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IMPLEMENTATION OF A MULTIQUBIT PHASE GATE WITH ONE QUBIT SIMULTANEOUSLY CONTROLLING N QUBITS IN THE ION-TRAP SYSTEM

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In this paper, we present a scheme for implementing a multiqubit phase gate with one control qubit simultaneously controlling n target qubits in the ion-trap system. In our scheme, there is no energy exchange between the internal and external degrees of freedom in the course of operation, and the vibrational mode is only virtually excited. The system is insensitive to changes in the vibrational motion. The proposed scheme is experimentally feasible based on the present ion-trap techniques.

Keywords: quantum computation, multiqubit phase gate, ion-trap system
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1 Introduction

Quantum gates could be implemented using various types of qubits [1, 2], quantum simulators [3], and hybrids [4]. Over the past decade, quantum gates have arisen much attention, and many schemes for realizing various types of one-qubit and two-qubit quantum gates via various physical systems were proposed [5, 6, 7, 8, 9, 10, 11, 12]. One-qubit gate and two-qubit controlled-phase gate are universal in constructing a quantum computer. The implementation of quantum algorithms and quantum error-correction protocols may involve multiqubit quantum gates. However, the procedure of decomposing multiqubit gates into a sequence of one-qubit gates and two-qubit controlled-phase gates becomes complicated as the number of qubits increases [13]. A direct implementation of a multiqubit gate without decomposing it into a series of one-qubit and two-qubit logic gates is an alternative approach with high efficiency. One of the principal multiqubit operations is multiqubit phase gate, which plays a key role in quantum computation. There are two types of multiqubit phase gates. One is a multiqubit phase gate with n-control qubits acting as one target qubit. During the past few years, several methods have been developed for directly constructing the multiqubit phase gate in cavity QED [14, 15] or ion traps [16, 17]. Another one is a multiqubit phase gate with one qubit simultaneously controlling n target qubits (NTCP). More recently, Yang et al presented a proposal to implement a NTCP gate in a cavity via three-step operations [18], and then gave a protocol for implementing a multiqubit tunable phase with one qubit simultaneously controlling n target qubits in a cavity [19]. Among various physical systems, ion-trap system is a qualified candidate for realizing quantum computation due to the long-lived internal states and precise manipulation of ionic inner states. NTCP gates for the ion-trap system have not been studied till now. In this study, the three-step operations similar to those in Ref [18] were used to implement the NTCP gates in an ion-trap system. The merit of our scheme is that no requirements are needed for the quantum information transfer from the internal degree of the ions to the external degree, and the vibrational mode is only virtually excited. The system is robust against decoherence, which is of importance for experimental implementation.

The paper was organized as follows. In section 2, the model and its evolution were introduced. In section 3, we gave a scheme to implement the NTCP in the ion-trap system. Finally, we analyzed the implementation of experiment and summarized the conclusions.

2 Model and unitary evolution

We consider that N identical two-level ions, which have a ground state $|g\rangle$ and an excited state $|e\rangle$, are confined in a linear trap. Then, the N ions are driven by a laser field with a frequency of ω_0, where ω_0 is the frequency of the transition $|e\rangle \to |g\rangle$. In addition, we simultaneously excite the N ions with two lasers of frequencies $\omega_0 + v + \delta$ and $\omega_0 - v - \delta$. Herein, v is the frequency of the center-of-mass mode of the collective motion of the ions, and δ is the detuning. Suppose $v \gg \delta$, and thus we can neglect other vibrational modes. In this case, the Hamiltonian for the system is given [20, 21] by

$$\begin{aligned} H_i = \quad & va^+a + \omega_0 \sum_{j=1}^{N} \sigma_{z,j}^+ + \\ & [\Omega_1 e^{-i\Phi_1} \sum_{j=1}^{N} \sigma_j^+ e^{i\eta(a^+ + a)} e^{-i\omega_0 t} + \\ & \Omega_2 e^{-i\Phi_2} \sum_{j=1}^{N} \sigma_j^+ e^{i\eta(a^+ + a)} e^{-i(\omega_0 + v + \delta)t} + \\ & \Omega_3 e^{-i\Phi_3} \sum_{j=1}^{N} \sigma_j^+ e^{i\eta(a^+ + a)} e^{-i(\omega_0 - v - \delta)t}] + H.C., \end{aligned} \tag{1}$$

where a^+ and a are the creation and annihilation operators for the collective vibrational mode. $\sigma_j^+ = |e_j\rangle \langle g_j|$ and $\sigma_j^- = |g_j\rangle \langle e_j|$ are the spin flip operators. η is the Lamb-Dicke parameter. Ω_l and Φ_l (l=1,2,3) are the Rabi frequency and the phase of the l-th laser. Furthermore, we consider the resolved sideband regime, where the vibrational frequency v is much larger than other characteristic frequencies [22]. In this case, we discard the rapidly oscillating terms and obtain the Hamiltonian in the interaction picture,

