Quantum Information and Computation, Vol. 14, No. 1&2 (2014) 0137–0143

ONE-STEP IMPLEMENTATION OF QUANTUM CONTROLLED-PHASE GATE VIA QUANTUM ZENO DYNAMICS

WEN-AN LI
Department of Physics, School of Physics and Electronic Engineering, Guangzhou University, Guangzhou 510006, China
State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, China
liwenan@126.com

LIAN-FU WEI
State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, China
Quantum Optoelectronics Laboratory, School of Physics and Technology, Southwest Jiaotong University, Chengdu 610031, China
weilianfu@gmail.com

Received January 14, 2013
Revised June 12, 2013

We propose a scheme to implement a quantum controllable-phase gate via quantum Zeno dynamics. The two qubits are asymmetrically encoded by two four-level atoms coupled via a quantized cavity mode. Under proper conditions, the desirable logic operation can be implemented in one step. Since the qubit is encoded by the ground and the metastable states of the atom and the cavity mode is not really excited, our protocol is robust against the spontaneous decays of the atoms and cavity. Specifically, the feasibility of our generic proposal is demonstrated with two nitrogen-vacancy centers coupled to whispering-gallery microresonator.

Keywords: quantum phase gate, quantum Zeno dynamics
Communicated by: I Cirac & P Zoller

Quantum computer has attracted much interest due to its powerful performance of quantum algorithm (compared with the classical one). For example, Shor [1] has shown that the problem of factorizing a large integer can be solved in polynomial time using a quantum computer. Also, Grover [2] has proved that finding an item from a large disordered system with a quantum computer needs significantly less time (than that with a classical computer).

Generally, various quantum operations (performed usually by the usual unitary evolutions) are required to implement scalable quantum computation. It was well-known that one-qubit gates and a two-qubit gate are universal for constructing a quantum computer, i.e., any quantum operation can be achieved by choosing appropriate set of these elementary gates. Many schemes for realizing these elementary gates have been proposed with various physical systems, such as Cavity QED [3, 4], ion traps [5], and quantum dots [6], etc..

On the other hand, quantum Zeno effect is an interesting subject in quantum physics. It holds that the system remains in the initial state under frequent measurements. Recently, Facchi et al showed that the dynamics of the measured system is not necessarily freezed under the quantum Zeno effect [7, 8], and the system can evolve within the "Zeno subspace". Such an evolution refers to the so-called quantum Zeno dynamics and is also induced under the continuous measurement (or driving) [8]. Until now, a few applications based on quantum Zeno dynamics have been found for quantum state engineerings [9, 10, 11, 12, 13], e.g., the preparations of entangled states, entanglement swapping and quantum state transfers, etc..

In this paper, we investigate how to implement a two-qubit logic operation by using the quantum Zeno dynamics. Our generic model consists of two four-level atoms coupled by a quantized field, and the relevant quantum Zeno dynamics is driven by applying an external classical field to only one of the atoms. A specifical solid-state counterpart of cavity QED systems, i.e., N-V centers in diamond are coupled by a whispering-gallery mode (WGM) microresonator [14, 15, 16], is demonstrated as the feasibility of our protocol. Remarkably, this composite system combines the advantage of the N-V centers and the WGM microresonators, e.g., the long electronic spin decoherence time even at room temperature [17, 18] and the ultrahigh quality factor of the WGM microresonators [19, 20].

Compared with previous proposals [3, 4, 5, 6] for implementing the controllable-phase gate (CPG), our scheme is based on quantum Zeno dynamics and consequently possesses the following advantages: (i) The gate one intends to realize can be established in only one step operation, which will effectively reduce the complexity of experimental demonstration. (ii) Under the Zeno condition, the cavity field is virtually excited. Thus our scheme is significantly robust against the cavity decay. (iii) In our proposal quantum information is encoded in the ground and the metastable states of the atoms, therefore it is immune to the spontaneous emission of the atomic levels. (iv) In principle, the scheme can be easily generalized to other physical system, such as Cavity QED, ion traps etc..

