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IMPROVING THE QUALITY OF NOISY SPATIAL QUANTUM CHANNELS

NING TANG, ZI-LONG FAN, HAO-SHENG ZENG*

Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Department of Physics, Hunan Normal University, Changsha 410081, China

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We show, for the non-Markovian or time-dependent Markovian model of noise, by breaking the noisy spatial quantum channel (SQC) into a series of periodically arranged subcomponents, that the quality of information transmission described by the purity, fidelity and concurrence of the output states can be improved. The physical mechanism and possible implementation of the idea have been discussed.

Keywords: Spatial quantum channels, Non-Markovian process, time-dependent Markovian process
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1. Introduction

Storage and transmission of quantum information are the foundation for quantum information processing. The former processes correspond to time quantum channels (TQCs), while the later one are fulfilled through SQCs. Due to the inevitable interaction of quantum systems with their environments, both TQCs and SQCs are noisy. Therefore, improving the quality of quantum information storage or transmission via noisy quantum channels becomes the vital problem in quantum information processing. For noisy TQCs, the stored quantum information (or quantum memory) touches or interacts with the same environment, which thus can be described by means of the normal open-quantum-system dynamics [1].

SQCs are the pipelines for quantum information transmission which are useful in many tasks of quantum communication, such as quantum teleportation [2], quantum cryptography [3] and distant entanglement distribution [4, 5]. During the transmission, the quantum entity that carries quantum information at different times locates different positions of the pipeline and thus possibly interacts with different environments. These environments could be independent or correlated of each other, depending on the property of the pipeline. Thus the transmission of quantum information through a noisy SQC should be described by a series of successively-proceeding open quantum dynamics, each with correlated or independent environments. This makes the treatment of noisy SQCs much more sophisticated than that of noisy TQCs. In a special situation where the pipeline is made up of a series of independent

*Corresponding author email: hszeng@hunnu.edu.cn

segments, each regarded as an independent local environment, then the process of information transmission over the whole SQC can be modeled by a series of successive but independent open dynamics. This special kind of noisy SQCs Make up actually the foundation of our study.

The evolution of open quantum systems may be distinguished into two basic types: Markovian and non-Markovian processes. For the Markovian process, the correlation time between the system and environment is considered to be infinitesimally small so that the dynamical map does not carry any memory effects, leading to the monotonic flow of information [6, 7] or the divisible dynamics [8] of the open system. For the non-Markovian process, the memory effect will give rise to, besides the backflow of information [6, 7], many new dynamical traits such as the non-monotonic reduction of entanglement [8] or correlation [9], the correlation quantum beat [10], the entanglement trapping [11], and the phenomenon of impossible thermalization [12], etc. Furthermore, the non-Markovianity may be used as useful resources in the context of quantum metrology [13], quantum key distribution [14], quantum teleportation [15], and improving the quantum capacity [16].

In the dynamical description of open quantum systems, the time-local master equation, $\dot{\rho}(t) = \mathcal{L}(t)\rho(t)$ with $\mathcal{L}(t)$ the time-dependent generator, is of very important. In fact, any reversible quantum map (including memory-kernel quantum dynamics [17, 18]) can always be cast into this form [19]. This is indeed the straightforward result of the time-convolutedness method [1, 20]. If the conditions of Hermiticity and trace-preserving are further imposed on the generator, then the time-local master equation is of the most general form,

$$\frac{\mathrm{d}\rho(\mathrm{t})}{\mathrm{d}t} = -\mathrm{i}[H(t), \rho(t)] + \sum_k \gamma_k(t) \left[V_k(t)\rho(t)V_k^+(t) - \frac{1}{2}\{V_k^+(t)V_k(t), \rho(t)\} \right].$$

When the Hamiltonian $H(t)$, the Lindblad operators $V_k(t)$ and the decay rates $\gamma_k(t)$ all become time-independent [i.e.,$\mathcal{L}(t)$ time-independent], the equation reduces to the well-known Lindblad master equation [21, 22] which describes the so-called (time-independent) quantum Markovian processes. For time-dependent $\mathcal{L}(t)$ but with $\gamma_k(t) \geq 0$ for every k and t, the dynamics is still divisible and thus is called time-dependent Markovian dynamics [23]. However when $\gamma_k(t)$ appears temporary negative values, the dynamics become indivisible [24] and are thus called non-Markovian processes.

In this paper, we call the noises that lead to above three kinds of different dynamics the time-independent Markovian, time-dependent Markovian and non-Markovian noises respectively. We will show that for the non-Markovian or time-dependent Markovian noises, making an integral noisy SQC into a series of segments can help to preserve the quantum information under transmission. We will take the very typical noise models–dephasing and amplitude damping noises–as exemplary examples to demonstrate our results.