$$
\begin{aligned}
H_i = {} & \Omega_1 e^{-i\Phi_1} e^{-\eta^2/2} \sum_{j=1}^{N} \sigma_j^+ \sum_{n=0}^{\infty} \frac{(i\eta)^{2n}}{(n!)^2} a^{+n} a^n + \\
& \Omega_2 e^{-i\Phi_2} e^{-\eta^2/2} \sum_{j=1}^{N} \sigma_j^+ \sum_{n=0}^{\infty} \frac{(i\eta)^{2n+1}}{n!(n+1)!} a^{+(n+1)} a^n e^{-i\delta t} + \\
& \Omega_3 e^{-i\Phi_3} e^{-\eta^2/2} \sum_{j=1}^{N} \sigma_j^+ \sum_{n=0}^{\infty} \frac{(i\eta)^{2n+1}}{n!(n+1)!} a^{+n} a^{n+1} e^{i\delta t}] + H.C..
\end{aligned} \tag{2}
$$

In the Lamb-Dicke regime, i.e., $\eta\sqrt{\bar{n}+1} \ll 1$, with $\bar{n}$ as the mean phonon number of the center-of-mass mode, the interaction Hamiltonian of Eq.(2) can be approximated by the expansion to the first order in η

$$H_i = H_{i1} + H_{i2}, \tag{3}$$

where

$$H_{i1} = \Omega_1 \sum_{j=1}^{N} (\sigma_j^+ e^{-i\Phi_1} + \sigma_j^- e^{i\Phi_1}), \tag{4}$$

$$
\begin{aligned}
H_{i2} = {} & i\eta\Omega_2 e^{-i\Phi_2} \sum_{j=1}^{N} \sigma_j^+ a^+ e^{-i\delta t} + \\
& i\eta\Omega_3 e^{-i\Phi_3} \sum_{j=1}^{N} \sigma_j^+ a e^{i\delta t} + H.C..
\end{aligned} \tag{5}
$$

We assume that the Rabi frequencies and the phases of the second and third lasers are equal, i.e., $\Omega_2 = \Omega_3 = \Omega$, and $\Phi_2 = \Phi_3 = \pi/2$. Then the Hamiltonian H_{i2} reduces to

$$H_{i2} = \eta\Omega \sum_{j=1}^{N} (\sigma_j^+ + \sigma_j^-)(a^+ e^{-i\delta t} + a e^{i\delta t}) \tag{6}$$

When $\delta \gg \eta\Omega$, for H_{i2}, the energy conserving transitions are $|e_j e_k n\rangle \longleftrightarrow |g_j g_k n\rangle$ and $|g_j e_k n\rangle \longleftrightarrow |e_j g_k n\rangle$, where n is the quantum number for the relevant vibrational mode of the trap. Fig.1 shows the energy conserving transition paths within a pair of ions. The transition of $|e_j e_k n\rangle \longleftrightarrow |g_j g_k n\rangle$ is mediated by the states of $|g_j e_k n \pm 1\rangle$ and $|e_j g_k n \pm 1\rangle$. The contributions of $|g_j e_k n \pm 1\rangle$ are equal to those of the states of $|e_j g_k n \pm 1\rangle$. The corresponding Rabi frequency is given by $\lambda = 2(\eta\Omega)^2/\delta$. Since the transition paths interfere destructively, there is no transfer of population in states with different vibrational excitation and thus the Rabi frequency is independent of the vibrational quantum number [20, 21]. The Rabi frequency of $|g_j e_k n\rangle \longleftrightarrow |e_j g_k n\rangle$ mediated by $|e_j e_k n \pm 1\rangle$ and $|g_j g_k n \pm 1\rangle$ is also given by $\lambda = 2(\eta\Omega)^2/\delta$.

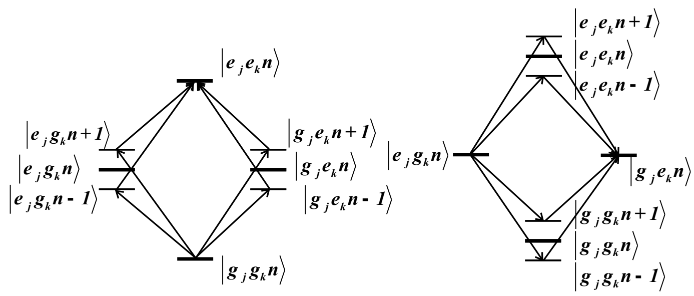

Fig. 1. Energy-level diagram for a pair of ions sharing an oscillator degree of freedom. The energy conserving transitions are from $|g_j g_k n\rangle$ to $|e_j e_k n\rangle$ (left) and from $|e_j g_k n\rangle$ to $|g_j e_k n\rangle$ (right).

