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ENTANGLEMENT TRANSFER BETWEEN ATOMIC QUBITS AND THERMAL FILEDS

SHI-BIAO ZHENG   RONG-XIN CHEN
Department of Physics, Fuzhou University, Fuzhou 350002, P. R. China
sbzheng@pub5.fz.fj.cn

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We investigate entanglement reciprocation between atomic qubits and cavity fields initially in a thermal state. We show that the entanglement between the atomic qubits can be fully transferred to the mixed fields through displacement operation and resonant atom-cavity interaction. This is a rare example, in which quantum systems in mixed states can be used as the memory for entanglement. The entanglement can be retrieved by another atomic pair. Apart from fundamental interest, the results are useful for implementation of quantum networking with atom-field interface in the microwave regime.

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1 Introduction

Quantum entanglement is not only an essential ingredient for test of quantum nonlocality, but also a key element for implementation of quantum information networks. Entanglement between two-level systems, referred to as qubits, has been extensively researched and used for many quantum information protocols, such as quantum cryptography [1] and teleportation [2]. Multiparticle entangled states are also important for Heisenberg-limit interferometry [3]. On the other hand, entanglement of continuous variable systems is also useful for quantum communication [4] and nonlocality test [5]. The Jaynes-Cummings model, the simplest system in quantum optics describing the interaction between a two-level atom and a single-mode quantized field, is toolbox for exploring many purely quantum-mechanical properties [6,7]. When the cavity mode is initially in the vacuum state, the Rabi oscillation results in atom-field entanglement [8] and the cavity can act as a memory for storing atomic states [9]. Entanglement between two atoms can be produced by real [10] or virtual [11] photon exchange in a high Q cavity. When the cavity contains a mesoscopic field with a few tens of photons, the atom-field interaction leads to the phase entanglement between the field and atomic dipole [12]. On the other hand, it has been shown that the coherence of an atomic qubit can be transferred to thermal fields through dispersive or resonant interactions and subsequent measurement of the atomic state [13,14]. The entanglement of two atomic qubits can be fully transferred to two cavity modes each of which is initially in the vacuum state and resonantly interacts with the respective atom. In this case the cavity modes behave like qubits since only 0 and 1 photon states are involved during the interaction.

Recently, much attention have been paid to the interface between qubits and continuous variable systems, which is useful for construction of quantum networks. Schemes have been proposed for entangling two atomic qubits using entangled Gaussian states of light [15,16]. Lee et al. have investigated the entanglement transfer between qubits and continuous variable systems [17]. It is shown that entanglement between two atomic qubits can be fully transferred to two cavity modes initially prepared in coherent states of high amplitudes through resonant interaction of the atoms with the respective cavities. Perfect entanglement between the cavities is achieved by the atom's depositing an extra photon to the cavities and thus the full entanglement transfer is conditioned on the atoms being detected in their ground states after leaving the cavities with the postselection probability being 25%. It has been shown that it is possible to transfer entanglement from qubits to cavity fields initially in coherent states without post selection [18]. All of the previous studies concentrate on the case that the cavity fields are initially in pure states. However, at finite temperature the cavity fields are actually in a mixed thermal state before interaction with the atoms. In particular, in the microwave regime thermal photons cannot be neglected. Therefore, entanglement reciprocation between atomic qubits and thermal fields is essential for implementation of quantum network in this regime.

In this paper, we show that the cavities, acting as the entanglement memory, do not need to be initially prepared in a pure state. Entanglement between two two-level atoms can be transferred to two cavities initially in displaced thermal states. The physics underlying the process is that the atomic dipole phase correlation is transferred to the displaced cavity fields through resonant atom-cavity interaction. The entanglement can be retrieved to another pair of qubits from the mixed field state, and then be measured using atomic state detectors plus classical pulses [10,11]. Numerical results demonstrate that the entanglement reciprocation can be achieved with a high efficiency for a moderate displacement amount. The present work offers the possibility to implement quantum information tasks using mixed states.