The paper is organized as follows. In Sec.2, we present the model of the toy noisy SQC and the physical quantities for describing the quality of the noisy SQC. In Sec.3 and Sec.4, we study respectively the properties of phase-damping and amplitude-damping SQCs. And in Sec.5, we present the illustration of the physical mechanism for improving the quality of noisy SQCs. The conclusion is arranged in Sec.6.

2. Model for a noisy SQC

We begin with our investigation by engineering a toy noisy SQC which is formed by a series of periodically arranged cavities (Fig.1). A two-level atom (qubit) passes from the left through all the cavities. Assuming in each cavity the atom stays a time Δt and the time between cavities can be ignored, thus the total time the atom spends over the SQC is $T = n\Delta t$. Assume that the properties of each cavity are completely same, so that in each cavity the atom evolves via an identical Hamiltonian H. According to the operator-sum representation of quantum operations [25], the evolution for the atom in each cavity can be described by the map

$$\varepsilon(\rho) = \sum_j K_j(t)\rho K_j^+(t), \tag{1}$$

where the Kraus operators $K_j(t)$ fulfill the trace-preserving condition $\sum_j K_j^+(t)K_j(t) = I$. After over the whole SQC, the atom state is thus given by

$$\rho(T) = \varepsilon_n\Big(\varepsilon_{n-1}\Big(\cdots\varepsilon_1\big(\rho(0)\big)\cdots\Big)\Big), \tag{2}$$

which means that the atom evolves from the initial state $\rho(0)$, with the output state of the former map as the input of the later one. The action time of each map is Δt.

In this paper, we describe the properties of the noisy SQC by the following quantities. As everyone knows, for a two-level system, the non-diagonal element ρ_{01} of the density matrix denotes the quantum coherence of the given quantum state. The non-diagonal element reaches its maximum for the pure quantum superposition state, and reduces after decoherence. Thus we define, by the value of the non-diagonal element of the density matrix of the atomic output state,

$$P = |\rho_{01}(T)|, \tag{3}$$

the purity of the atomic output state, which describes the coherence-preserving ability of the atomic states after over the noisy SQC. The second quantity is the fidelity defined by the overlap of the output state $\rho(T)$ with the input state $\rho(0)$

$$F = tr\sqrt{\sqrt{\rho(0)}\rho(T)\sqrt{\rho(0)}}, \tag{4}$$

which describes the state-preserving ability for transmitting quantum states through the noisy SQC. The last quantity used in this paper is the output entanglement which describes the entanglement-preserving ability for entanglement distribution through the noisy SQC. Suppose we have a two-qubit locally entangled state $\rho_{12}(0)$ and wish to establish remote entanglement from it, we thus let one of the qubit transmit to the remote receiver via the SQC. After this, a nonlocal entanglement state $\rho_{12}(T)$ between the two qubits is formed, whose entanglement may be described by the well-known concurrence [26]

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}. \tag{5}$$

Here $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the square roots of the eigenvalues of the matrix $R = \rho_{12}(T)\widetilde{\rho}_{12}(T)$, with $\widetilde{\rho}_{12}(T) = \sigma_y^{(1)} \otimes \sigma_y^{(2)} \rho_{12}^*(T) \sigma_y^{(1)} \otimes \sigma_y^{(2)}$ and the sign "$*$" standing for complex conjugate. The larger the concurrence is, the better the ability for the preservation of entanglement will be.

At the end of this section, we would like to make some explanations for the toy SQC depicted in Fig.1. The two-level atom in the model is just a substitution of a qubit which may be any other two-state system in practice. Correspondingly, the SQC may then be constituted by a series of segmental "pipelines" that guide the qubit motion. Of course, the gaps between neighboring "pipelines" must be very less so that the time the qubit through them can be neglected.

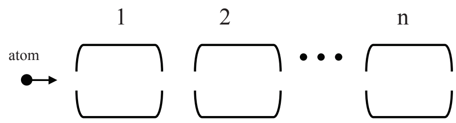

Fig. 1. A toy noisy SQC formed by n cavities is passed by a two-level atom. Denoted by Δt the time the atom stays in each cavity, the total time over the whole SQC is thus $T = n\Delta t$.

3. Dephasing SQC

Now let us proceed in detail with the case of dephasing SQC. The Hamiltonian for the interaction of the two-level atom with each cavity is given by

$$H = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^+ b_k + \sum_k \sigma_z(\lambda_k b_k^+ + \lambda_k^* b_k). \tag{6}$$

Where ω_0 and σ_z are respectively the transition frequency and Pauli operator of the atom, and ω_k, b_k are respectively the frequency and annihilation operator for the k-th harmonic oscillator of the reservoir. The coupling strength λ_k is assumed to be complex in general. This dephasing model, which is extensively used to simulate the decoherence of a qubit coupled to its environment in quantum information science, can be solved exactly. For the initial product states of the qubit plus its environment, the evolution of the reduced density matrix of the qubit, in the basis $\{|0\rangle, |1\rangle\}$, may be written as [1]

$$\rho(t) = \begin{pmatrix} \rho_{00} & \rho_{01}e^{-\Gamma(t)} \\ \rho_{10}e^{-\Gamma(t)} & \rho_{11} \end{pmatrix}. \tag{7}$$

