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QUANTUM CODES FROM CODES OVER GAUSSIAN INTEGERS WITH RESPECT TO THE MANNHEIM METRIC

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In this paper, some nonbinary quantum codes using classical codes over Gaussian integers are obtained. Also, some of our quantum codes are better than or comparable with those known before, (for instance $[[8,2,5]]_{4+i}$).

Keywords: Nonbinary quantum codes, MDS codes, Mannheim metric, Hamming metric
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1. Introduction

An important class of quantum codes are Calderbank-Shor-Steane (shortly CSS) codes. In fact, CSS codes are obtained from two classical codes such that one of these codes contains the other code. Moreover, the bit flip and the phase flip error correcting capacities of a CSS code depends on the classical code that contains the other code and the dual code of the other classical code, respectively [1, pp. 450-451]. The possibility of correcting decoherence errors in entangled states was discovered by Shor [2] and Steane [3]. Binary quantum CSS codes have been constructed in several ways (for instance [3, 10, 11, 12]). In [10], good quantum codes of minimum distance three and four for such length n are obtained via Steane's construction and the CSS construction. In [11], a large number of good quantum codes of minimum distance five and six by Steane's Construction were given. In [12], some quantum error correcting codes, including an optimal quantum code $[[27,13,5]]$, were presented. Later, some results were generalized to the case of nonbinary stabilizer codes [5, 6, 7, 8]. A connection between classical codes and nonbinary quantum codes was given in [5, 6, 7]. However, the theory explained in [5, 6, 7] is not nearly as complete as in the binary case. The closest theory to the binary case of nonbinary stabilizer codes was presented in [8].

On the other hand, the Mannheim metric was introduced by Huber in [9]. It is well known that the Euclidean metric is the relevant metric for maximum-likelihood decoding. Although

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the Mannheim metric is a reasonable approximation to it, it is not a priori, a natural choice. However, the codes being proposed are very useful in coded modulation schemes based on quadrature amplitude modulation (QAM)-type constellations for which neither Hamming nor Lee metric is appropriate.

The rest of this paper is organized as follows. In section 2, classical codes over Gaussian integer ring with respect to the Mannheim metric are given. In Section 3, error bases are defined and quantum codes with respect to the Mannheim distance are constructed.

2. Codes over Gaussian integers

Gaussian integers are a subset of complex numbers which have integers as real and imaginary parts. Let $p = a^2 + b^2 = \pi\overline{\pi} = N(\pi) \equiv 1 \pmod 4$, where $\pi = a + ib$ is a Gaussian integer, $\overline{\pi} = a - ib$ denotes the conjugate of π and p is an odd prime integer. Here, $N(\pi)$ denotes the norm of π. Let G denotes the Gaussian integers and G_π the residue class of G modulo π, where the modulo function $\mu : G \to G_\pi$ is defined according to

$$\mu(\varsigma) = \varsigma \mod \pi = \varsigma - \left[\frac{\varsigma\overline{\pi}}{\pi\overline{\pi}}\right]\pi. \tag{1}$$

$[\cdot]$ denotes rounding of complex numbers. The rounding of complex numbers to Gaussian integers can be done by rounding the real and imaginary parts separately to the closest integer. Hence, G_π becomes a finite field with characteristic p. Let $\alpha, \beta \in G_\pi$ and $\gamma = \beta - \alpha \pmod \pi$. Then, the Mannheim weight of γ is defined as $w_M(\gamma) = |\mathrm{Re}\,(\gamma)| + |\mathrm{Im}\,(\gamma)|$. Also, the Mannheim distance d_m between α and β is defined as $d_M(\alpha, \beta) = w_M(\gamma)$. Let C be code of length n over G_π and let $c = \begin{pmatrix} c_0, & c_1, & \cdots, & c_{n-1} \end{pmatrix}$ be a codeword. Then the Mannheim weight of c is equal to $\sum_{i=0}^{n-1} (|\mathrm{Re}(c_i)| + |\mathrm{Im}(c_i)|)$. Note that A Mannheim error of weight one takes on one of the four values ± 1, $\pm i$ [9]. It is well known that the Hamming weight of c is the number of the non-zero entries of c. We give an example to compare a classical code with respect to these metrics.