2 Theoretical model

We consider two two-level atoms initially in the maximally entangled states

$$|\psi_a(0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 |g\rangle_2 - |g\rangle_1 |e\rangle_2), \tag{1}$$

where $|e\rangle$ and $|g\rangle$ are the excited and ground states of the atoms, and the subscripts label the atoms. These two atoms are placed in different cavities and resonantly interact with respective field modes. In the rotating-wave approximation, the Hamiltonian for the interaction between the jth atom and the respective cavity is (assuming $\hbar = 1$)

$$H_j = \lambda(a_j^\dagger S_j^- + a_j S_j^+), \tag{2}$$

where $S_j^+ = |e\rangle_j \langle g|$, $S_j^- = |g\rangle_j \langle e|$, $a_j^\dagger$ and a_j are the creation and annihilation operators for the jth cavity mode, and λ is the atom-cavity coupling strength. Each cavity initially contains a thermal field, whose state is described by the density operator

$$\rho_j(0) = \frac{1}{\pi\, \bar{n}_{th}} \int e^{-|\alpha|^2/\bar{n}_{th}} |\alpha_j\rangle_j \langle \alpha_j| \, d^2\alpha_j, \tag{3}$$

where $\bar{n}_{th} = 1/(e^{\hbar\omega/k_B T} - 1)$ is the mean photon-number of the thermal field. We first displace the each cavity field by an amount β, leading to displaced thermal state

$$\begin{aligned} \rho_j'(0) &= D_j(\beta)\rho_j(0)D_j^+(\beta) \\ &= \frac{1}{(\pi\,\bar{n}_{th})^2}\int e^{-|\alpha|^2/\bar{n}_{th}} \left|\beta+\alpha_j\right\rangle_j \left\langle \alpha_j+\beta\right| d^2\alpha_j, \end{aligned} \quad (4)$$

with $D_j(\beta)$ being the displacement operator $D_j(\beta) = e^{\beta a_j^\dagger - \beta^* a_j}$. We here assume that β is a positive real number. The coherent state component $\left|\alpha_j+\beta\right\rangle_j$ can be expressed in terms of Fock states as

$$\left|\alpha_j+\beta\right\rangle_j = \sum_{n=0}^{\infty} c_{\alpha_j,n}\left|n\right\rangle_j, \quad (5)$$

where $c_{\alpha_j,n} = e^{-|\alpha_j+\beta|^2/2}\frac{(\alpha_j+\beta)^n}{\sqrt{n!}}$. Then the evolution of the whole system is given by

$$\rho(t) = \frac{1}{(\pi\,\bar{n}_{th})^2}\int e^{-(|\alpha_1|^2+|\alpha_2|^2)/\bar{n}_{th}} \left|\psi_{\alpha_1,\alpha_2}(t)\right\rangle \left\langle \psi_{\alpha_1,\alpha_2}(t)\right| d^2\alpha_1 d^2\alpha_2, \quad (6)$$

where

$$\begin{aligned} \left|\psi_{\alpha_1,\alpha_2}(t)\right\rangle &= \frac{1}{\sqrt{2}}(\left|\psi_{\alpha_1,e}(t)\right\rangle_1 \left|\psi_{\alpha_2,g}(t)\right\rangle_2 - \left|\psi_{\alpha_1,g}(t)\right\rangle_1 \left|\psi_{\alpha_2,e}(t)\right\rangle_2), \\ \left|\psi_{\alpha_j,e}(t)\right\rangle_j &= \sum_{n=0}^{\infty}[c_{\alpha_j,n}\cos(\sqrt{n+1}\lambda t)\left|e\right\rangle_j - ic_{\alpha_j,n-1}\sin(\sqrt{n}\lambda t)\left|g\right\rangle_j]\left|n\right\rangle_j, \\ \left|\psi_{\alpha_j,g}(t)\right\rangle_j &= \sum_{n=0}^{\infty}[c_{\alpha_j,n}\cos(\sqrt{n}\lambda t)\left|g\right\rangle_j - ic_{\alpha_j,n+1}\sin(\sqrt{n+1}\lambda t)\left|e\right\rangle_j]\left|n\right\rangle_j. \end{aligned} \quad (7)$$

Due to the Jaynes-Cummings interaction, each atoms is correlated to the respective cavity mode. In general, the whole system would evolve to a complex mixed entangled state.