We consider a generic cavity QED system, which can be specifically realized (see, e.g., Fig. 1) by two negatively charged N-V centers positioned near the equator of a high-Q microsphere cavity. Each N-V center includes two ground states ($|g\rangle$ and $|s\rangle$), an excited state ($|e\rangle$) and an auxiliary state ($|f\rangle$). The transition between the levels $|e_i\rangle \leftrightarrow |g_i\rangle$ $(i = 1, 2)$ is coupled to the cavity mode with the coupling constant g, and the transition $|e_1\rangle \leftrightarrow |s_1\rangle$ is driven by the classical field with the Rabi frequency Ω. In the figure, Δ represents the detuning. For the simplicity, our dynamics is limited in the asymmetrically computational subspace spanned by

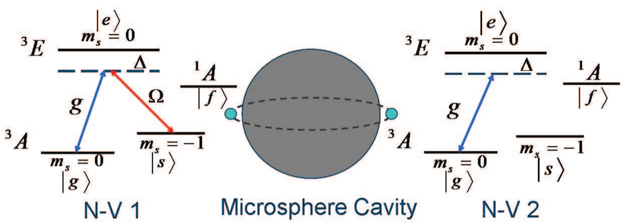

Fig. 1. Simplified cavity-QED system, generated by two N-V centers coupled to a whispering-gallery mode (WGM) microresonator. Here Ω is the external classical field coupled to the first N-V center.

$\{|f_1(=0_1)f_2(=0_2)\rangle, |f_1g_2(=1_2)\rangle, |s_1(=1_1)f_2\rangle, |s_1g_2\rangle\}$, and the two-qubit gate (we want to implement) reads

$$\begin{aligned} |0_1 0_2\rangle &\to |0_1 0_2\rangle \\ |0_1 1_2\rangle &\to |0_1 1_2\rangle \\ |1_1 0_2\rangle &\to |1_1 0_2\rangle \\ |1_1 1_2\rangle &\to e^{i\delta}|1_1 1_2\rangle \end{aligned} \tag{1}$$

with δ being the phase. Obviously, in our model the interactions only occur among the states $|s_1\rangle$, $|e_i\rangle$ and $|g_i\rangle$ $(i=1,2)$. So the auxiliary states $|f_i\rangle$ $(i=1,2)$ and $|s_2\rangle$ are not involved in the interactions throughout our scheme. As a result, the state $|f_1f_2\rangle$ and $|f_1g_2\rangle$ remain unchanged in the evolution. In what follows, we discuss how to implement the last two evolutions in Eq. (1) based on the quantum Zeno dynamics.

First, it is easily seen that, the Hamiltonian of the present driven cavity-QED system can be written as

$$H = H_1 + H_2 \tag{2}$$

with

$$H_1 = \Delta \sum_{i=1,2} |e_i\rangle\langle e_i| + (\Omega|e_1\rangle\langle s_1| + \text{h.c.}) \tag{3}$$

$$H_2 = g a^\dagger \sum_{i=1,2} |g_i\rangle\langle e_i| + \text{h.c.}. \tag{4}$$

in the interaction picture. Initially, the system is assumed to be prepared in the state $|s_1f_2\rangle|0_c\rangle$ ($|0_c\rangle$ denotes the vacuum state of the cavity field). As a consequence, it will be constrained in the subspace spanned by $\{|s_1f_2\rangle|0_c\rangle, |e_1f_2\rangle|0_c\rangle, |g_1f_2\rangle|1_c\rangle\}$. In such a subspace, we can rewrite the Hamiltonian as

$$\begin{aligned} H_1' &= \frac{\Delta}{2}(-|\varphi_1\rangle + |\varphi_2\rangle)(-\langle\varphi_1| + \langle\varphi_2|) \\ &\quad + \left[\frac{\Omega}{\sqrt{2}}|s_1f_2\rangle|0_c\rangle(-\langle\varphi_1| + \langle\varphi_2|) + \text{h.c.}\right], \end{aligned} \tag{5}$$

$$H_2' = -g|\varphi_1\rangle\langle\varphi_1| + g|\varphi_2\rangle\langle\varphi_2|, \tag{6}$$

