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QUANTUM DISCORD OF A THREE-QUBIT W-CLASS STATE IN NOISY ENVIRONMENTS

HUI GUO
State Key Laboratory of Precision Spectroscopy, Department of Physics
East China Normal University, Shanghai 200062, China

JIN-MING LIU[a]
State Key Laboratory of Precision Spectroscopy, Department of Physics
East China Normal University, Shanghai 200062, China
and
Department of Physics and Centre for Quantum Technologies, National University of Singapore
2 Science Drive 3, Singapore 117542, Singapore

CHENG-JIE ZHANG   C. H. OH
Department of Physics and Centre for Quantum Technologies, National University of Singapore
2 Science Drive 3, Singapore 117542, Singapore

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We study the dynamics of the pairwise quantum discord (QD), classical correlation (CC), and entanglement of formation (EOF) for the three-qubit W-class state $|W\rangle_{123} = \frac{1}{2}\left(|100\rangle_{123} + |010\rangle_{123} + \sqrt{2}\,|001\rangle_{123}\right)$ under the influence of various Markovian noises by analytically solving the master equation in the Lindblad form. Through numerical analysis, we find that EOF decreases asymptotically to zero with time for the dephasing noise, but it undergoes sudden death for the bit-flip noise, the isotropic noise, as well as the dissipative and noisy environments. Moreover, QD decays to zero in an asymptotical way for all the noises we investigated. Thus, when the W-class state $|W\rangle_{123}$ is subject to the above Markovian noises, QD is more robust than EOF against decoherence excluding the phase-flip noise, implying that QD is more useful than entanglement to characterize the quantum correlation. We also find a remarkable character for the CC in the presence of the phase-flip noise, *i.e.*, CC displays the behavior of sudden transition and then keeps constant permanently, but the corresponding QD just exhibits a very small sudden change. Furthermore, we verify the monogamic relation between the pairwise QD and EOF of the W-class state.

Keywords: Quantum discord, entanglement of formation, W-class state, noisy environments.
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1 Introduction

Quantum entanglement plays an important role in quantum communication and quantum computation [1]. However, entanglement is not the only type of quantum correlation applied to quantum information processing, there are some quantum correlations other than

[a]Corresponding author, E-mail: jmliu@phy.ecnu.edu.cn

entanglement [2–4] which also offer some specific advantages for achieving tasks of quantum information. Distinguishing classical and quantum correlations in quantum systems is therefore an important problem. In 2001, Ollivier and Zurek [5] introduced the concept of the so called quantum discord (QD) to describe the difference between the total and classical correlations for a given state as measured by von Neumann entropy. QD may be still present for some separate states [6, 7], implying that it is a more proper measure of the quantumness of correlation than entanglement. In 2008, QD was used to characterize the correlations in the model of deterministic quantum computation with one quantum bit [8, 9]. Subsequently, quantum correlations in spin-half chain [10, 11] and the potential of QD to detect critical points of quantum phase transition at finite temperature [12, 13] were discussed. The behavior of QD in the process of the Grover search algorithm [14] and the relation between QD and entanglement irreversibility [15] were also investigated. Moreover, Parashar and Rana [16] calculated the analytic expression of QD via relative entropy for superposition of some orthonormal Greenberger-Horne-Zeilinger (GHZ) states and conjectured the discord for W states. In addition, some algorithms to analytically evaluate the QD for two-qubit X states or general two-qubit states were derived in Refs. [17–20].

On the other hand, much attention has been paid to study the dynamics of QD in realistic physical systems under the influence of the environment. For example, the authors of Refs. [21–24] investigated the influence of Markovian and non-Markovian environment on the QD dynamics of two qubits. Their results indicated that QD is more robust than entanglement against decoherence. Later, Li *et al.* [25] examined the dynamics of two-qubit quantum and classical correlations in local and global environments, and the relations in these correlations. Li *et al.* [26] studied the entanglement and QD dynamics of two identical atoms resonantly coupled to a single-mode cavity under feedback control. Furthermore, Pei *et al.* [27] analyzed the dynamics of quantum coherence and QD in two coupled semiconductor double-dot molecules separated by a distance and indirectly coupled via a transmission line resonator. Altintas and Eryigit [28] considered the analytic dynamics of QD, concurrence and Bell nonlocalities for initial GHZ- or W-type mixed states in the presence of Ornstein-Uhlenbeck noise [29]. Apart from the relevant theoretical progress listed above, Xu *et al.* [30] experimentally demonstrated the dynamics of classical and quantum correlations between biqubit systems in a one-sided phase-damping channel. However, the above mentioned papers mainly referred to the time evolution of quantum correlations for the systems of two qubits. To the best of our knowledge, few studies [28] were pursued with the QD dynamics of three-qubit or multi-qubit states in various noisy environments.