Where the decoherence function is defined by

$$\Gamma(t) = -\sum_k \ln\langle \exp[\alpha_k b_k^+ - \alpha_k^* b_k]\rangle, \tag{8}$$

with $\alpha_k = 2\lambda_k(1 - e^{-i\omega_k t})/\omega_k$, and the angular brackets denoting the expectation value with respect to the bath state. The evolution of Eq.(7) satisfies master equation

$$\frac{d\rho}{dt} = \frac{1}{2}\gamma_p(t)[\sigma_z\rho\sigma_z - \rho], \tag{9}$$

with dephasing rate $\gamma_p(t) = \dot{\Gamma}(t)$. In the operator-sum representation, one gets two Kraus operators

$$K_1(t) = \sqrt{\frac{1+e^{-\Gamma(t)}}{2}} I, K_2(t) = \sqrt{\frac{1-e^{-\Gamma(t)}}{2}} \sigma_z. \tag{10}$$

Now for the case under consideration, the atom passes over the whole SQC which includes n completely identical cavities, thus the output state of the atom can be evaluated from Eq.(2) as

$$\rho(T) = \begin{pmatrix} \rho_{00} & \rho_{01} e^{-T\frac{\Gamma(\Delta t)}{\Delta t}} \\ \rho_{10} e^{-T\frac{\Gamma(\Delta t)}{\Delta t}} & \rho_{11} \end{pmatrix}. \tag{11}$$

It shows that the coherence decays exponentially with the total time T, with the decay rate given by $R(\Delta t) \equiv \Gamma(\Delta t)/\Delta t$. In the following, we will show through exemplary examples that $R(\Delta t)$ may be reduced by decreasing Δt. For this dephasing SQC, the purity and the output-to-input fidelity are

$$P = |\rho_{01}| e^{-T\frac{\Gamma(\Delta t)}{\Delta t}}, \tag{12}$$

and

$$\begin{aligned} F &= \rho_{00}^2 + \rho_{11}^2 + 2\rho_{01}\rho_{10} e^{-T\frac{\Gamma(\Delta t)}{\Delta t}} \\ &+ 2\sqrt{(\det\rho)[\rho_{00}\rho_{11} - \rho_{01}\rho_{10} e^{-T\frac{\Gamma(\Delta t)}{\Delta t}}]}, \end{aligned} \tag{13}$$

respectively, where $\det\rho = \rho_{00}\rho_{11} - \rho_{01}\rho_{10}$ is the determinant of ρ. In order to discuss the entanglement-preserving ability of the SQC, we assume the initial local entanglement state of the two qubits is an arbitrary pure state

$$|\Psi(0)\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle, \tag{14}$$

with the complex superposition coefficients satisfying normalization $|a|^2+|b|^2+|c|^2+|d|^2 = 1$. Note that this pure state has concurrence $C(0) = 2|ad - bc|$. After the first qubit transmits through the SQC to another receiver, a nonlocal entanglement state

$$\rho_\Psi(T) = \begin{pmatrix} |a|^2 & ab^* & ac^*E(T) & ad^*E(T) \\ a^*b & |b|^2 & bc^*E(T) & bd^*E(T) \\ a^*cE(T) & b^*cE(T) & |c|^2 & cd^* \\ a^*dE(T) & b^*dE(T) & c^*d & |d|^2 \end{pmatrix} \tag{15}$$

is obtained, where $E(T) = e^{-T\frac{\Gamma(\Delta t)}{\Delta t}}$. This nonlocal entanglement state has a concurrence

$$C = 2|ad - bc| e^{-T\frac{\Gamma(\Delta t)}{\Delta t}}. \tag{16}$$

Eqs.(12) and (16) show that both the purity and the concurrence take on simple role of exponentially decay. The normalized quantities, $P(T)/P(0)$ and $C(T)/C(0)$, are same and independent of the initial states. While the fidelity of Eq.(13) has a complicated rule of decay which depends on the initial states of the atom.