3 Entanglement transfer from qubits to cavity modes

The contribution of coherent state components with $|\alpha|^2 \gg \bar{n}_{th}$ to the expansion of the thermal state is negligible. When $\beta^2 \gg \bar{n}_{th}$ the coherent states that contribute significantly to the expansion of ρ_j' are those with $|\alpha_j| \ll |\beta|$. We further assume that β is large enough so that $|\alpha_j+\beta| \gg 1$ for these components. The width Δn_{α_j} of the photon number distributions of the strong coherent state $\left|\alpha_j+\beta\right\rangle_j$ obey the inequality $1 \ll \Delta n_{\alpha_j} \ll \bar{n}_{\alpha_j}$, with $\bar{n}_{\alpha_j} = |\alpha_j+\beta|^2$ being the mean photon number. In this case the coefficients of the components that dominate in the Fock state expansion (5) satisfy [19]

$$c_{\alpha_j,n+1} = e^{-|\alpha_j+\beta|^2/2}\frac{(\alpha_j+\beta)^n\left|\alpha_j+\beta\right|e^{i\theta_{\alpha_j}}}{\sqrt{n+1}\sqrt{n!}} \simeq e^{i\theta_{\alpha_j}}c_{\alpha_j,n}, \quad (8)$$

where $e^{i\theta_{\alpha_j}} = (\alpha_j+\beta)/\left|\alpha_j+\beta\right|$. We can express $\sqrt{n+1}$ in Eq. (7) as $\sqrt{n}(1+\frac{1}{n})^{1/2}$ and expand the terms in the bracket to the second order in $1/n$, leading to

$$\begin{aligned}\sqrt{n+1} &\simeq n^{1/2}(1+\frac{1}{2n}-\frac{1}{8n^2}) \\ &= n^{1/2}+\frac{1}{2n^{1/2}}-\frac{1}{8n^{3/2}}.\end{aligned} \tag{9}$$

We can further expand the second and third terms right-hand of Eq. (9) in power of $n-\bar{n}_{\alpha_j}$ and discard the terms containing a factors $\bar{n}_{\alpha_j}^{-l}$ with l> 3/2, obtaining

$$\sqrt{n+1}\lambda t \simeq [n^{1/2}+\frac{1}{2\,\bar{n}_{\alpha_j}^{1/2}}-\frac{n-\bar{n}_{\alpha_j}+1/2}{4\,\bar{n}_{\alpha_j}^{3/2}}]\lambda t. \tag{10}$$

At the time $T=\beta\pi/\lambda$, we have

$$\sqrt{n+1}\lambda T \simeq n^{1/2}\lambda T+\frac{\pi}{2}\frac{\beta}{\bar{n}_{\alpha_j}^{1/2}}-\frac{n-\bar{n}_{\alpha_j}^{1/2}+1/2}{4\,\bar{n}_{\alpha_j}}\frac{\beta}{\bar{n}_{\alpha_j}^{1/2}}\pi. \tag{11}$$

In the Jaynes-Cummings model, if the cavity mode is initially in the coherent state $|\beta\rangle$ with $\beta \gg 1$, the time T corresponds to the middle of the collapse regime of the atomic inversion, where the atom is disentangled with the cavity [19].

For the state components that contribute significantly to the sum (5), the last term in (11) is on the order of $1/\bar{n}_{\alpha_j}^{1/2}$. The deviation of $\beta/\bar{n}_{\alpha_j}^{1/2}$ from 1 is on the order of $|\alpha_j|/\bar{n}_{\alpha_j}^{1/2}$. Neglecting terms on the order of $1/\bar{n}_{\alpha_j}^{1/2}$ and $|\alpha_j|/\bar{n}_{\alpha_j}^{1/2}$, $\sqrt{n+1}\lambda T$ approximates $n^{1/2}\lambda T+\frac{\pi}{2}$. Then we have