Example 1 *Let $p = 17$. Then,*

$$G_{4+i} = \{0, \pm 1, \pm i, \pm 2, \pm 2i \pm (1+i), \pm(1-i), \pm(2-i), \pm(1+2i)\}.$$

Let the generator matrix of C over G_{4+i} be $\begin{pmatrix} -1+i, & 1 \end{pmatrix}$. Then, the set of the codewords of C is

$$C = \left\{ \begin{array}{cccc} (0,0), & (-1+i,1), & (1-i,-1), & (-1-i,i), \\ (1+i,-i), & (-1-2i,2), & (1+2i,-2), & (2-i,2i), \\ (-2+i,-2i), & (-2,1+i), & (2i,1-i), & (2,-1+i), \\ (2,-1-i), & (-i,2-i), & (i,-2+i), & (1,1+2i), \\ (-1,-1-2i) & & & \end{array} \right\}.$$

The minimum Mannheim distance of the code C is 3 and the minimum Hamming distance of the code C is 2. Let us assume that at the receiving end we get the vector $r = (\ -1+i,\ 0\ )$. The minimum Mannheim distance between r and the codewords of C is 1, namely, $d_M(r, (\ -1+i,\ 1\ )) = 1$. Thus, we can correct this error with respect to the Mannheim metric. But, we can not correct this error with respect to the Hamming metric since $d_H(r, (\ -1+i,\ 1\ )) = 1$ and $d_H(r, (\ 0,\ 0\ )) = 1$.

We now define a block code C of length n over G_π as a set of codewords

$$c = \left(\begin{array}{cccc} c_0, & c_1, & \cdots & , c_{n-1} \end{array} \right)$$

with coefficients $c_i \in G_\pi$. Let $\alpha_1, \alpha_2 \in G_\pi$ be two different elements of orders $p-1$ such that $\alpha_1^{p-1/4} = i$ and $\alpha_2^{p-1/4} = -i$. Hence, $x^{p-1/4} - i$ and $x^{p-1/4} + i$ are factored as $(x-\alpha_1)\left(x-\alpha_1^5\right)\cdots\left(x-\alpha_1^{p-4}\right)$ and $(x-\alpha_2)\left(x-\alpha_2^5\right)\cdots\left(x-\alpha_2^{p-4}\right)$, respectively. Also, the polynomials $x^{p-1/2}+1$ and $x^{p-1}-1$ are factored as

$$(x-\alpha_1)\left(x-\alpha_1^5\right)\cdots\left(x-\alpha_1^{p-4}\right)(x-\alpha_2)\left(x-\alpha_2^5\right)\cdots\left(x-\alpha_2^{p-4}\right) \tag{2}$$

and

$$\left(x^{p-1/2}+1\right)\left(x^{p-1/2}-1\right), \tag{3}$$

respectively. A monic polynomial $g(x)$ in $G_\pi[x]$ is the generator polynomial for a cyclic code if and only if $g(x)|\, x^n \pm 1$, where $G_\pi[x]$ is the set of all polynomials with coefficients in G_π. Hence, Using (2), we always can construct two classical codes C_1, C_2 of length $n=(p-1)/2$ over G_π such that $C_2 \subset C_1$.