The decoherence function $\Gamma(t)$ of Eq. (8) depends on the properties of environments. In the following, we will consider two kinds of commonly encountered baths: thermal bath and squeezed vacuum bath. For the thermal bath, the decoherence function becomes [1]

$$\Gamma(t) = \int_0^\infty d\omega J(\omega) \coth(\frac{\omega}{2k_B T_B}) \frac{1-\cos(\omega t)}{\omega^2}, \tag{17}$$

with k_B the Boltzmann constant and T_B the bath temperature. The spectral density is defined as $J(\omega) = \sum_k 4|\lambda_k|^2\delta(\omega - \omega_k)$. The corresponding decay rate can be written as

$$\gamma_p^t(t) = \int_0^\infty d\omega J(\omega)\coth(\frac{\omega}{2k_BT_B})\frac{\sin(\omega t)}{\omega}. \tag{18}$$

For the squeezed vacuum bath $\rho_B = |\phi\rangle\langle\phi|$ with $|\phi\rangle = \prod_k S_k(\xi_k)|0\rangle$ and unitary squeeze operator $S_k(\xi_k) = \exp[\frac{1}{2}\xi_k^* b_k^2 - \frac{1}{2}\xi_k b_k^{+2}]$, the decoherence function becomes

$$\begin{aligned}\Gamma(t) &= \int_0^\infty d\omega J(\omega)\frac{1-\cos(\omega t)}{\omega^2} \\ &\times \{\cosh[2r(\omega)] - \sinh[2r(\omega)]\cos[\omega t - \theta(\omega)]\},\end{aligned} \tag{19}$$

where $r(\omega)$ and $\theta(\omega)$ are respectively the amplitude and argument of the squeeze parameter ξ_k. The corresponding decay rate in this case is

$$\gamma_p^s(t) = \int_0^\infty \frac{d\omega}{\omega}J(\omega)\left\{\cosh[2r(\omega)]\sin(\omega t) + 2\sinh[2r(\omega)]\sin(\frac{\omega t}{2})\cos(\frac{3}{2}\omega t - \theta)\right\}. \tag{20}$$

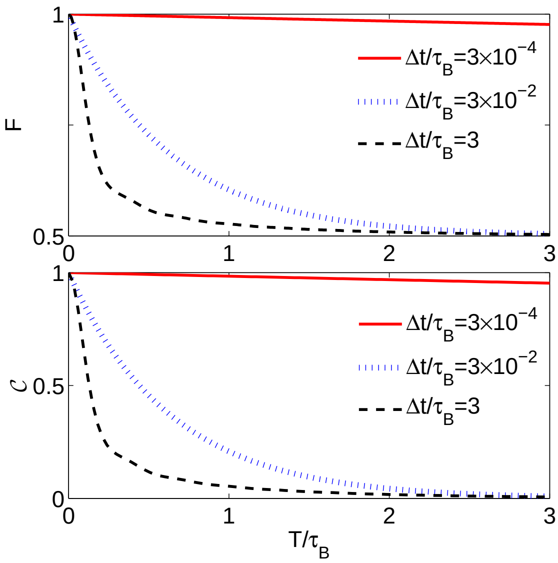

Fig. 2. (Color online) Evolutions of the fidelity and the normalized concurrence versus the dimensionless time T/τ_B for thermal bath and Ohmic spectrum $J(\omega) = \omega e^{-\omega/\Omega}$ with $\Omega\tau_B = 20$. The fidelity is calculated under the initial state $|\varphi(0)\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ of the atom.

In Fig.2, we show, for the thermal bath and Ohmic spectrum $J(\omega) = \omega e^{-\omega/\Omega}$, the evolutions of the fidelity and the normalized concurrence versus total time T for three kinds of Δt. Where the dimensionless time is realized through the thermal correlation time $\tau_B = 1/k_BT_B$ and we take $\Omega\tau_B = 20$ in the figure. The evolution of normalized purity is the same as that of the normalized concurrence and thus not shown. In the calculation of fidelity, we take the initial state of the atom to be $|\varphi(0)\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. We see, for given Δt, that both the fidelity and concurrence decrease with the total time T, which agree with our intuitive senses. However, when the total time T is fixed, the fidelity and concurrence increase with the decreasing of Δt. In the limit of $\Delta t \to 0$, we have from Eq.(17) that the decay rate $\Gamma(\Delta t)/\Delta t \to 0$, so that the fidelity and the normalized concurrence approach to unity. The output states of Eqs.(11) and (15) also return respectively to their initial states.

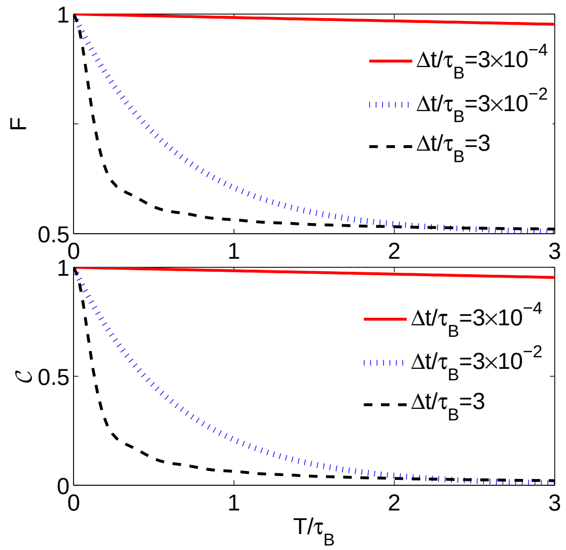

Fig. 3. (Color online) Evolutions of the fidelity and the normalized concurrence versus the dimensionless time T/τ_B for squeezed vacuum bath and Ohmic spectrum $J(\omega) = \omega e^{-\omega/\Omega}$ with $\Omega\tau_B = 20$. The fidelity is calculated under the initial state $|\varphi(0)\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ of the atom.