It is well accepted that there are two nonequivalent classes for the tripartite entangled states [31], the GHZ class and the W class, even under stochastic local operation and classical communication. In particular, an interesting property of W-class state is that it remains bipartite entanglement when any one of the three qubits is traced out. In 2006, Agrawal and Pati [32] showed that there exists a W-class state, taking the form of $|W\rangle_{123} = \frac{1}{2}\left(|100\rangle_{123} + |010\rangle_{123} + \sqrt{2}\,|001\rangle_{123}\right)$, which can be used for perfect quantum teleportation and superdense coding. Because of the unavoidable interaction between a quantum system and its external environment, Jung *et al.* [33] considered the quantum teleportation process through the noisy W-class and GHZ channels by solving analytically the master equation in the Lindblad form. Later, Hu [34] studied the robustness between the W-class state and the

GHZ state in terms of their teleportation capacity under the influence of different external environments.

In this paper, we study the dynamics of the three physical quantities, namely, pairwise QD, classical correlation (CC), and entanglement of formation (EOF) of the W-class state $|W\rangle_{123}$ coupled to different Markovian reservoirs (say, Pauli noises, zero temperature and infinite temperature environments). Our results show that the three physical quantities are independent measures of correlation without simple relative ordering between them. Moreover, we find a remarkable property of pairwise CC for the dephasing noise case, namely, CC displays the behavior of sudden transition at a critical time and then remains a fixed value after the time while QD just exhibits a very small sudden change, which is disagreement with the result that the sudden transition between classical and quantum decoherence occurs in Ref. [35]. We also find that the EOF of the W-class state suffers the phenomenon of entanglement sudden death (ESD) [36–39] apart from the phase-flip noise case, but QD always decays to zero asymptotically for all the noises discussed in the present paper. Therefore, when the W-class state $|W\rangle_{123}$ is affected by the above Markovian noises, QD is more robust than EOF against decoherence except for the dephasing noise, implying that QD is more useful than the EOF to describe the quantum correlation. To enrich our discussion for the pairwise quantum correlations of the three-qubit system, the monogamic relation of the noisy W-class state between the QD and the EOF is also verified through numerical analysis.

The paper is organized as follows. We review the concept of QD and EOF in Section 2. In Section 3, we study the dynamics of the three physical quantities (pairwise QD, CC, and EOF) for the W-class state $|W\rangle_{123}$ exposed to different Pauli noises. Then the monogamic relation of the noisy W-class state is verified through numerical analysis. In Section 4 a similar process mentioned in the preceding section is investigated by replacing the Pauli noises with different sources of decoherence. Section 5 ends with a summary.

2 Quantum Discord and Entanglement of Formation

Generally, a bipartite quantum system contains both classical and quantum correlations. The total correlation [40, 41] of the bipartite system can be expressed by quantum mutual information with $I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$ [42], where ρ_{AB} denotes the density operator of the composite bipartite system AB, ρ_A (ρ_B) denotes the reduced density matrix over the subsystem B (A), and $S(\rho) = -tr(\rho \log_2 \rho)$ is the von Neumann entropy. Moreover, quantum mutual information can also be written as a sum of classical correlation $C(\rho_{AB})$ and quantum correlation $D(\rho_{AB})$, that is, $I(\rho_{AB}) = C(\rho_{AB}) + D(\rho_{AB})$ [5, 17, 18]. The quantum part $D(\rho_{AB})$ is called QD, but it is not easy to be computed. The reason lies in the complicated maximization procedure for calculating the classical correlation because the maximization is to be done over all possible von Neumann measurements of one subsystem. Depending on the maximal information of ρ_{AB} with measurement on one of the subsystems, classical correlation (CC) is defined as $C(\rho_{AB}) = S(\rho_A) - \min_{B_i^+ B_i} \sum_i q_i S\left(\rho_A^i\right)$, where $B_i^+ B_i$ is a positive-operator-valued measure performed on the subsystem B and $q_i = tr_{AB}(B_i \rho_{AB} B_i^+)$, and $\rho_A^i = tr_B(B_i \rho_{AB} B_i^+)/q_i$ is the postmeasurement state of A after obtaining outcome i on particle B. Once $C(\rho_{AB})$ is in hand, QD is simply gained by $D(\rho_{AB}) = I(\rho_{AB}) - C(\rho_{AB})$. In fact, the QD and CC are not symmetric quantities with respect to the measurements performed on subsystem A or B. In the subsequent discussions of QD and CC, we use the subscript f to denote the measurement on the first subsystem and the subscript s to denote the measurement on the second.

The entanglement of formation (EOF) [43] for any two-qubit systems ρ is defined by the analytical formula $E = H\left(\frac{1+\sqrt{1-C'^2}}{2}\right)$, where the function $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$ and the concurrence $C' = \max\{0, \Lambda\}$. Here $\Lambda = \sqrt{\mu_1} - \sqrt{\mu_2} - \sqrt{\mu_3} - \sqrt{\mu_4}$, and μ_j ($j = 1$, 2, 3, 4) are the eigenvalues in decreasing order of the matrix $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, with σ_y representing the Pauli matrix and ρ^* the complex conjugate of ρ. The concurrence $C' = 0$ corresponds to a separable state and $C' = 1$ corresponds to a maximally entangled state.