$$\begin{aligned}\left|\psi_{\alpha_j,e}(T)\right\rangle_j &= -\sum_{n=0}^{\infty}[c_{\alpha_j,n}\sin(\sqrt{n}\lambda T)\left|e\right\rangle_j + ic_{\alpha_j,n-1}\sin(\sqrt{n}\lambda T)\left|g\right\rangle_j]\left|n\right\rangle_j \\ &= -\left|\psi_{\alpha_j,e}(T)\right\rangle_{f,j}\left|\phi_{\alpha_j}(T)\right\rangle_{a,j}, \\ \left|\psi_{\alpha_j,g}(T)\right\rangle_j &= \sum_{n=0}^{\infty}[c_{\alpha_j,n}\cos(\sqrt{n}\lambda T)\left|g\right\rangle_j - ic_{\alpha_j,n+1}\cos(\sqrt{n}\lambda T)\left|e\right\rangle_j]\left|n\right\rangle_j \\ &= -ie^{i\theta_{\alpha_j}}\left|\psi_{\alpha_j,g}\right\rangle_{f,j}\left|\phi_{\alpha_j}(T)\right\rangle_{a,j}.\end{aligned} \tag{12}$$

where

$$\begin{aligned}\left|\psi_{\alpha_j,e}(T)\right\rangle_{f,j} &= \sqrt{2}\sum_{n=0}^{\infty}c_{\alpha_j,n}\sin(\sqrt{n}\lambda T)\left|n\right\rangle_j, \\ \left|\psi_{\alpha_j,g}(T)\right\rangle_{f,j} &= \sqrt{2}\sum_{n=0}^{\infty}c_{\alpha_j,n}\cos(\sqrt{n}\lambda T)\left|n\right\rangle_j, \\ \left|\phi_{\alpha_j}(T)\right\rangle_{a,j} &= (\left|e\right\rangle_j + ie^{-i\theta_{\alpha_j}}\left|g\right\rangle_j)/\sqrt{2}.\end{aligned} \tag{13}$$

Thus we have

$$|\psi_{\alpha_1,\alpha_2}(T)\rangle = |\psi_{\alpha_1,\alpha_2}(T)\rangle_f\,|\phi_{\alpha_1}(T)\rangle_{a,1}\,|\phi_{\alpha_2}(T)\rangle_{a,2}, \tag{14}$$

where

$$|\psi_{\alpha_1,\alpha_2}(T)\rangle_f = \frac{i}{\sqrt{2}}[e^{i\theta_{\alpha_2}} |\psi_{\alpha_1,e}(T)\rangle_{f,1} |\psi_{\alpha_2,g}(T)\rangle_{f,2} - e^{i\theta_{\alpha_1}} |\psi_{\alpha_1,g}(T)\rangle_{f,1} |\psi_{\alpha_2,e}(T)\rangle_{f,2}]. \quad (15)$$

The total density operator is

$$\begin{aligned}\rho(T) &= \frac{1}{(\pi \bar{n}_{th})^2} \int e^{-(|\alpha_1|^2+|\alpha_2|^2)/\bar{n}_{th}} |\psi_{\alpha_1,\alpha_2}(T)\rangle_f \langle\psi_{\alpha_1,\alpha_2}(T)| \\ &\quad \otimes |\phi_{\alpha_1}(T)\rangle_{a,1} \langle\phi_{\alpha_1}(T)| \otimes |\phi_{\alpha_2}(T)\rangle_{a,2} \langle\phi_{\alpha_2}(T)| \, d^2\alpha_1 d^2\alpha_2, \quad (16)\end{aligned}$$

As has been pointed out, for the state components that contribute significantly to the coherent state expansion of the thermal state, $|\alpha_j| \ll \beta$ and thus $e^{i\theta_{\alpha_j}} \simeq 1$. This indicates that each atom is almost in the pure state $(|e\rangle_j + i\,|g\rangle_j)/\sqrt{2}$ at the time T. The atoms are not only disentangled to each other but also to the cavity fields at this time. The entanglement initially carried by the atoms is fully transferred to the cavity modes. As opposed to the study of Ref. [17], the average excitation number of the atoms in the final state is the same as that in the initial state, indicating the entanglement transfer does not arise from excitation exchange between the atoms and the fields.