Fig.3 shows, for the squeezed vacuum bath and Ohmic spectrum $J(\omega) = \omega e^{-\omega/\Omega}$, the evolutions of the fidelity and the normalized concurrence versus total time T for three kinds of Δt. Where we also take $\Omega\tau_B = 20$ and the initial state of the atom to be $|\varphi(0)\rangle = (|0\rangle+|1\rangle)/\sqrt{2}$ for the calculation of fidelity. In addition, we take the mode-dependent squeezed amplitude to be a Gaussian distribution

$$r(\omega) = \frac{r_0}{\sqrt{2\pi}\sigma} e^{-\frac{(\omega-\omega_0)^2}{2\sigma^2}},$$

with $r_0 = 3$, $\omega_0 = 10^9\mathtt{Hz}$ and width $\sigma = 100\mathtt{Hz}$. We assume $\theta(\omega) \equiv \pi/4$ which is independent of modes. The figure also shows the similar properties as that of Fig.2. In the limit of $\Delta t \to 0$, we also have from Eq.(19) that the decay rate $\Gamma(\Delta t)/\Delta t \to 0$, so that the fidelity and the normalized concurrence approach to unity.

4. Amplitude-damping SQC

In the previous section, we discussed the situation where the noise of the SQC is the purely dephasing type. Another typical noise is the so-called dissipation,

$$H = \omega_0\sigma_z + \sum_k \omega_k b_k^+ b_k + \sum_k (g_k b_k \sigma_+ + g_k^* b_k^+ \sigma_-), \tag{21}$$

with σ_+, σ_- the ladder operators of the atom, and g_k the coupling constant of the atom with the bath mode k. All other parameters are the same as before. For the initial vacuum state of environment, this model can be solved exactly which gives the evolution of the reduced density of the atom [7],

$$\rho(t) = \begin{pmatrix} 1-|G(t)|^2\rho_{11} & G(t)\rho_{01} \\ G^*(t)\rho_{10} & |G(t)|^2\rho_{11} \end{pmatrix}, \tag{22}$$

where the function $G(t)$ fulfils the integro-differential equation

$$\dot{G}(t) = -\int_0^t dt_1 f(t-t_1)G(t_1), \tag{23}$$

with initial condition $G(0) = 1$. The two-point reservoir correlation function $f(t-t_1)$ relates to the spectral density $J(\omega)$ via the Fourier transformation $f(t-t_1) = \int d\omega J(\omega)\exp[i(\omega_0 - \omega)(t-t_1)]$ with ω_0 the transition frequency of the atom. The dynamical state of Eq.(22) satisfies the time-local master equation

$$\frac{d\rho}{dt} = -\frac{i}{2}S(t)[\sigma_+\sigma_-, \rho] + \gamma_a(t)\left[\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\right], \tag{24}$$

with

$$S(t) = -2\mathtt{Im}\left[\frac{\dot{G}(t)}{G(t)}\right], \gamma_a(t) = -2\mathtt{Re}\left[\frac{\dot{G}(t)}{G(t)}\right]. \tag{25}$$

In operator-sum representation, there are accordingly two Kraus operators,

$$K_1(t) = \begin{pmatrix} 1 & 0 \\ 0 & G^*(t) \end{pmatrix}, K_2(t) = \begin{pmatrix} 0 & \sqrt{1-|G(t)|^2} \\ 0 & 0 \end{pmatrix}. \tag{26}$$

After the atom passes over the whole SQC, the output state becomes

$$\rho(T) = \begin{pmatrix} 1-\rho_{11}|G(\Delta t)|^{\frac{2T}{\Delta t}} & \rho_{01}[G(\Delta t)]^{\frac{T}{\Delta t}} \\ \rho_{10}[G^*(\Delta t)]^{\frac{T}{\Delta t}} & \rho_{11}|G(\Delta t)|^{\frac{2T}{\Delta t}} \end{pmatrix}. \tag{27}$$

