extended receiver. As a result, we increase the distance of the high probability ($\geq 90\%$) state transfer from $N = 4$ to $N = 17$ (for the model with all node interactions). The length of the chain allowing us to create any eigenvalue of the receiver is increased from $N_c = 37$ ($N_c = 34$ for the case of nearest neighbor interactions, see [36]) to $N_c = 109$, as shown in Fig.2. As a consequence, the creatable region is also extended.

We emphasize that the local unitary transformations of the two-qubit extended receiver do not essentially increase the creatable region in the case of nearest-neighbor approximation. These transformations become useful if the spin dynamics is governed by the Hamiltonian taking into account interactions among all nodes, which, obviously, is more natural in the case of dipole-dipole interactions. Thus, we have an example demonstrating the advantage of all-node interaction in comparison with nearest-neighbor approximation. Remember that all-node interaction makes obstacle for the PST.

The algorithm constructing the optimized unitary transformation of the extended receiver V^{max} is described in Sec.3 in detail. Doing this we also obtain the initial sender's state (a^{opt} in eq.(31) with $U = U^{max}$) maximizing the excitation transfer probability. It is this probability that is responsible for the area of creatable region. It is remarkable that the optimized unitary transformation (i.e., transformation maximizing the creatable region) of the extended receiver V^{max} can be constructed in terms of the transition amplitudes p_{ij} ($i = 1, 2$, $j = N-1, N$) between the nodes of the sender and extended receiver (amplitudes p_{ij} are inherent characteristics of the communication line). More exactly, this transformation can be obtained from the singular value decomposition of the matrix P (23) whose elements are the above probabilities. This observation is very useful for numerical simulation of the remote state creation. Results of such simulations are shown in Fig.3.

Summarizing our results we conclude that, in spite of the poor state-transfer and state-creation characteristics of the homogeneous spin chain itself, there are methods significantly improving them yielding the optimized homogeneous chain. The further development of such methods is a problem of practical importance because homogeneous chains are much simpler for implementation than non-homogeneous ones. Although the photons remain privileged information carriers in long-distance communication lines, the optimized homogeneous spin chains can be widely applied in compact quantum devises where they can serve as transmission lines transferring the information among different modules.

There is a natural question whether the larger extended receiver improves the results. Our study shows that this question is not trivial. For instance, the three-node extended receiver yields the minor improvement: $N_c = 110$, i.e., the critical length increases by one in comparison with the two-node extended receiver. But the four-node extended receiver yields $N_c = 191$, this is a valuable increase. We shall also remark that the spin chain governed by the XY Hamiltonian with the nearest neighbor interactions yields significantly different result: $N_c = 34$ for the communication line without the extended receiver, the extended receiver of two nodes doesn't change it, but the extended receiver of three nodes increases it up to $N_c = 77$, and the extended receiver of four nodes yields the same value. The effect of the extended receiver's dimension on the length of both HPST and effective state creation will be considered in a different paper.

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