where $|\varphi_1\rangle = (-|e_1f_2\rangle|0_c\rangle + |g_1f_2\rangle|1_c\rangle)/\sqrt{2}$ and $|\varphi_2\rangle = (|e_1f_2\rangle|0_c\rangle + |g_1f_2\rangle|1_c\rangle)/\sqrt{2}$ are the eigenvectors of H_2 corresponding to eigenvalues $-g$ and g. Under the unitary transformation $e^{iH_2't}$, we further obtain

$$\begin{aligned} H_1'' &= \frac{\Delta}{2}(|\varphi_1\rangle\langle\varphi_1| + |\varphi_2\rangle\langle\varphi_2| \\ &- |\varphi_1\rangle\langle\varphi_2|e^{-2igt} - |\varphi_2\rangle\langle\varphi_1|e^{2igt}) \\ &+ \left[\frac{\Omega}{\sqrt{2}}|s_1f_2\rangle|0_c\rangle(-\langle\varphi_1|e^{igt} + \langle\varphi_2|e^{-igt}) + \text{h.c.}\right]. \end{aligned} \tag{7}$$

Once the Zeno conditions $g \gg \Omega$ are satisfied, we can readily discard the fast-oscillating terms in H_1'', and then obtain the effective Hamiltonian

$$H_{1,eff}'' = \frac{\Delta}{2}(|\varphi_1\rangle\langle\varphi_1| + |\varphi_2\rangle\langle\varphi_2|). \tag{8}$$

This effective Hamiltonian takes no any action on the evolution of the initial state $|s_1 f_2\rangle|0_c\rangle$, and thus the third evolution in Eq. (1) is implemented.

Next, we consider another situation related to the initial state $|s_1 g_2\rangle|0_c\rangle$. For this case the invariant subspace is spanned by $\{|s_1 g_2\rangle|0_c\rangle, |e_1 g_2\rangle|0_c\rangle, |g_1 g_2\rangle|1_c\rangle, |g_1 e_2\rangle|0_c\rangle\}$, and the relevant Hamiltonian H of the system can be rewritten as

$$\begin{aligned}\tilde{H}_1 &= \Delta|\phi_1\rangle\langle\phi_1| + \frac{\Delta}{2}|\phi_2\rangle\langle\phi_2| + \frac{\Delta}{2}|\phi_3\rangle\langle\phi_3| \\ &+ \frac{\Delta}{2}|\phi_2\rangle\langle\phi_3| + \frac{\Delta}{2}|\phi_3\rangle\langle\phi_2| \\ &+ \left[\frac{\Omega}{2}|s_1 g_2\rangle|0_c\rangle(-\sqrt{2}\langle\phi_1| + \langle\phi_2| + \langle\phi_3|) + \text{h.c.}\right] \quad (9)\\ \tilde{H}_2 &= -\sqrt{2}g|\phi_2\rangle\langle\phi_2| + \sqrt{2}g|\phi_3\rangle\langle\phi_3|. \quad (10)\end{aligned}$$

Here,

$$|\phi_1\rangle = \frac{1}{\sqrt{2}}(-|e_1 g_2\rangle|0_c\rangle + |g_1 e_2\rangle|0_c\rangle), \quad (11)$$

and

$$\begin{aligned}|\phi_2\rangle &= \frac{1}{2}(|e_1 g_2\rangle|0_c\rangle - \sqrt{2}|g_1 g_2\rangle|1_c\rangle + |g_1 e_2\rangle|0_c\rangle), \quad (12)\\ |\phi_3\rangle &= \frac{1}{2}(|e_1 g_2\rangle|0_c\rangle + \sqrt{2}|g_1 g_2\rangle|1_c\rangle + |g_1 e_2\rangle|0_c\rangle), \quad (13)\end{aligned}$$

are the eigenstates of H_2 with the eigenvalues 0, $-\sqrt{2}g$ and $\sqrt{2}g$, respectively. Similarly, under the unitary transformation $e^{i\tilde{H}_2 t}$ and the condition $g \gg \Omega$, we have

$$\tilde{H}_1' = \Delta|\phi_1\rangle\langle\phi_1| - \left(\frac{\Omega}{\sqrt{2}}|s_1 g_2\rangle|0_c\rangle\langle\phi_1| + \text{h.c.}\right). \quad (14)$$