3 W-class State subject to Pauli Noises

In general, a real quantum system will unavoidably interact with its surrounding environment. To describe the interaction process, we assume that the environment is Markovian. In quantum optics, the evolution of an open quantum system can be described with the following master equation of the density operator $\rho(t)$ in Lindblad form [44],

$$\frac{\partial\rho}{\partial t} = -\mathrm{i}[H,\rho] + \sum_{j,k}\frac{\gamma_k}{2}(2L_{j,k}\rho L_{j,k}^{+} - \{L_{j,k}^{+}L_{j,k},\rho\}), \tag{1}$$

where H is the system Hamiltonian and $\hbar = 1$, γ_k denotes the coupling strengths of the qubits with their respective environments, the Lindblad operator $L_{j,k}$ acts on the jth qubit, and the brace $\{\}$ means the anticommutator.

In what follows, we will consider the dynamics of pairwise QD and EOF of the W-class state $|W\rangle_{123}$ infected by various kinds of Pauli noises. We assume that each individual particle of the W-class state interacts independently with its surrounding environment. In this situation, the Lindblad operator is given by $L_{j,k} = \sigma_k^{(j)}$ with $\sigma_k^{(j)}$ being the Pauli matrix acting on the jth qubit ($k = x$, y, z). To demonstrate the time evolution of the W-class state, it is more convenient to use the form of its density matrix $\rho(W) = |W\rangle_{123}\langle W|$ with its matrix elements χ_{mn} ($m = 0, 1, 2, \ldots, 7$, $n = 0, 1, 2, \ldots, 7$). Then the initial matrix elements of the W-class state are $\chi_{11} = 1/2$, $\chi_{12} = \chi_{21} = \chi_{14} = \chi_{41} = \sqrt{2}/4$, $\chi_{22} = \chi_{24} = \chi_{42} = \chi_{44} = 1/4$ under the basis $\{|000\rangle, |001\rangle, |010\rangle, \ldots, |111\rangle\}$, and the rest matrix elements of $\rho(W)$ are $\chi_{mn} = 0$ if m and n are equal to other values.

(i). We first examine the situation that the three particles 1, 2, 3 are simultaneously subject to the phase-flip noise or the dephasing noise described by the Lindblad operators $L_{1,z}$, $L_{2,z}$, $L_{3,z}$, respectively, because it is simplest to solve the master equation (1) with the assumption of $H = 0$ here and after. After the interaction of the three particles with the environment, the initial pure state $|W\rangle_{123}$ will become a noisy W-class state. For simplicity, assuming that all the environment coupling strengths $\gamma_k = \gamma$ throughout this paper, we can obtain the density matrix elements $\rho_{evo}(W)$ of the time evolution of the W-class state

$$\begin{aligned}
\chi_{11} &= \frac{1}{2},\ \chi_{22} = \chi_{44} = \frac{1}{4}, \\
\chi_{12} &= \chi_{21} = \chi_{14} = \chi_{41} = \frac{\sqrt{2}}{4}e^{-4\gamma t},\ \chi_{24} = \chi_{42} = \frac{1}{4}e^{-4\gamma t},
\end{aligned} \tag{2}$$

and $\chi_{mn} = 0$ for the other values of m, n.

It is easy to obtain the reduced density matrix ρ_{12} of $\rho_{evo}(W)$ by tracing out the particle 3,

$$\rho_{12} = tr_3\left(\rho_{evo}\left(W\right)\right) = \frac{1}{4}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & e^{-4\gamma t} & 0 \\ 0 & e^{-4\gamma t} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

where tr_3 is the partial trace over particle 3. Due to the exchange invariability of the particles 1 and 2, the expressions for the two reduced density matrices ρ_{13} and ρ_{23} are identical and given by

$$\rho_{13} = tr_2\left(\rho_{evo}\left(W\right)\right) = \rho_{23} = tr_1\left(\rho_{evo}\left(W\right)\right) = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & \sqrt{2}e^{-4\gamma t} & 0 \\ 0 & \sqrt{2}e^{-4\gamma t} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4}$$

From Eqs. (3) and (4), we can see that the states of the reduced density matrices ρ_{12} and ρ_{13} (ρ_{23}) take the form of the so-called X states, *i.e.*, the biqubit density matrices ρ_{12} and ρ_{13} only contain nonzero elements along the main diagonal and anti-diagonal. By numerical calculations, we plot QD, CC and EOF [43] versus γt under the influence of the dephasing noise for the density matrix ρ_{12} in Fig. 1(a) and those for ρ_{13} in Fig. 1(d) and Fig. 1(h). In Fig.1, D_f and D_s (C_f and C_s) represent the QD (the CC) considering the measurement over the first subsystem and the second subsystem, respectively. Due to the symmetry of ρ_{12} under the exchange of particles 1 and 2, $D_f(\rho_{12})$ is identical with $D_s(\rho_{12})$. So in the following we only analyze the time evolution of $D_f(\rho_{12})$ for simplicity.