Therefore, the effective Hamiltonian for H_{i2} can be described by [20, 21, 23]

$$H_{e2} = \lambda \sum_{j,k=1,j\neq k}^{N} \sigma_{x,j}\sigma_{x,k}. \tag{7}$$

Herein, $\sigma_{x,j} = \sigma_j^+ + \sigma_j^-$, the operator σ_x satisfies the eigenequation: $\sigma_{x,j}|\pm_j\rangle = \pm|\pm_j\rangle$, and $|\pm\rangle$ is defined by $|\pm\rangle = (|g\rangle \pm |e\rangle)/\sqrt{2}$. Under the conditions $\delta \gg \eta\Omega$ and $\delta \gg \Omega_1$, the evolution of the whole system is governed by the effective Hamiltonian

$$H_e = \Omega_1 \sum_{j=1}^{N} (\sigma_j^+ e^{-i\Phi_1} + \sigma_j^- e^{i\Phi_1}) + \lambda \sum_{j,k=1,j\neq k}^{N} \sigma_{x,j}\sigma_{x,k} \quad . \tag{8}$$

3 Implementation of a NTCP gate

We now show how to utilize the above model to implement a NTCP gate in the ion-trap system. The three-step operations previously presented by Yang et al [18] were used during the implementation process. Assuming (n+1) ions are confined in a linear trap. Firstly, we simultaneously excite the (n+1) ions using three lasers with frequencies ω_0, $\omega_0 + v + \delta$ and $\omega_0 - v - \delta$, respectively. The Rabi frequencies and phases of the second and third lasers are equal, i.e., $\Omega_2 = \Omega_3 = \Omega$ and $\Phi_2 = \Phi_3 = \pi/2$. The evolution of the whole system is governed by the Hamiltonian Eq.(8). When the phase Φ_1 is zero, the corresponding evolution operator $U(\tau)$ for an interaction time τ is

$$U(\tau) = \exp[-i\tau(\Omega_1 \sum_{j=1}^{n+1} \sigma_{x,j} + \lambda \sum_{j,k=1,j\neq k}^{n+1} \sigma_{x,j}\sigma_{x,k})]. \tag{9}$$

During the second stage, similarly, we simultaneously excite the ions 2, 3,..., n+1 using three lasers with frequencies ω_0, $\omega_0 + v + \delta^{'}$ and $\omega_0 - v - \delta^{'}$, respectively. The interaction is also described by the Hamiltonian Eq.(8). With the choice of $\Phi_1 = \pi$, the evolution operator of this system for an interaction time $\tau^{'}$ can be expressed as

$$U(\tau^{'}) = \exp[i\tau^{'}(\Omega_1 \sum_{j=2}^{n+1} \sigma_{x,j} - \lambda' \sum_{j,k=2,j\neq k}^{n+1} \sigma_{x,j}\sigma_{x,k})], \tag{10}$$

where $\lambda^{'} = 2(\eta\Omega)^2/\delta^{'}$. Set the detunings $\delta^{'} = -\delta$ and $\tau^{'} = \tau$, after the above two steps, the joint time-evolution operation of the whole system can be written:

$$U(2\tau) = \exp[-i\tau(\Omega_1\sigma_{x,1} + \lambda\sigma_{x,1} \sum_{j=2}^{n+1} \sigma_{x,j})]. \tag{11}$$

At last, the first ion is excited by a laser with a frequency of ω_0, the Rabi frequency of $\Omega_1^{'}$,and the phase of $\Phi_1^{'}$. And, the other ions, 2, 3,..., and (n+1) are excited by another laser with a frequency of ω_0, the Rabi frequency of Ω_r, and the phase of Φ_r. In doing this way, the interaction Hamiltonian for the qubit system and the pulses is given by

$$\begin{aligned} \widetilde{H}_i = \Omega_1^{'}(\sigma_1^{+}e^{-i\Phi_1^{'}} + \sigma_1^{-}e^{i\Phi_1^{'}}) + \\ \Omega_r \sum_{j=2}^{n+1}(\sigma_j^{+}e^{-i\Phi_r} + \sigma_j^{-}e^{i\Phi_r}). \end{aligned} \tag{12}$$

Let $\Phi_1^{'} = \Phi_r = \pi$, then the corresponding evolution operator $\widetilde{U}(\tau)$ for a duration τ is

$$\widetilde{U}(\tau) = \exp[i\tau(\Omega_1^{'}\sigma_{x,1} + \Omega_r \sum_{j=2}^{n+1} \sigma_{x,j})]. \tag{13}$$