3. Nonbinary quantum CSS codes

Let p be an odd prime, let $p = \pi\overline{\pi} \equiv 1 \pmod 4$. A p-ary quantum code Q of length n and size K is a $K-$dimensional subspace of a p^n-dimensional Hilbert space. This Hilbert space is identified with the $n-$fold tensor product of $p-$dimensional Hilbert space, that is, $\left(\mathcal{C}\surd\right)^{\otimes n} = \mathcal{C}\surd \otimes \mathcal{C}\surd \cdots \mathcal{C}\surd$, where $\mathcal{C}$ denotes complex numbers. We denote by $|u\rangle$ the vectors of a distinguished orthonormal basis of $\mathcal{C}\surd$, where the labels u range over the elements of the finite field H_π. For $u = \left(\begin{array}{cccc} u_0, & u_1, & \cdots & , u_{n-1} \end{array} \right), v = \left(\begin{array}{cccc} v_0, & v_1, & \cdots & , v_{n-1} \end{array} \right) \in G_\pi^n$, let $u \cdot v = \sum u_i v_i$ be the usual inner product on G_π^n. For $(u|\, v), (u'|\, v') \in G_\pi^{2n}$, set $(u|\, v) * (u'|\, v') = Tr(vu' - v'u)$, where $Tr : G_{\pi^k} \to G_\pi$ is the trace map. For the integer $k=1$ then $(u|\, v) * (u'|\, v') = (vu' - v'u)$. Let $w = (u|\, v) - (u'|\, v') = (u_i - u_i^{'} | v_i - v_i^{'}) = (w_i | w_i^{'}) \; (mod\, \pi)$, for $i = 0, 1, 2, ..., n-1$, we define the Mannheim weight of w and the Mannheim distance between $(u|\, v)$ and $(u'|\, v')$ as

$$wt_M(w) = \left[\left[\begin{array}{c} |\mathrm{Re}(w_0)| + |\mathrm{Im}(w_0)| + \cdots + |\mathrm{Re}(w_{n-1})| + |\mathrm{Im}(w_{n-1})| \\ + \left|\mathrm{Re}\left(w_0^{'}\right)\right| + \left|\mathrm{Im}\left(w_0^{'}\right)\right| + \cdots + \left|\mathrm{Re}\left(w_{n-1}^{'}\right)\right| + \left|\mathrm{Im}\left(w_{n-1}^{'}\right)\right| \end{array} \right] \Big/ 2 \right]$$

$d_M((u|\, v), (u'|\, v')) = wt_M(w)$, respectively. Let $C \subset G_\pi^{2n}$. Then the dual code $C^{\perp_*}$ of C is defined to be

$$C^{\perp_*} = \left\{ (u\,|v) \in G_\pi^{2n} : (u|\, v) * (u'|\, v') = 0 \text{ for all } (u'|\, v') \in C \right\}.$$

Definition 1 *The unitary operators were defined in [8] as* $X_a|u\rangle = |(a+u)\rangle$, $Z_b|u\rangle = \xi^{(bu)}|u\rangle$ *, where* a, b *are elements of the finite fields* F_p*, and* ξ *is a primitive pth root of unity.*

Definition 2 *We define the unitary operators as* $X_a|u\rangle = |\mu(a+u)\rangle$, $Z_b|u\rangle = \xi^{\mu^{-1}(bu)}|u\rangle$ *, where* $a, b \in G_\pi$*,* ξ *is a primitive pth root of unity, and the function* $\mu : F_p \to G_\pi$ *defines* $\mu(g) = g - [g\overline{\pi}/p]\,\pi$.

Also, we define the Hadamard gate as

$$H_{gate} = \frac{1}{\sqrt{N(\pi)}} \left(a_{s,t}\right), \ a_{s,t} = \xi^{(s-1)(t-1) \ (\bmod \ p)}, \ \ 1 \le s,t \le p = \pi.\overline{\pi} = N(\pi).$$

For example, let $\pi = 2+i$. Then,

$$H_{gate} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & \xi & \xi^2 & \xi^3 & \xi^4 \\ 1 & \xi^2 & \xi^4 & \xi & \xi^3 \\ 1 & \xi^3 & \xi & \xi^4 & \xi^2 \\ 1 & \xi^4 & \xi^3 & \xi^2 & \xi \end{pmatrix}.$$

Note that $H_{gate}H_{gate}^{\dagger} = H_{gate}^{\dagger}H_{gate} = I_p$, where $H_{gate}^{\dagger}$ denotes the conjugate transpose of H_{gate} and I_p denotes the identity matrix in p dimensions.