(ii). Next, we consider the situation where the three particles are simultaneously subject to a bit-flip noise described by the Lindblad operators $L_{1,x}$, $L_{2,x}$, $L_{3,x}$. By analytically solving the master equation (1), we can get the density matrix elements of the noisy W-class state $\rho_{evo}(W)$,

$$\begin{aligned}
\chi_{00} &= \frac{1}{8}\left(1+e^{-2\gamma t}-e^{-4\gamma t}-e^{-6\gamma t}\right),\ \chi_{11}=\frac{1}{8}\left(1+e^{-2\gamma t}+e^{-4\gamma t}+e^{-6\gamma t}\right),\\
\chi_{22} &= \chi_{44}=\frac{1}{8}\left(1+e^{-6\gamma t}\right),\ \chi_{33}=\chi_{55}=\frac{1}{8}\left(1-e^{-6\gamma t}\right),\\
\chi_{66} &= \frac{1}{8}\left(1-e^{-2\gamma t}+e^{-4\gamma t}-e^{-6\gamma t}\right),\ \chi_{77}=\frac{1}{8}\left(1-e^{-2\gamma t}-e^{-4\gamma t}+e^{-6\gamma t}\right),\\
\chi_{03} &= \chi_{05}=\sqrt{2}\chi_{06}=\frac{\sqrt{2}}{16}\left(1+e^{-2\gamma t}-e^{-4\gamma t}-e^{-6\gamma t}\right),\\
\chi_{12} &= \chi_{14}=\sqrt{2}\chi_{24}=\frac{\sqrt{2}}{16}\left(1+e^{-2\gamma t}+e^{-4\gamma t}+e^{-6\gamma t}\right),\\
\chi_{27} &= \chi_{47}=\sqrt{2}\chi_{17}=\frac{\sqrt{2}}{16}\left(1-e^{-2\gamma t}-e^{-4\gamma t}+e^{-6\gamma t}\right),\\
\chi_{36} &= \chi_{56}=\sqrt{2}\chi_{35}=\frac{\sqrt{2}}{16}\left(1-e^{-2\gamma t}+e^{-4\gamma t}-e^{-6\gamma t}\right),
\end{aligned} \tag{5}$$

with $\chi_{mn}=\chi_{nm}$ and $\chi_{mn}=0$ for the other values of m, n.

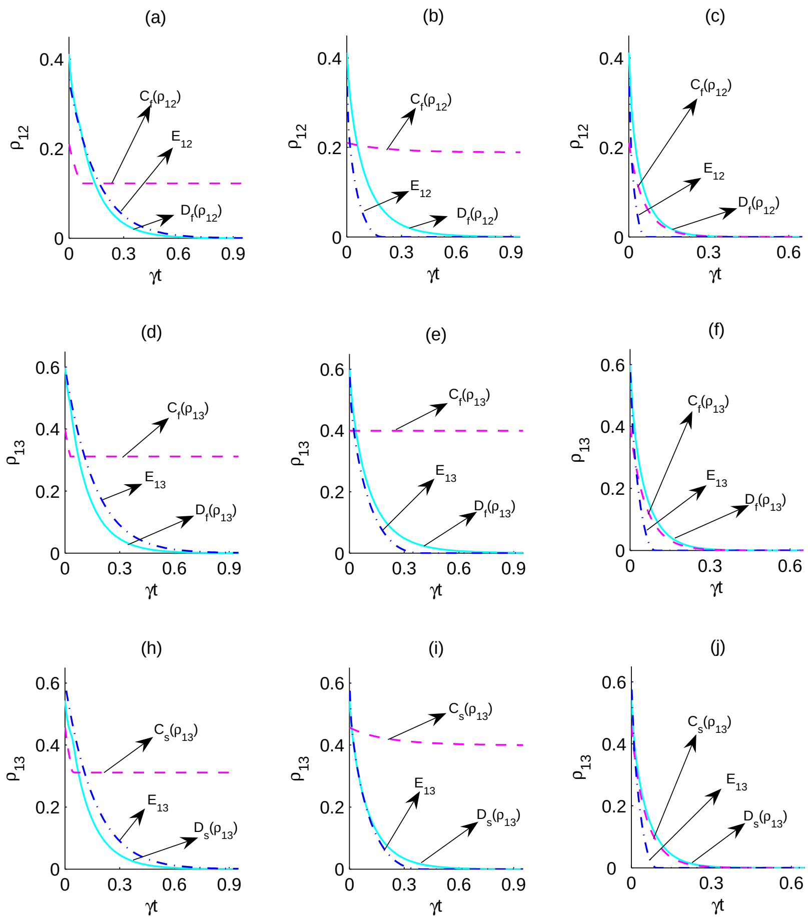

Fig. 1. For the density matrices ρ_{12} and ρ_{13}, QD, CC, and EOF are plotted as a function of γt, where (a, d, h) correspond to phase-flip noise, (b, e, i) to bit-flip noise; (c, f, j) to isotropic noise. The solid line stands for QD marked as D_f or D_s, the dashed line for CC marked as C_f or C_s, and the dot-dashed line for EOF marked as E_{12} or E_{13}.