It can be seen that after the three-step operation, the joint time-evolution operator of the qubit system is

$$\begin{aligned} U(3\tau) = \exp[i\tau(\Omega_1^{'} - \Omega_1)\sigma_{x,1} + i\tau\Omega_r \sum_{j=2}^{n+1} \sigma_{x,j} - \\ i\tau\lambda\sigma_{x,1} \sum_{j=2}^{n+1} \sigma_{x,j}]. \end{aligned} \tag{14}$$

Under the following conditions,

$$\Omega_1^{'} - \Omega_1 = n\lambda = 2n(\eta\Omega)^2/\delta, \Omega_r = \lambda = 2(\eta\Omega)^2/\delta, \tag{15}$$

Eq.(14) can be written as follows:

$$U(3\tau) = \Pi_{j=2}^{n+1} U(1, j), \tag{16}$$

where $U(1,j) = \exp[i\tau\lambda(\sigma_{x,1} + \sigma_{x,j} - \sigma_{x,1}\sigma_{x,j})]$. According to the operator $U(1,j)$, the evolution of the four computational basis states $|+_1\rangle|+_j\rangle, |+_1\rangle|-_j\rangle, |-_1\rangle|+_j\rangle$ and $|-_1\rangle|-_j\rangle$ can be obtained as follows,

$$\begin{aligned} U(1,j)|+_1\rangle|+_j\rangle &= |+_1\rangle|+_j\rangle, \\ U(1,j)|+_1\rangle|-_j\rangle &= |+_1\rangle|-_j\rangle, \\ U(1,j)|-_1\rangle|+_j\rangle &= |-_1\rangle|+_j\rangle, \\ U(1,j)|-_1\rangle|-_j\rangle &= e^{-i4\lambda\tau}|-_1\rangle|-_j\rangle. \end{aligned} \tag{17}$$

Herein, an overall phase factor $e^{i\lambda\tau}$ is omitted. Let $4\lambda\tau = \pi$, then we can get the following equation:

$$\begin{aligned} U(1,j)|+_1\rangle|+_j\rangle &= |+_1\rangle|+_j\rangle, \\ U(1,j)|+_1\rangle|-_j\rangle &= |+_1\rangle|-_j\rangle, \\ U(1,j)|-_1\rangle|+_j\rangle &= |-_1\rangle|+_j\rangle, \\ U(1,j)|-_1\rangle|-_j\rangle &= -|-_1\rangle|-_j\rangle. \end{aligned} \tag{18}$$

It shows that a two-qubit controlled phase gate $U(1,j)$ for the qubit pair (1, j) is achieved. According to Eqs.(16) and (18), we can simultaneously implement n two-qubit controlled phase gates for the qubit pairs (1, 2), (1, 3),..., and (1, n+1) in the ion-trap system. Note that each qubit pair contains the same control qubit (qubit 1) and a different target qubit (qubit 2, 3,...,or n + 1).

4 Discussion and conclusions

We address the experimental feasibility of the proposed scheme. The level configuration under our consideration can be found in $^{40}Ca^+$. One Zeeman level of the $S_{1/2}$ ground state of $^{40}Ca^+$ ions acts as the ground state, and one Zeeman level of the metastable $D_{5/2}$ state can act as the excited state [24]. The lifetime of the metastable state τ_l , is about 1.16s. Set $\Omega = 0.1v, \eta = 0.1$ and $\delta = 0.1v$, so the condition, $v \gg \delta \gg \eta\Omega$, can be satisfied. The scheme requires the condition of the Lamb-Dicke regime, i.e. $\eta\sqrt{\bar{n}+1} \ll 1$. Set $\bar{n} = 3$, then we have $\eta\sqrt{\bar{n}+1} = 0.2$. The effective Hamiltonian of Eq. (8) is valid if the mean phonon number does not exceed 3. In our protocol, the condition of the detunings, $\delta^{'} = -\delta$, can be met with via adjusting the frequencies of excited lasers. It is easy to implement experimentally. With the accessible center-of-mass mode frequency, v = 1.2MHz [25], the time of the gate operation is $3\tau = 3\pi/4\lambda = 3\pi\delta/8(\eta\Omega)^2 \approx 9.81 \times 10^{-4}$s, which is much shorter than τ_l. Thus, the loss due to the ionic spontaneous emission can be neglected. Furthermore, with the advanced development of ion-trap techniques [24, 25, 26], it is achievable to control the laser-illuminating time and locate the ions in a trap with high accuracy.

In summary, we first presented an efficient scheme for the implementation of a multiqubit phase gate via one control qubit simultaneously controlling n qubits in the ion-trap system. Our work indicates that the transfer of quantum information from the internal degrees of the ions to the external degree is not required. The vibrational mode is only virtually excited. The scheme is insensitive to both the initial vibrational state and changes in the vibrationl motion, which is of importance from the experimental point of view. Furthermore, the gate time is independent of the number n of the qubits. Therefore, the proposed scheme is realizable with current techniques and will be extremely helpful for quantum computation.

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