The purity and the output-to-input fidelity are thus

$$P = |\rho_{01}| \cdot |G(\Delta t)|^{\frac{T}{\Delta t}}, \tag{28}$$

and

$$\begin{aligned} F &= \rho_{00} + \rho_{11}(\rho_{11}-\rho_{00})|\Theta|^2 + 2\mathrm{Re}[\rho_{01}\rho_{10}\Theta] \\ &+ 2\sqrt{(\det\rho)[\rho_{11}|\Theta|^2(1-\rho_{11}|\Theta|^2) - \rho_{01}\rho_{10}|\Theta|^2]}, \end{aligned} \tag{29}$$

respectively, where $\Theta = [G(\Delta t)]^{T/\Delta t}$. For the entanglement distribution, we still assume the local initial state of Eq.(14) and let the first atom over the SQC, then the resultant nonlocal two-atom state becomes

$$\rho_\Psi(T) = \begin{pmatrix} |a|^2+|c|^2(1-|\Theta|^2) & ab^*+cd^*(1-|\Theta|^2) & ac^*\Theta & ad^*\Theta \\ a^*b+c^*d(1-|\Theta|^2) & |b|^2+|d|^2(1-|\Theta|^2) & bc^*\Theta & bd^*\Theta \\ a^*c\Theta^* & b^*c\Theta^* & |c|^2|\Theta|^2 & cd^*|\Theta|^2 \\ a^*d\Theta^* & b^*d\Theta^* & c^*d|\Theta|^2 & |d|^2|\Theta|^2 \end{pmatrix}, \tag{30}$$

which has the concurrence,

$$C = 2|ad - bc| \cdot |G(\Delta t)|^{\frac{T}{\Delta t}}. \tag{31}$$

For the amplitude-damping SQC, we only discuss the case where the spectral density of each cavity is Lorentzian

$$J(\omega) = \frac{\gamma_0 \lambda^2}{2\pi[(\omega_0 - \omega - \Delta)^2 + \lambda^2]}, \tag{32}$$

where $\Delta = \omega_0 - \omega_c$ is the detuning between the atomic frequency ω_0 and cavity-mode ω_c, γ_0 and λ are respectively the atomic free decay rate and the photon-leakage rate of the cavity. For this Lorentzian structured environment, the function $G(t)$ in Eq. (23) may be evaluated as

$$G(t) = e^{-(\lambda - i\Delta)t/2}\left[\cosh(\frac{\delta t}{2}) + \frac{\lambda - i\Delta}{\delta}\sinh(\frac{\delta t}{2})\right], \tag{33}$$

with $\delta = \sqrt{(\lambda - i\Delta)^2 - 2\gamma_0\lambda}$. The corresponding decay rate and Lamb shift become

$$\gamma_a(t) = \texttt{Re}\left[\frac{2\gamma_0\lambda\sinh(\delta t/2)}{\delta\cosh(\delta t/2) + (\lambda - i\Delta)\sinh(\delta t/2)}\right], \tag{34}$$

and

$$S(t) = \texttt{Im}\left[\frac{2\gamma_0\lambda\sinh(\delta t/2)}{\delta\cosh(\delta t/2) + (\lambda - i\Delta)\sinh(\delta t/2)}\right], \tag{35}$$

respectively.

In Fig.4, we show the evolutions of the fidelity and the normalized concurrence versus the total dimensionless time for the vacuum Lorentzian environment, Where $\lambda = 50\gamma_0$ and $\Delta = 20\gamma_0$. In the calculation of fidelity, we again take the initial state of the atom to be $|\varphi(0)\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. It again shows the similar behaviors as that of Fig.2. In the limit of $\Delta t \to 0$, we have from Eq.(33) that $\Theta = [G(\Delta t)]^{T/\Delta t} \to 1$, so that the fidelity and the normalized concurrence approach to unity.

5. Physical mechanism

From the above discussion about dephasing and amplitude-damping SQC model, we find that if we break a noisy SQC into a series of sub-components, the quality of information transmission would be improved. Especially in the limit of $\Delta t \to 0$, the atomic state will preserve unchanged and the quantum information can be transmitted losslessly. Now we further analyze the physical mechanism of this phenomena. In Fig.5, we plot the evolution of the decay rates of the dephasing SQC for thermal and squeezed vacuum baths, where the parameters are respectively in accordance with Figs.2 and 3. It is shown that the decay rates increase from zero and may be small if we restrict the dynamical evolution into a small time interval at the initial stage. Smaller decay rate certainly leads to slower evolution of the atomic state. By breaking an integral SQC into a series of short sub-components, the evolution in each sub-component is independent of each other and each evolution in itself starts from zero time. Therefore the effective decay rate of the integral SQC reduces and the quality of quantum information transmission gets improved. This can be seen from Eq.(11), where the effective decay rate $\Gamma(\Delta t)/\Delta t = \int_0^{\Delta t}\gamma_p(\tau)d\tau/\Delta t$ is actually the average of the decay rate $\gamma_p(t)$ in the initial interval Δt. In the limit of the length of each sub-component approaches zero,