Set $\Delta \gg \Omega$ for further simplicity, then there are no any energy-exchange between state $|s_1 g_2\rangle|0_c\rangle$ and $|\phi_1\rangle$. Consequently, another effective Hamiltonian

$$\tilde{H}_{1,eff}' = \frac{\Omega^2}{2\Delta}|s_1 g_2\rangle|0_c\rangle\langle 0_c|\langle s_1 g_2|. \quad (15)$$

is obtained. Under the action of $\tilde{H}_{1,eff}'$, we obtain $|s_1 g_2\rangle|0_c\rangle \rightarrow \exp\left(i\Omega^2 t/2\Delta\right)|s_1 g_2\rangle|0_c\rangle$, which is nothing but the fourth evolution in Eq. (1).

In order to validate the feasibility of the above theoretical analysis, we perform a direct numerical simulation of the Schrödinger equation with the original Hamiltonian (2) (without decoherence). We choose the typical parameters: $\Omega = 0.1g$ and $\Delta = g$. In the simulation, we calculated the temporal evolutions of the system beginning with two distinct initial states $|s_1 f_2\rangle|0_c\rangle$ and $|s_1 g_2\rangle|0_c\rangle$. As shown in Fig. 2, the red and blue lines describe the real parts of the coefficients of the basic states $|s_1 f_2\rangle|0_c\rangle$ and $|s_1 g_2\rangle|0_c\rangle$, respectively. It is seen that, the system returns to its initial state but obtains a global phase shift π at the time $\tau = 2\pi\Delta/\Omega^2$ when the system is initially prepared in the state $|s_1 g_2\rangle|0_c\rangle$, while it is almost unchanged for the initial state $|s_1 f_2\rangle|0_c\rangle$. Under realistic environment, two main decoherence processes (i.e.,

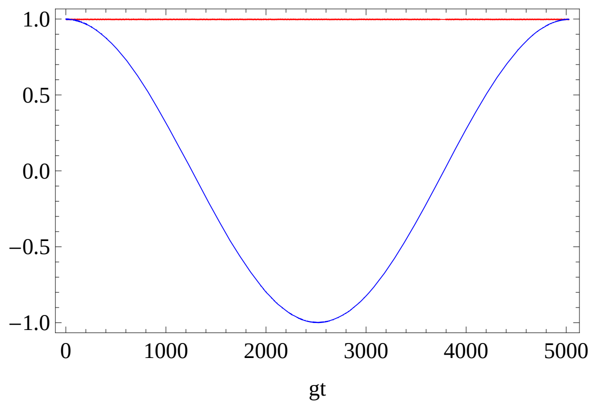

Fig. 2. Real parts of the coefficients of the states $|s_1 f_2\rangle|0_c\rangle$ (Red line) and $|s_1 g_2\rangle|0_c\rangle$ (Blue line) versus the evolution time. The parameters are typically set as: $\Omega = 0.1g$, $\Delta = g$.

cavity photon loss with decay rate κ and the decay γ of the N-V center) should be taken into consideration. In this case the evolution of the system is governed formally by the following master equation

$$\begin{aligned}\dot{\rho} &= -i[H,\rho] - \frac{\kappa}{2}(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) \\ &\quad - \sum_{i=1,2}\sum_{n=g,s}\frac{\gamma_{en}^i}{2}(\sigma_{ee}^i\rho - 2\sigma_{ne}^i\rho\sigma_{en}^i + \rho\sigma_{ee}^i),\end{aligned} \tag{16}$$

where γ_{en}^i denotes the spontaneous decay from level $|e_i\rangle$ to $|n_i\rangle$ $(i = 1, 2)$ and we assume $\gamma_{eg}^i = \gamma_{es}^i = \gamma$ for simplicity. The fidelity of the two-qubit controlled-phase gate implemented in the presence of the decoherence can be defined as

$$F = \langle\Psi(0)|U_p^\dagger\rho'(t=\tau)U_p|\Psi(0)\rangle, \tag{17}$$

where $\rho'(t)$ represents the temporal reduced density matrix (obtained by tracing out the cavity mode part), $U_p = e^{i\pi|s_1g_2\rangle\langle s_1g_2|}$ is the ideal quantum phase gate operation, and $|\Psi(0)\rangle$ the initial state of the qubits, which is selected as $|\Psi(0)\rangle = (|f_1\rangle + |s_1\rangle)/\sqrt{2}\otimes(|f_2\rangle + |g_2\rangle)/\sqrt{2}$ for the present simulation. In Fig.3, we plot the fidelity F versus the decays κ and γ. We can see that the fidelity is still larger than 80% for $\kappa = \gamma = 0.1g$.