Using the similar method as case (i), the reduced density matrix ρ_{12} can be obtained as

$$\rho_{12} = tr_3\left(\rho_{evo}\left(W\right)\right) = \frac{1}{8}\begin{pmatrix} 2+2e^{-2\gamma t} & 0 & 0 & 1-e^{-4\gamma t} \\ 0 & 2 & 1+e^{-4\gamma t} & 0 \\ 0 & 1+e^{-4\gamma t} & 2 & 0 \\ 1-e^{-4\gamma t} & 0 & 0 & 2-2e^{-2\gamma t} \end{pmatrix}. \tag{6}$$

Moreover, ρ_{13} and ρ_{23} are given by

$$\begin{aligned} \rho_{13} &= tr_2\left(\rho_{evo}\left(W\right)\right) = \rho_{23} = tr_1\left(\rho_{evo}\left(W\right)\right) \\ &= \frac{1}{8}\begin{pmatrix} 2+e^{-2\gamma t}-e^{-4\gamma t} & 0 & 0 & \sqrt{2}-\sqrt{2}e^{-4\gamma t} \\ 0 & 2+e^{-2\gamma t}+e^{-4\gamma t} & \sqrt{2}+\sqrt{2}e^{-4\gamma t} & 0 \\ 0 & \sqrt{2}+\sqrt{2}e^{-4\gamma t} & 2-e^{-2\gamma t}+e^{-4\gamma t} & 0 \\ \sqrt{2}-\sqrt{2}e^{-4\gamma t} & 0 & 0 & 2-e^{-2\gamma t}-e^{-4\gamma t} \end{pmatrix} \end{aligned} \tag{7}$$

Similarly, the QD, CC and EOF for the density matrix ρ_{12} under the circumstance of the bit-flip noise are depicted in Fig. 1(b), while the same three physical quantities for ρ_{13} (ρ_{23}) are displayed in Fig. 1(e) and Fig. 1(i). Furthermore, if every particle of the W-class state is subject to the noise acting on y direction, namely, the bit-phase-flip noise ($L_{1,y}$, $L_{2,y}$, $L_{3,y}$), we find that the evolution of QD, CC, and EOF for the density matrix ρ_{12} (and ρ_{13}) is the same as that infected by the bit-flip noise. This is because, to the W-class state, a bit-phase-flip noise is like a bit-flip noise.

(iii). Now we turn our attention to the case that the three particles are simultaneously subject to the isotropic noise, *i.e.*, the Lindblad operators are expressed as $L_{1,k}$, $L_{2,k}$ and $L_{3,k}$ with $k = x$, y, z. In this situation, according to equation (1), we can get the density matrix elements of the noisy W-class state $\rho_{evo}(W)$,

$$\begin{aligned} \chi_{00} &= \frac{1}{8}\left(1+e^{-4\gamma t}-e^{-8\gamma t}-e^{-12\gamma t}\right), \ \chi_{11} = \frac{1}{8}\left(1+e^{-4\gamma t}+e^{-8\gamma t}+e^{-12\gamma t}\right), \\ \chi_{22} &= \chi_{44} = \frac{1}{8}\left(1+e^{-12\gamma t}\right), \ \chi_{33} = \chi_{55} = \frac{1}{8}\left(1-e^{-12\gamma t}\right), \\ \chi_{66} &= \frac{1}{8}\left(1-e^{-4\gamma t}+e^{-8\gamma t}-e^{-12\gamma t}\right), \ \chi_{77} = \frac{1}{8}\left(1-e^{-4\gamma t}-e^{-8\gamma t}+e^{-12\gamma t}\right), \\ \chi_{12} &= \chi_{14} = \sqrt{2}\chi_{24} = \frac{\sqrt{2}}{8}\left(e^{-8\gamma t}+e^{-12\gamma t}\right), \\ \chi_{36} &= \chi_{56} = \sqrt{2}\chi_{35} = \frac{\sqrt{2}}{8}\left(e^{-8\gamma t}-e^{-12\gamma t}\right), \end{aligned} \tag{8}$$

with $\chi_{mn} = \chi_{nm}$ and $\chi_{mn} = 0$ for the other values of m, n. As a result, ρ_{12} becomes