Theorem 1 *(CSS Code Construction) Let C_1 and C_2 denote two classical linear codes over G_π with the parameters $[n,k_1,d_{M_1}]_\pi$ and $[n,k_2,d_{M_2}]_\pi$ such that $C_2 \subseteq C_1$. Then, there exists an $[[n,k_1-k_2,d_M]]_\pi$ quantum code with minimum distance $d_M = \min\left\{d_{M_1}, d_{M_2}^{\perp}\right\}$, where $d_{M_2}^{\perp}$ denotes the minimum Mannheim distance of the dual code $C_2^{\perp}$ of the code C_2.*

Proof.

□.

Let x be a codeword of C_1. Then, we define the quantum state

$$|x+C_2\rangle = \frac{1}{\sqrt{C_2}} \sum_{y \in C_2} |x+y\rangle,$$

where $+$ is bitwise addition modulo π. If $x^{'}$ is an element of C_1 such that $x - x^{'} \in C_2$ then, $|x+C_2\rangle = \left|x^{'}+C_2\right\rangle$, and thus the state $|x+C_2\rangle$ depends only upon the coset of C_1/C_2. The number of cosets of C_2 in C_1 is equal to $|C_1|/|C_2|$ so the dimension of the quantum code is $N(\pi)^{k_1-k_2}$. Hence, we define the quantum code Q_{C_1,C_2} as the vector space spanned by t he state $|x+C_2\rangle$ for all $x \in C_1$. Therefore, the quantum code Q_{C_1,C_2} is an $[[n,k_1-k_2,d_M]]_\pi$.

We now explain the minimum Mannheim distance d_M of the quantum code Q_{C_1,C_2} equals $\min\left\{d_{M_1}, d_{M_2}\right\}$. Suppose that a bit flip error occurs at only one qubit in n qubit and a phase flip error occurs at only one qubit in n qubit. If $|x+C_2\rangle$ was the original state then the corrupted state is

$$\frac{1}{\sqrt{C_2}} \sum_{y \in C_2} \xi^{\mu^{-1}\left((x+y)\widehat{e_2}\right)} \left|(x+y+\widehat{e}_1) \mod \pi\right\rangle.$$

To detect where bit flip error occurred it is convenient to introduce an ancilla containing sufficient qubits to store the syndrome for the code C_1, and initially in the all zero state $|0\rangle$. We use reversible computation to apply the parity check matrix H_1 for the code C_1, taking $|x+y+\widehat{e}_1\rangle|0\rangle$ to $|x+y+\widehat{e}_1\rangle|H_1(x+y+\widehat{e}_1)\rangle = |x+y+\widehat{e}_1\rangle|H_1(\widehat{e}_1)\rangle$, since $(x+y) \in C_1$ is annihilated by the parity check matrix. The effect of this operation is to produce the state:

$$\frac{1}{\sqrt{C_2}} \sum_{y \in C_2} \xi^{\mu^{-1}\left((x+y)\widehat{e_2}\right)} |x+y+\widehat{e}_1\rangle|H_1(\widehat{e}_1)\rangle.$$

Error detection for the bit flip error is completed by measuring the ancilla to obtain the result $H_1(\widehat{e}_1)$ and discarding the ancilla. This shows that the bit flip error correcting capacity of the quantum code Q_{C_1,C_2} depends on the classical code C_1. We now show the phase flip error correcting capacity of the quantum code Q_{C_1,C_2} depends on the dual code $C_2^{\perp}$ of the classical code C_2. The latest state of the corrupted state, discarding the ancilla, is:

$$\frac{1}{\sqrt{C_2}} \sum_{y \in C_2} \xi^{\mu^{-1}\left((x+y)\widehat{e}_2\right)} |x+y\rangle.$$