We now briefly analyze the experimental feasibility of the proposed scheme. Practically, the energy-level configuration involved in our scheme can be implemented in the N-V center with two unpaired electrons located at the vacancy, usually treated as electron spin-1 [21, 22]. The ground state $|g\rangle$ and $|s\rangle$ correspond to $|^3A, m_s = 0\rangle$ and $|^3A, m_s = -1\rangle$ states of spin triplet, respectively. $|^3E, m_s = 0\rangle$ and the metastable singlet state $|^1A\rangle$ could be selected as the excited state $|e\rangle$ and the auxiliary state $|f\rangle$, respectively. Also, the decoherence rate of the zero-phonon N-V transition at frequency $637nm$ is about $\gamma \sim 2\pi \times 13$MHz, and the Q factor of the WGM microresonator can have a value exceeding 10^9, which can lead to less photon decay rate $\kappa = \omega/Q \sim 2\pi \times 0.5$MHz. Strong coupling between the individual N-V

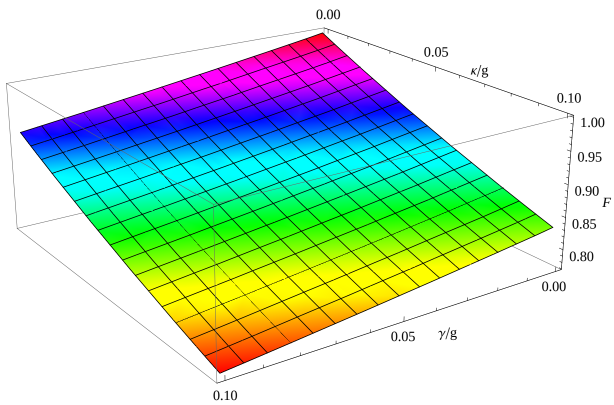

Fig. 3. The influences of atomic spontaneous emission γ/g and cavity field decay κ/g on the fidelity F of the two-qubit gate.

center in diamond and the WGM in a microsphere or microdisk resonator reaches [14, 15, 16] to $g/2\pi \sim 0.3$ GHz. Based on these experimental parameters, the fidelity could be estimated as larger than 96.2%.

In summary, we have presented an experimentally-feasible protocol to implement a quantum controllable-phase gate between two N-V centers coupled by a WGM microresonator. With the induced quantum Zeno dynamics, this gate can be realized by only one step, which greatly reduces the complexity in practical experiment. Moreover, our numerical simulation showed that the protocol is robust against the spontaneous decays. The required energy-level configuration is rather simple, so the approach can be easily generalized to other physical system, such as the ion trap and Josephson junctions.

This work was supported in part by the National Natural Science Foundation of China, under Grants Nos. 90921010 and 11174373, and the National Fundamental Research Program of China, through Grant No. 2010CB923104.

References

1. P. W. Shor (1994), *Proceedings of the 35th Annual Symposium on Foundations of Computer Science*, IEEE Computer Society, Los Alomitos, CA, pp. 116.
2. L. K. Grover (1998), *Quantum Computers Can Search Rapidly by Using Almost Any Transformation*, Phys. Rev. Lett. 80, pp. 4329-4332.
3. A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche (1999), *Coherent Operation of a Tunable Quantum Phase Gate in Cavity QED*, Phys. Rev. Lett. 83, pp. 5166-5169.
4. Y.-F. Xiao, X.-B. Zou, and G.-C. Guo (2007), *One-step implementation of an N-qubit controlled-phase gate with neutral atoms trapped in an optical cavity*, Phys. Rev. A 75, pp. 054303.
5. X.-B. Zou, K. Pahlke, and W. Mathis (2002), *Quantum phase-gate implementation for trapped ions in thermal motion*, Phys. Rev. A 66, pp. 044307.
6. L.-B. Chen, L. J. Sham, and E. Waks (2012), *Optically controlled phase gate for two spin qubits in coupled quantum dots*, Phys. Rev. B 85, pp. 115319.
7. P. Facchi and S. Pascazio (2002), *Quantum Zeno Subspaces*, Phys. Rev. Lett. 89, pp. 080401.
8. P. Facchi, G. Marmo, and S. Pascazio (2009), *Quantum Zeno dynamics and quantum Zeno sub-*