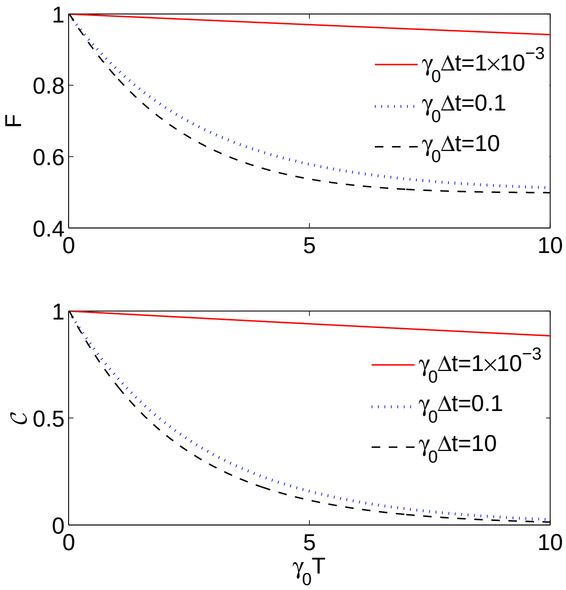

Fig. 4. (Color online) Evolutions of the fidelity and the normalized concurrence versus the dimensionless time $\gamma_0 T$ for vacuum Lorentzian environment with $\lambda = 50\gamma_0$ and $\Delta = 20\gamma_0$. The fidelity is calculated under the initial state $|\varphi(0)\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ of the atom.

the effective decay rate becomes the transient decay rate at time zero, $\gamma_p(0) = 0$. Therefore the atomic state preserves unchanged. In Fig.6, we show the evolutions of the decay rate and Lamb shift for the amplitude-damping environment, where the parameters are in accordance with Fig.4. It shows that the decay rate also increases from zero and the same justification is thus valid in this case.

The method for protecting quantum information is something like the quantum Zeno effect [27, 28]. The function of breaking the SQC into segments is to constantly interrupt the evolution and reset the initial time of the evolution in each segment to zero, which corresponds to the constant measurements in quantum Zeno effect. However, the two phenomena are different in nature: In quantum Zeno effect, the system state after each measurement returns to its initial state probably and then restarts its revolution. While in our method, the interruption never made the system state back, but continuously evolve in a (much) smaller decay rate.

It is required to be emphasized that the above result only occurs for the Non-Markovian or time-dependent Markovian model of noise, where the decay rates increase in time from zero. For the time-independent Markovian model of noise, the idea for improving the quality of quantum information transmission is obviously invalid. We can explain this in a more general way. For the time-independent Markovian model of noise, the evolution of the atomic state is governed by the standard Lindblad master equation $\dot{\rho} = \mathcal{L}\rho$ with a time-independent generator $\mathcal{L}$. Such a master equation leads to a dynamical semigroup of one-parameter, completely positive and trace-preserving maps, $\varepsilon(t) = \exp(\mathcal{L}t)$. If one breaks such a dynamical map with evolutional time T into a series of sub-maps, or equivalently breaks an integral

SQC into a series of sub-components as in Fig.1, then the time translation invariance of the semigroup dynamics would still lead to $\rho(T) = \exp(\mathcal{L}T)\rho(0)$, though each sub-map in itself is independent.

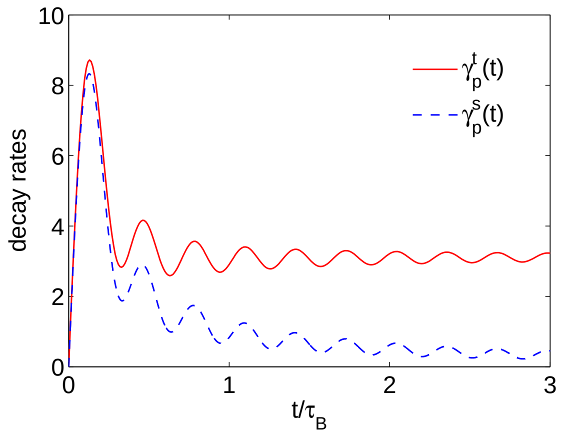

Fig. 5. (Color online) Time evolution of the decay rates for dephasing SQCs. The solid red and dash blue lines respectively correspond to thermal and squeezed vacuum baths. The parameters are respectively in accordance with Figs.2 and 3.

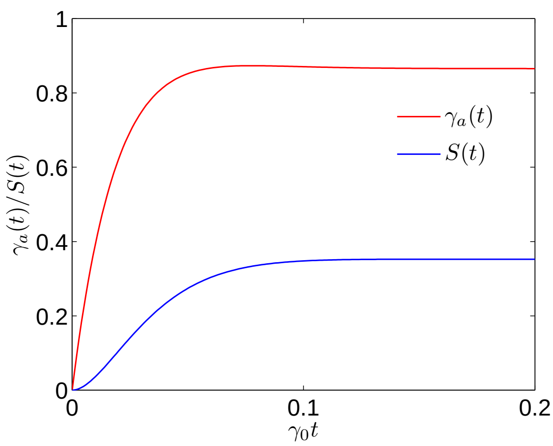

Fig. 6. (Color online) Time evolution of the decay rate and Lamb shift for amplitude-damping SQC. The parameters are same with Fig.4.