$$\rho_{12} = tr_3\left(\rho_{evo}\left(W\right)\right) = \frac{1}{4}\begin{pmatrix} 1+e^{-4\gamma t} & 0 & 0 & 0 \\ 0 & 1 & e^{-8\gamma t} & 0 \\ 0 & e^{-8\gamma t} & 1 & 0 \\ 0 & 0 & 0 & 1-e^{-4\gamma t} \end{pmatrix}, \tag{9}$$

and ρ_{13} (ρ_{23}) is given by

$$\begin{aligned}\rho_{13} &= tr_2\left(\rho_{evo}\left(W\right)\right) = \rho_{23} = tr_1\left(\rho_{evo}\left(W\right)\right)\\ &= \frac{1}{8}\begin{pmatrix} 2+e^{-4\gamma t}-e^{-8\gamma t} & 0 & 0 & 0\\ 0 & 2+e^{-4\gamma t}+e^{-8\gamma t} & 2\sqrt{2}e^{-8\gamma t} & 0\\ 0 & 2\sqrt{2}e^{-8\gamma t} & 2-e^{-4\gamma t}+e^{-8\gamma t} & 0\\ 0 & 0 & 0 & 2-e^{-4\gamma t}-e^{-8\gamma t}\end{pmatrix}\end{aligned} \tag{10}$$

We now plot QD, CC and EOF versus γt under the action of the isotropic noise for the density matrices ρ_{12} in Fig. 1(c) and those for ρ_{13} in Fig. 1(f) and Fig. 1(j). From Fig. 1(a, b, d, e, h, i), one can see that for the phase-flip and bit-flip noises, CC is less than EOF and QD for the small γt, but when γt is large there are going in the opposite direction. Moreover, in the case of large γt, QD is smaller than EOF for the dephasing noise, however, for the bit-flip noise the situation becomes completely reversed. Meanwhile, it can be seen from Fig. 1(c, f, j) that the three physical quantities decay more rapidly than those for the other two kinds of noises, thus the isotropic noise is more harmful to the W-class state. In this case, the QD is always not less than the classical correlation and the EOF. We also notice that for the dephasing noise, CC decreases until a critical time and then keeps unchanged while QD just displays a very small sudden change at the critical time and then continues to diminish to zero asymptotically, which is different from the behavior of sudden transition between classical and quantum correlation obtained in Ref. [35]. It is worth emphasizing that the dynamics of mutual information, namely, the sum of CC and QD, is always continuous and its derivative exists for all time. On the other hand, it should be mentioned that EOF decreases asymptotically to zero for the phase-flip noise, the reason of no entanglement sudden death (ESD) is that there exists a zero diagonal element in the density matrices of Eqs. (3) and (4), which is consistent with the results of Refs. [45, 46]. But for the bit-flip noise, EOF suffers the phenomenon of ESD in a finite time $\gamma t = 0.189$ for the density matrix ρ_{12} (and at $\gamma t = 0.378$ for the density matrix ρ_{13}), while for the isotropic noise, EOF permanently vanishes at $\gamma t = 0.064$ for ρ_{12} (and at $\gamma t = 0.093$ for ρ_{13}). Furthermore, QD for both ρ_{12} and ρ_{13} always decreases to zero in an asymptotical way under the influence of the three Pauli noises. Therefore, for the situation of the W-class state $|W\rangle_{123}$ subject to Pauli noises, QD is more robust than EOF against decoherence aside from the dephasing noise, indicating that QD is more useful than the entanglement to describe the quantum correlation. Besides, from Fig. 1(d, e, f) and Fig. 1(h, i, j), we can observe that under the influence of the same noises, the time evolution behavior of D_f and C_f by measuring the first subsystem is similar to that of D_s and C_s by measuring the second subsystem, respectively.

Recently, many papers have been devoted to studying the unique property about the monogamy of quantum entanglement in a quantitative way [47, 48]. For a pure state ρ_{ABE} with A, B representing two subsystems and E representing the environment, Fanchini *et al.* [48] obtained a very simple but powerful result $E_{AB} = D_{A(B)E} + S_{A(B)|E}$, where $S_{A(B)|E} = S_{A(B)E} - S_E$ is the conditional entropy, E_{AB} and $D_{A(B)E}$ are EOF and QD, respectively. Moreover, considering the unavoidable interaction between a quantum system and its surrounding environment, the monogamic inequality $E_{AB} \leq D_{A(B)E} + S_{A(B)|E}$ was derived in Ref. [48]. In what follows, through straightforward calculation we will verify whether EOF, QD and the corresponding von Neumann entropy satisfy the monogamic relation or not, namely,

$$E_{12} + E_{13} \leq D_{12} + D_{13} + S(\rho_{12}) + S(\rho_{13}) - S(\rho_3) - S(\rho_2), \tag{11}$$