We apply the Hadamard gates to each qubit, taking the state to

$$\frac{1}{\sqrt{C_2 N(\pi)^n}} \sum_{z} \sum_{y \in C_2} \xi^{\mu^{-1}\left((x+y)\left(\widehat{e}_2+z\right)\right)} |z\rangle,$$

where the sum is over all possible values for n bit z. Setting $z^{'} \equiv z + \widehat{e}_2(\mathrm{mod}\pi)$, we obtain

$$\frac{1}{\sqrt{N(\pi)^n / C_2}} \sum_{z^{'} \in C_2^{\perp}} \xi^{\mu^{-1}\left(xz^{'}\right)} \left|z^{'} + \widehat{e}_2\right\rangle.$$

Note that if $z^{'} \in C_2^{\perp}$ then $\sum_{y \in C_2} \xi^{\mu^{-1}(yz^{'})} = |C_2|$, and if $z^{'} \notin C_2^{\perp}$ then $\sum_{y \in C_2} \xi^{\mu^{-1}(yz^{'})} = 0$. This looks just like a bit flip error described by the vector $\widehat{e}_2$. To determine the error $\widehat{e}_2$, we introduce an ancilla qubit and reversibly apply the parity check matrix H_2 for $C_2^{\perp}$ to obtain $H_2\widehat{e}_2$, and correct the error $\widehat{e}_2$, obtaining the state

$$\frac{1}{\sqrt{N(\pi)^n / |C_2|}} \sum_{z^{'} \in C_2^{\perp}} \xi^{xz^{'}} \left|z^{'}\right\rangle.$$

The error correcting is completed by applying the inverse Hadamard gates, $H_{gate}^{\dagger}$, to each qubit. This takes us back to the initial state with $\widehat{e}_2 = 0$. Hence, the proof is completed.

We use the Mannheim metric to determine the positions and the value of the errors $\widehat{e}_1, \widehat{e}_2$.

Let the minimum Mannheim distance of C_1 and $C_2^{\perp}$ be d_m. Then, the number of the errors corrected by the quantum code Q_{C_1,C_2} obtained from the classical codes C_1, C_2 is equal to

$$4\begin{pmatrix} n \\ 1 \end{pmatrix} + 4^2 \begin{pmatrix} n \\ 2 \end{pmatrix} + \cdots + 4^t \begin{pmatrix} n \\ t \end{pmatrix},$$

where $t = \lfloor (d_m - 1)/2 \rfloor$ and the symbol $\begin{pmatrix} \cdot \\ \cdot \end{pmatrix}$ gives the binomial coefficient.

Theorem 2 *Let* $C = \left(C_2 \middle| C_1^{\perp}\right)$ *be code in* G_{π}^{2n} *such that* $C \subset C^{\perp_*}$, *where* C_1 *and* C_2 *denote two classical codes, and* $C_1^{\perp}$ *denotes the dual code of* C_1. *Then, there exists an* $[[n, K, d_M]]_{\pi}$ *quantum code with the minimum distance*

$$d_M = \min\left\{wt_M(w) : \ w \in C^{\perp_*} \backslash C\right\},$$

where $K = dim(C^{\perp_*}) - dim(C)$.

The proof of Theorem 2 can be easily seen from the proof of Theorem 1.

Example 2 *Let $\pi = 4 + i$. Let the generator polynomial of the code C_1 be $g_1(x) = 1 + 2i + (-1+i)x - ix^2 + x^3$ and let the generator polynomial of the code C_2 be $g_2(x) = 1 - i + (2-i)x + (-1+i)x^2 - ix^3 - ix^4 + x^5$. Hence, using the codes C_1 and C_2 we obtain a quantum code with parameters $[[8, 2, 5]]_{4+i}$ with respect to the Mannheim metric since the minimum distance of C_1 and $C_2^\perp$ are 5. Let the quantum state $|\psi\rangle = \left| \; 1-i, \quad 2-i, \quad -1+i, \quad -i, \quad -i, \quad 1, \quad 0, \quad 0 \; \right\rangle$. If the operator $IIIX_1X_1III$ acts on this state, then the corrupted state becomes*

$$\left| \; 1-i, \quad 2-i, \quad -1+i, \quad 1-i, \quad 1-i, \quad 1, \quad 0, \quad 0 \; \right\rangle.$$

The quantum code $[[8, 2, 5]]_{4+i}$ with respect to the Mannheim metric overcomes this error since the minimum Mannheim distance of the classical code C_1 is equal to 5. Also, the number of the bit flip errors corrected by this quantum code is 480.