There are two optional ways in going practically to the limit of $\Delta t \to 0$. One is to make each segment as shorter as possible, the other is to enhance the speed of the atom. For the former way, an important fact must be pointed out: When each segment (cavity) becomes smaller, the mode structure of the cavity will change. For example for the Lorentzian spectrum, the characteristic frequency ω_c will increase, leading to smaller detuning Δ for given ω_0. As in the limit of $t \to 0$, Eq.(33) may be written as $|G(t)| \simeq e^{-\frac{\lambda t}{2}} \sqrt{(1 + \frac{\lambda t}{2})^2 + (\frac{\Delta}{2})^2 t^2}$, thus the speed of $|G(\Delta t)|^{\frac{T}{\Delta t}} \to 1$ would not be affected severely as long as the conditions $\lambda \cdot \Delta t \ll 1$ and $\Delta \cdot (\Delta t) \ll 1$ are met. This is to say, for the Lorentzian environment, the change of the cavity size would not obstruct severely the execution of our method. Of course, we have also some approaches to avoid the problem. For example, by adjusting the atomic frequency ω_0 to offset the change of ω_c, or by making the standing wave direction of the cavity field perpendicular to the movement of the atom, one can maintain the detuning Δ unchanged while shorten the length of cavities. For the Ohmic spectrum in the thermal bath, it is shown [1] that in the limit of $t \to 0$, Eq.(17) leads to $\Gamma(t)/t \simeq \frac{1}{2}\Omega^2 t$. If assume that the shortening of the cavity length enlarges the cutoff frequency Ω (in fact we do not know yet the detailed relation between Ω and the cavity size), then the speed for $\Gamma(\Delta t)/\Delta t \to 0$ when $\Delta t \to 0$ would slow down and the ability for the protection of quantum state would be weakened. Similar analysis applies to the decoherence function Eq.(19) of the squeezed vacuum bath.

6. Conclusion

In conclusion, we have investigated the properties of a noisy SQC constituted by a series of identical segments, each segment modeled by a local environment. For the non-Markovian or time-dependent Markovian model of noise, we found when the length of each segment, i.e., Δt, keeps fixed, that the fidelity, the purity and concurrence of the output states decrease with the increasing of the SQC length (quantified by the time T), which agree with our intuitive senses. However, for the fixed length T of the SQC, these physical quantities increase as Δt gets shorter. It implies, by breaking a noisy SQC into a series of periodically arranged sub-components, that the quality of information transmission can be improved. In the limit of $\Delta t \to 0$, the quantum Zeno-like effect may be observed and the information can be transmitted losslessly. For the time-independent Markovian model of noise, the method for improving the quality of quantum information transmission becomes invalid. We have explained the physical mechanism of these phenomena. Our result, on the one hand, presents a possible method for improving the quality of quantum information transmission via noisy quantum channels. And on the other hand, it further supplements the application of non-Markovian effect.

In the discussion, we take the dephasing and amplitude-damping noisy SQCs as the exemplary examples, because these two noisy models are encountered frequently. We guess that this phenomenon also exists in other types of noisy models.

We point out that the idea for improving information transmission quality also applies to the information storage. In fact, if all the cavities in Fig.1 are located in a small area (say in a laboratory), then the device corresponds to a process of information storage. That is to say, frequently changing storage environments is more beneficial for the storage of quantum information than staying in one cavity all the time.

Our method may be demonstrated in current experimental techniques. We specify this by considering the Lorentzian environment along with the experiment performed by Brune et al. [29]. Note that the circular Rydberg levels with principal quantum number 51 and 50 of rubidium atom have a lifetime as long as 30 ms, corresponding to decay rate $\gamma_0 = 33\mathtt{Hz}$. If we require $\gamma_0 \Delta t = 10^{-3}$ [which means a fairly large improvement in the quality of quantum information transmission, see the reference data in Fig.4], then $\Delta t = 30\mu s$. For the atomic speed of about $v = 400\mathtt{m/s}$, one has the width of each cavity [Note that the axis of the

Fabry-Perot cavity, i.e., the standing wave direction is normal to the atomic movement.], $v\cdot\Delta t = 12\mathtt{mm}$, which matches up with the standing wave waist 6 mm of the Brune's experiment. In the experiment sketched by Brune et al., the cavity frequency ω_c can be tuned continuously around the transition frequency of the two Rydberg levels (about $\omega_0 = 51\mathtt{GHz}$) to produce a detuning from 70 to 800Hz, which completely fits our assumption $\Delta = 20\gamma_0 \simeq 660\mathtt{Hz}$.

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