where E_{12} (E_{13}) denotes the EOF of the systems consisting of particles 1 and 2 (particles 1 and 3), D_{12} (D_{13}) denotes the QD between particles 1 and 2 (particles 1 and 3). The inequality (11) is plotted with respect to γt in Fig. 2. Clearly, one can see that the right hand of the inequality is no less than the left hand for any time t. Only at the beginning of $t = 0$, the inequality becomes an equality $E_{12} + E_{13} = D_{12} + D_{13}$ because particles 1, 2 and 3 do not interact with the external environment at this moment, *i.e.* they are still in the pure state $|W\rangle_{123}$, which implies that $S(\rho_{12}) = S(\rho_3)$ and $S(\rho_{13}) = S(\rho_2)$. On the other hand, in a recent literature [49], the authors showed that for pure states, the EOF can be written exclusively as a function of the QD. Actually, for a tripartite pure state, they found that $E_{12} = D_{12} + D_{23} - D_{32}$, where D_{23} and D_{32} denote the QD of particles 2 and 3 with the measurements over particle 3 and particle 2, respectively. But if the tripartite state is mixed, what is the difference between E_{12} and $D_{12} + D_{23} - D_{32}$? To examine this difference, we plot the dynamics of E_{12} and $D_{12} + D_{23} - D_{32}$ with respect to γt in Fig. 2. From Fig. 2(a), it can be seen that E_{12} is more than $D_{12} + D_{23} - D_{32}$ except for the situation of the time γt quite approaching to 0.043. In Fig. 2(b), $D_{12} + D_{23} - D_{32}$ is first larger than E_{12} with the increasing γt, but an opposite situation appears through numerical analysis, namely, $D_{12} + D_{23} - D_{32} < 0$ after $\gamma t > 0.54$ (as was stated above, E_{12} occurred the sudden death when $\gamma t > 0.189$). Nevertheless, in Fig. 2(c) $D_{12} + D_{23} - D_{32}$ is always greater than E_{12} for any $\gamma t > 0$. As a consequence, we can learn from Fig. 2 that as for the different Pauli noises, the relation between E_{12} and $D_{12} + D_{23} - D_{32}$ for the tripartite mixed states is uncertain. It should be noted that in Fig. 2 and the subsequent Fig. 4, the dynamics of QD was only calculated by measuring the second subsystem.

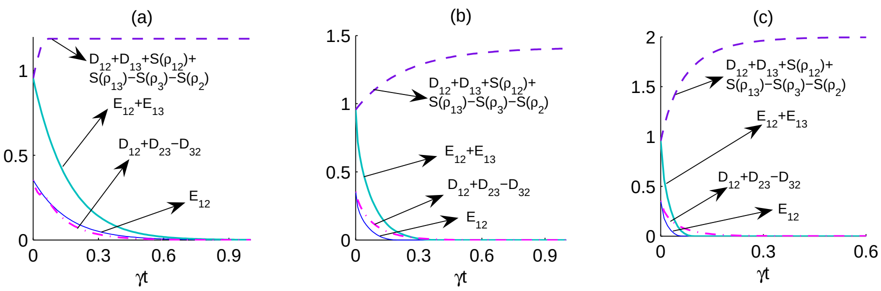

Fig. 2. $E_{12} + E_{13}$ (thick solid line), $D_{12} + D_{13} + S(\rho_{12}) + S(\rho_{13}) - S(\rho_3) - S(\rho_2)$ (dashed line), E_{12} (thin solid line) and $D_{12} + D_{23} - D_{32}$ (dot-dashed line) are plotted versus γt with the W-class state $|W\rangle_{123}$ infected by (a) phase-flip noise, (b) bit-flip noise, and (c) isotropic noise.