On the other hand, let F_{17} be a finite field of characteristic 17 and let C_1, C_2 be the classical codes with respect to the Hamming metric such that $C_2 \subset C_1$. Then a quantum code with parameters $[[8, 2, 4]]_{17}$ can be obtained. The code $[[8, 2, 4]]_{17}$ is a maximum distance separable (shortly MDS) since this code attains the quantum singleton bound, namely, $17^2 = 17^{8-2.4+2}$. Also, the number of the bit flip errors corrected by this quantum code is 128. For the length $n = 8$, a quantum code having the minimum distance greater than 4 is not obtained with respect to the Hamming metric. So, it is obvious that the quantum code obtained here is better than the quantum code of the same length with respect to the Hamming metric.

In Table I, some CSS codes compared with respect to the Hamming metric and the Mannheim metric are given. The CSS codes constructed from classical codes with respect to the Hamming metric can be found in the HM column of Table I. The CSS codes constructed from classical codes with respect to the Mannheim metric can be found in the MM column of Table I. It is obvious that, some of the quantum codes obtained in this paper can correct more errors than the quantum MDS codes of the same length given in Table I. Using a computer program, we compute the minimum Mannheim distance of the codes given in Table I.

4. Tables

Table I: Some CSS codes compared with respect to the Hamming metric and the Mannheim metric.

p	α_1	α_2	h_1	g_2	HM	MM
5	i	$-i$	$x^3 - ix^2 - x + i$	$x^3 + ix^2 - x - i$	$[[4,2,2]]_{2+i}$	$[[4,2,2]]_{2+i}$
13	2	-2	$(1-i) - x + x^2$	$(1-i) + x + x^2$	$[[6,2,3]]_{3+2i}$	$[[6,2,4]]_{3+2i}$
13	2	-2	$-i - 2x + 2ix^2 + x^3$	$1 - i + x + x^2$	$[[6,1,3]]_{3+2i}$	$[[6,1,4]]_{3+2i}$
13	2	-2	$x - 2$	$x + 2$	$[[6,4,2]]_{3+2i}$	$[[6,4,2]]_{3+2i}$
13	2	-2	$-1 + ix + x^2$	$-i + (-1+i)x + x^2$	$[[6,2,2]]_{3+2i}$	$[[6,2,2]]_{3+2i}$
17	$1+i$	$-2+i$	$-1 + i + (2-i)x$ $+(1-i)x^2 - ix^3$ $+ix^4 + x^5$	$-i + (-2i)x$ $+x^3 + x^4$	$[[8,1,4]]_{4+i}$	$[[8,1,5]]_{4+i}$
17	$1+i$	$-2+i$	$(-2+i) + (1+i)x$ $+(2-i)x^2 + x^3$	$(2-i) + (1+i)x$ $-(2-i)x^2 + x^3$	$[[8,2,4]]_{4+i}$	$[[8,2,5]]_{4+i}$
17	$1+i$	$-2+i$	$-1 + (1+i)x + x^2$	$-1 - (1+i)x + x^2$	$[[8,4,3]]_{4+i}$	$[[8,4,\geq 3]]_{4+i}$
17	$1+i$	$-2+i$	$-(1+i) + x$	$1 + i + x$	$[[8,6,2]]_{4+i}$	$[[8,6,\geq 2]]_{4+i}$
19	$-1+i$	$-2+i$	$-2 + x$	$2 + x$	$[[14,12,2]]_{5+2i}$	$[[14,12,\geq 2]]_{5+2i}$

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