spaces, J. Phys: Conf. Ser. 196, pp. 012017.
9. X. B. Wang, J. Q. You, and F. Nori (2008), *Quantum entanglement via two-qubit quantum Zeno dynamics*, Phys. Rev. A 77, pp. 062339.
10. W.-A. Li and G.-Y. Huang (2011), *Deterministic generation of a three-dimensional entangled state via quantum Zeno dynamics*, Phys. Rev. A 83, pp. 022322.
11. X.-Q. Shao, L. Chen, S. Zhang, Y.-F. Zhao, and K.-H. Yeon (2010), *Deterministic generation of arbitrary multi-atom symmetric Dicke states by a combination of quantum Zeno dynamics and adiabatic passage*, Europhys. Lett. 90, pp. 50003.
12. A. Beige, D. Braun, B. Tregenna, and P. L. Knight (2000), *Quantum Computing Using Dissipation to Remain in a Decoherence-Free Subspace*, Phys. Rev. Lett. 85, pp. 1762-1765.
13. J. D. Franson , B. C. Jacobs, and T. B. Pittman (2004), *Quantum computing using single photons and the Zeno effect*, Phys. Rev. A 70, pp. 062302.
14. S. Schietinger, T. Schroder, and O. Benson (2008), *One-by-one coupling of single defect centers in nanodiamonds to high-Q modes of an optical microresonator*, Nano Lett. 8, pp. 3911-3915.
15. P. E. Barclay, K.-M. C. Fu, C. Santori, and R. G. Beausoleil (2009), *Chip-based microcavities coupled to nitrogen-vacancy centers in single crystal diamond*, Appl. Phys. Lett. 95, pp. 191115.
16. C. Santori, P. E. Barclay, K.-M. Fu, R.G. Beausoleil, S. Spillane, and M. Fisch (2010), *Nanophotonics for quantum optics using nitrogen-vacancy centers in diamond*, Nanotechnology 21, pp. 274008.
17. G. Balasubramanian et al. (2009), *Ultralong spin coherence time in isotopically engineered diamond*, Nature Mater. 8, pp. 383-387.
18. A. Batalov, V. Jacques, F. Kaiser, P. Siyushev, P. Neumann, L. J. Rogers, R. L. McMurtrie, N. B. Manson, F. Jelezko, and J. Wrachtrup (2009), *Low Temperature Studies of the Excited-State Structure of Negatively Charged Nitrogen-Vacancy Color Centers in Diamond*, Phys. Rev. Lett. 102, pp. 195506.
19. K. J. Vahala (2003), *Optical microcavities*, Nature (London) 424, pp. 839.
20. S. M. Spillane, T. J. Kippenberg, K. J. Vahala, K. W. Goh, E. Wilcut, and H. J. Kimble (2005), *Ultrahigh-Q toroidal microresonators for cavity quantum electrodynamics*, Phys. Rev. A 71, pp. 013817.
21. C. Santori, P. Tamarat, P. Neumann, J. Wrachtrup, D. Fattal, R. G. Beausoleil, J. Rabeau, P. Olivero, A. D. Greentree, S. Prawer, F. Jelezko, and P. Hemmer (2006), *Coherent Population Trapping of Single Spins in Diamond under Optical Excitation*, Phys. Rev. Lett. 97, pp. 247401.
22. P. Neumann, R. Kolesov, V. Jacques, J. Beck, J. Tisler, A. Batalov, L. Rogers, N. B. Manson, G. Balasubramanian, F. Jelezko, and J. Wrachtrup (2009), *Excited-state spectroscopy of single NV defects in diamond using optically detected magnetic resonance*, New J. Phys. 11, pp. 013017.