To verify the validity of our approximations, we present numerical simulations. The efficiency of entanglement transfer is characterized by the purity of the atomic system. If each atom evolves to a pure state, the entanglement is fully transferred to the cavity system. The purity of atom j can be measured by the quantity $Tr\rho_{aj}^2$, where ρ_{aj} is the reduced density operator for atom j. When $Tr\rho_{aj}^2 = 1/2$, this atom is in a maximally mixed state. On the other hand, $Tr\rho_{aj}^2 = 1$ corresponds to a pure atomic state. During the evolution, the two atoms have the same purity. In Fig. 1 we plot the quantity $Tr\rho_{a1}^2$ against the value of the displacement parameter β at the time $T = \beta\pi/\lambda$. The average thermal photon number in each cavity is $\bar{n}_{th}= 2$. It shows that this quantity increases with the value of β, and it can be above 90% for $\beta \sim 6$, implying that the approximation is quite good even for a moderate β. In Fig. 2, we plot $Tr\rho_{a1}^2$ against the interaction time. The amplitude of the displacement is $\beta = 8$ and the average thermal photon number is $\bar{n}_{th}= 2$. As shown in this figure each atom is almost in a pure state around the time $T = \beta\pi/\lambda$, indicating the entanglement initially deposited in the atoms has been transferred to the cavities.

To gain deeper insight into the underlying physics, we rewrite the initial atomic state as the dipole entangled state

$$|\psi(0)\rangle_a = \frac{1}{\sqrt{2}}(|\phi^+(0)\rangle_{a,1} |\phi^-(0)\rangle_{a,2} - |\phi^-(0)\rangle_{a,1} |\phi^+(0)\rangle_{a,2}), \quad (17)$$

where

$$|\phi^\pm(0)\rangle_{a,j} = (|e\rangle_j \pm |g\rangle_j)/\sqrt{2}. \quad (18)$$

To see how each dipole state component evolves due to interaction with the respective cavity we rewrite $|\psi_{\alpha_1,\alpha_2}(t)\rangle$ as

$$|\psi_{\alpha_1,\alpha_2}(t)\rangle = \frac{1}{\sqrt{2}}(|\psi^+_{\alpha_1}(t)\rangle_1 |\psi^-_{\alpha_2}(t)\rangle_2 - |\psi^-_{\alpha_1}(t)\rangle_1 |\psi^+_{\alpha_2}(t)\rangle_2), \quad (19)$$

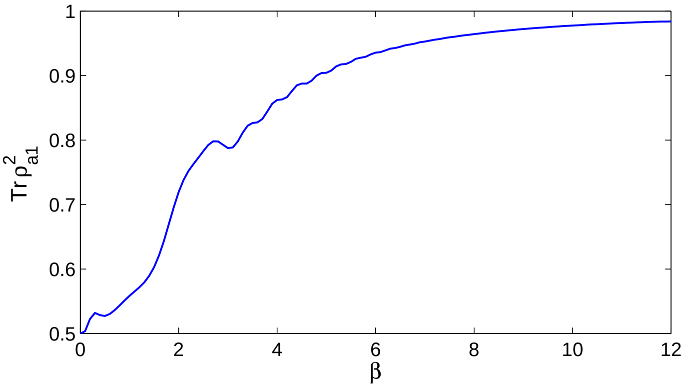

Fig. 1. The purity of atom 1 as a function of the displacement amplitude β at the time $T = \beta\pi/\lambda$. The average thermal photon number in each cavity is $\bar{n}_{th}= 2$.

where

$$\begin{aligned}\left|\psi^{\pm}_{\alpha_j}(t)\right\rangle_j &= \frac{1}{\sqrt{2}}\sum_{n=0}^{\infty}[c_{\alpha_j,n}\cos(\sqrt{n+1}\lambda t)\left|e\right\rangle_j - ic_{\alpha_j,n-1}\sin(\sqrt{n}\lambda t)\left|g\right\rangle_j]\left|n\right\rangle_j \\ &\pm\frac{1}{\sqrt{2}}\sum_{n=0}^{\infty}[c_{\alpha_j,n}\cos(\sqrt{n}\lambda t)\left|g\right\rangle_j - ic_{\alpha_j,n+1}\sin(\sqrt{n+1}\lambda t)\left|e\right\rangle_j]\left|n\right\rangle_j \\ &\simeq \frac{1}{\sqrt{2}}\{\sum_{n=0}^{\infty}c_{\alpha_j,n}e^{\mp i\sqrt{n+1}\lambda t}\left|e\right\rangle_j \pm \sum_{n=0}^{\infty}c_{\alpha_j,n}e^{\mp i\sqrt{n}\lambda t}\left|g\right\rangle_j\}\left|n\right\rangle_j . \end{aligned} \quad (20)$$

For $t \leq \beta\pi/\lambda$ we can neglect the third term of Eq. (10) and use the approximation

$$n^{1/2} \simeq \bar{n}_{\alpha_j}^{1/2} + \frac{n-\bar{n}_{\alpha_j}}{2\,\bar{n}_{\alpha_j}^{1/2}} - \frac{(n-\bar{n}_{\alpha_j})^2}{8\,\bar{n}_{\alpha_j}^{3/2}}. \quad (21)$$

This leads to

$$\begin{aligned}\left|\psi^{\pm}_{\alpha_j}(t)\right\rangle_j &\simeq \frac{1}{\sqrt{2}}\{\sum_{n=0}^{\infty}c_{\alpha_j,n}e^{\mp i[\bar{n}_{\alpha_j}^{1/2}/2+(n+1)/2\bar{n}_{\alpha_j}^{1/2}-(n-\bar{n}_{\alpha_j})^2/8\bar{n}_{\alpha_j}^{3/2}]\lambda t}\left|e\right\rangle_j \\ &\pm\sum_{n=0}^{\infty}c_{\alpha_j,n}e^{\mp i[\bar{n}_{\alpha_j}^{1/2}/2+n/2\bar{n}_{\alpha_j}^{1/2}-(n-\bar{n}_{\alpha_j})^2/8\bar{n}_{\alpha_j}^{3/2}]\lambda t}\left|g\right\rangle_j\}\left|n\right\rangle_j \\ &= e^{\mp i\bar{n}_{\alpha_j}^{1/2}\lambda t/2}\sum_{n=0}^{\infty}c_{\alpha_j,n}e^{\mp i[n/2\bar{n}_{\alpha_j}^{1/2}-(n-\bar{n}_{\alpha_j})^2/8\bar{n}_{\alpha_j}^{3/2}]\lambda t}\left|n\right\rangle_j\left|\phi^{\pm}_{\alpha_j}(t)\right\rangle_{a,j}, \end{aligned} \quad (22)$$

where

$$\left|\phi^{\pm}_{\alpha_j}(t)\right\rangle_{a,j} = (e^{\mp i\lambda t/2\bar{n}_{\alpha_j}^{1/2}}\left|e\right\rangle_j \pm \left|g\right\rangle_j)/\sqrt{2}. \quad (23)$$

Due to the resonant interaction, the phase of the two initial atomic dipole states $|\phi^+(0)\rangle_{a,j}$ and $|\phi^-(0)\rangle_{a,j}$ rotate in opposite directions. Meanwhile, the field is split into two quasicoherent components whose phases are correlated with the corresponding atomic dipole states

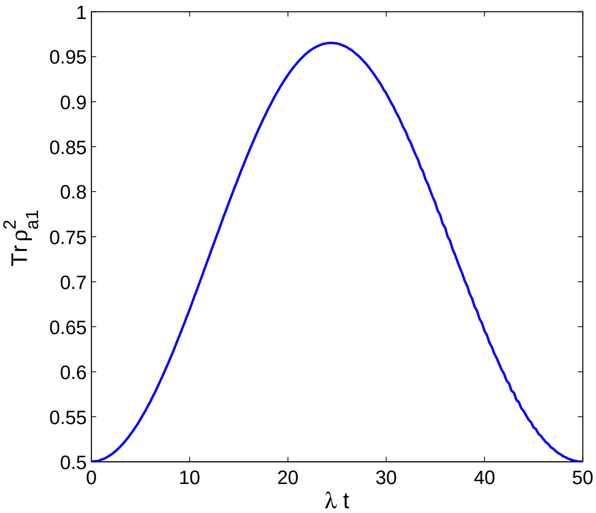

Fig. 2. The purity of atom 1 as a function of the interaction time. The average thermal photon number in each cavity is $\bar{n}_{th}= 2$ and the displacement amplitude is $\beta = 8$.

and also rotate in opposite directions in phase space. As the phases of the dipole states for the two atoms are initially correlated, the two fields and the two atoms are all phase entangled due to the atom-cavity interaction. For $\beta \gg \alpha_j$, we have $\bar{n}_{\alpha_j} \simeq \beta$. During the interval $[0,\beta\pi/\lambda]$, as time proceeds, the phase separation of the two atomic dipole states becomes smaller and smaller, while that of the quasicoherent field states becomes larger and larger. When $t = \beta\pi/\lambda$, the two atomic dipole states merge together so that each atom is disentangled with other subsystems. On the other hand, the phase separation of the two quasicoherent components for each field approaches the maximum at this time. As a result, the phase correlation initially carried by the atomic dipoles is completely transferred to the quasicoherent states of the two fields so that they are maximally phase entangled. It should be noted that the fields are not in an entangled coherent states due to the presence of the factor $e^{\pm i(n-\bar{n}_{\alpha_j})^2 \lambda t/8\bar{n}_{\alpha_j}^{3/2}}$, which leads to phase spreading.

4 Retrival of entanglement from cavities

We note that the entanglement imprinted in the two mesoscopic fields can also be transferred to two atomic qubits using the echo technique proposed by Morigi et al. [20,21]. The atom-cavity system evolves from time 0 to t under the Jaynes-Cummings evolution operator $U_1 = e^{-iH_1 t} e^{-iH_2 t}$. Then each atom undergoes a phase kick $\sigma_{j,z} = |e\rangle_j \langle e| - |g\rangle_j \langle g|$. For a second Jaynes-Cummings interaction time t' the evolution operator of the system is $U_2 = e^{-iH_1 t'} e^{-iH_2 t'}$. The whole evolution operator is

$$U = U_2 \sigma_{1,z} \sigma_{2,z} U_1 = \sigma_{1,z} \sigma_{2,z} e^{-iH_1(t-t')} e^{-iH_2(t-t')}. \tag{24}$$

We here have used the relation $\sigma_{j,z} H_j \sigma_{j,z} = -H_j$. Therefore, the phase kick leads to the reversal of the unitary evolution of the system. After the duration $2t$, the system evolves back to the initial state, i.e., the entanglement is retrieved from the mesoscopic fields back to the two atoms. If these atoms leave the respective cavities at the time $T = \beta\pi/\lambda$, the entanglement of the mesoscopic fields can be transferred to a second pair of atoms, each of which is initially in the state

$$\left|\phi^{'}(0)\right\rangle_{a,j} = (i\left|e\right\rangle_j + \left|g\right\rangle_j)/\sqrt{2}. \tag{25}$$

The initial state of the second atomic pair is equivalent to that of the first atomic pair after interacting with the cavities for a time $\beta\pi/\lambda$ and then undergoing the phase kick. After an interaction time $T^{'} = \beta\pi/\lambda$ with the respective cavities, the second pair of atoms evolve to the maximally entangled state in which the first atomic pair is initially.

To confirm that the entanglement can indeed be transferred to the second pair of atoms, we numerically simulate the entanglement dynamics for these atoms in Fig.3. The degree of entanglement for the second pair of atoms after an interaction time $t^{'}$ with the respective cavities can be quantified by concurrence [22], which is defined as

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{26}$$

where $\lambda_i^{'}s$ are the square roots in decreasing order of the eigenvalues of the operator $\rho_a^{'}(\sigma_y \otimes \sigma_y)\rho_a^{'*}(\sigma_y \otimes \sigma_y)$, with σ_y being the pauli operator and $\rho_a^{'}$ the reduced density operator for the second atomic pair after interaction with the cavities. In Fig. 3 we plot the concurrence for second atomic pair against the interaction time $t^{'}$. As one can see from this figure, the second pair of atoms are in a highly entangled state at the time $t^{'} = \beta\pi/\lambda$, demonstrating the retrieval of entanglement from the cavities back to the qubits. This further verifies that the two cavity modes carry about one ebit of entanglement before interaction with the second atomic pair.

5 Discussion and conclusion

To estimate the effect of decoherence, we take parameters from microwave cavity QED experiments with circular Rydberg atoms and a superconducting millimeter-wave cavity, which has a remarkably long damping time $T_c = 0.13$ s [23]. The states $|f\rangle$, $|g\rangle$, and $|e\rangle$ are the circular states with principal quantum numbers 49, 50, and 51, respectively. The corresponding atomic radiative time is about $T_r = 3 \times 10^{-2} s$. The transition $|g\rangle \longleftrightarrow |e\rangle$ is strongly coupled to the cavity mode with the coupling strength $\lambda = 2\pi \times 25$ kHz. For $\beta = 8$, the required interaction time is $T = 8\pi/\lambda = 1.6 \times 10^{-4}$ s, much shorter than the atomic radiative time. The effective lifetime for the cavity coherence is $\mathrm{T}_{c,eff} \sim T_c/|\beta|^2 = 2.03 \times 10^{-3}$ s, much longer than the interaction time. This implies that the entanglement transfer can be completed before decoherence seriously affects the state evolution.

In conclusion, we show that it is possible to transfer the entanglement from two atomic qubits to two cavity fields initially in thermal states and retrieve it to the next pair of atomic qubits, indicating that continuous-variable systems in mixed states can be used as memory for entanglement. The entanglement reciprocation is not restricted to bipartite entangled states. When n entangled two-level atoms are sent through n respective cavities initially prepared in strongly displaced thermal states, after an interaction time $T = \beta\pi/\lambda$ each atom is in a pure state and the multipartite entanglement is transferred to the mixed cavity fields. The

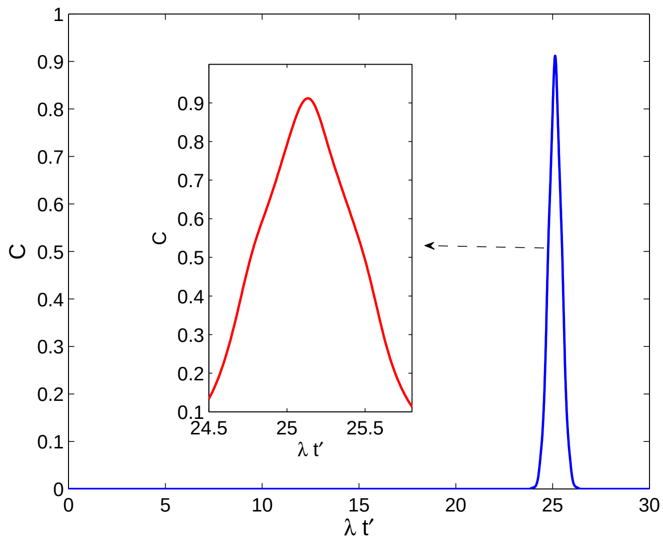

Fig. 3. The concurrence of the second atomic pair as a function of the interaction time. The parameters are the same as those in Fig. 2.

entanglement can be retrieved to the next set of atoms. This process may also be realized with circuit QED, in which controlled coupling between qubits and resonators have been reported [24,25]. The result offers the possibility of realizing interface between qubits and mixed continuous-variable systems. It may have important application in quantum networking in the microwave regime where the thermal photons are not negligible.

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