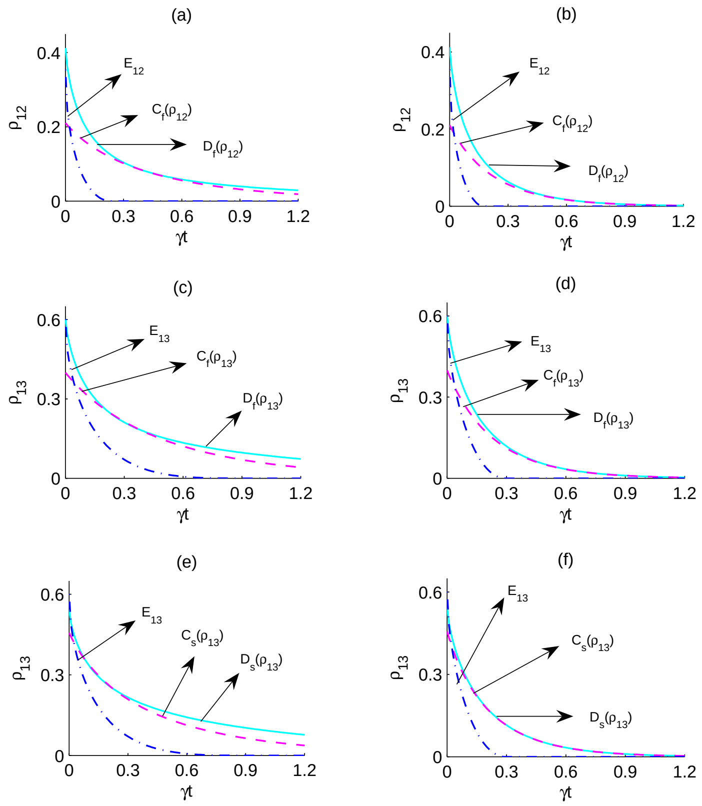

Fig. 3. For the reduced density matrices ρ_{12} and ρ_{13}, QD, CC and EOF are plotted as a function of γt, where (a, c, e) correspond to zero temperature environment, (b, d, f) to infinite temperature environment. The solid line stands for QD marked as D_f or D_s, the dashed line for CC marked as C_f or C_s, and the dot-dashed line for EOF marked as E_{12} or E_{13}.

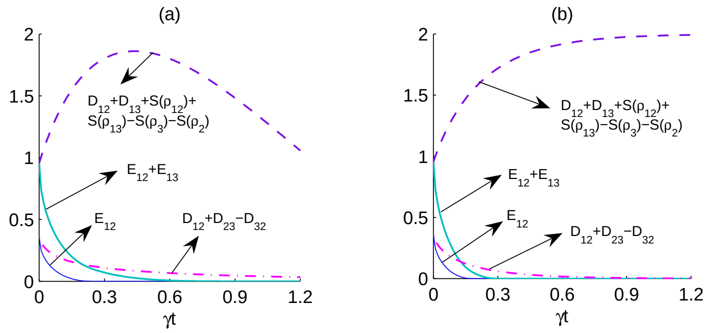

Fig. 4. $E_{12}+E_{13}$ (thick solid line), $D_{12}+D_{13}+S(\rho_{12})+S(\rho_{13})-S(\rho_3)-S(\rho_2)$ (dashed line), E_{12} (thin solid line) and $D_{12}+D_{23}-D_{32}$ (dot-dashed line) are plotted as a function of γt with the W-class state $|W\rangle_{123}$ infected by (a) zero temperature environment and (b) infinite temperature environment.

4 W-class State Subject to Dissipative and Noisy Environments

In this section, we investigate the dynamics of pairwise QD, CC and EOF of the W-class state $|W\rangle_{123}$ exposed to zero temperature and infinite temperature environments [34,50], for which the decoherence dynamics of the system can also be described by the general master equation (1). In contrast to the Pauli noises of Refs. [33,45,51,52], here the Lindblad operator $L_j=\sigma^-$ is for the dissipative environment (*i.e.*, zero temperature reservoir), the Lindblad operators $L_{j,1}=\sigma^-$ and $L_{j,2}=\sigma^+$ for the noisy environment (*i.e.*, infinite temperature reservoir), where $\sigma^\pm$ are the raising and lowering operators $\sigma^\pm=(\sigma_x+i\sigma_y)/2$, σ_x and σ_y are the usual Pauli matrices.

(i). We first consider the situation that the W-class state is infected by a zero temperature environment. Through analytical derivation of the master equation (1), the only nonvanishing components of $\rho_{evo}(W)$ can be calculated explicitly as

$$\begin{aligned}
\chi_{11} &= \frac{1}{2}e^{-2\gamma t},\ \chi_{22}=\chi_{44}=\frac{1}{4}e^{-2\gamma t},\ \chi_{33}=\chi_{55}=\frac{3}{4}\left(e^{-\gamma t}-e^{-2\gamma t}\right),\\
\chi_{66} &= \frac{1}{2}\left(e^{-\gamma t}-e^{-2\gamma t}\right),\ \chi_{77}=1-2e^{-\gamma t}+e^{-2\gamma t},\\
\chi_{12} &= \chi_{14}=\sqrt{2}\chi_{24}=\frac{\sqrt{2}}{4}e^{-2\gamma t},\\
\chi_{36} &= \chi_{56}=\sqrt{2}\chi_{35}=\frac{\sqrt{2}}{4}(e^{-\gamma t}-e^{-2\gamma t}),
\end{aligned} \tag{12}$$

and $\chi_{mn}=\chi_{nm}$.

With the similar method as section 3, the reduced density matrix ρ_{12} can be obtained as

$$\rho_{12}=tr_3\left(\rho_{evo}\left(W\right)\right)=\frac{1}{4}\begin{pmatrix} 2e^{-2\gamma t} & 0 & 0 & 0\\ 0 & 3e^{-\gamma t}-2e^{-2\gamma t} & e^{-\gamma t} & 0\\ 0 & e^{-\gamma t} & 3e^{-\gamma t}-2e^{-2\gamma t} & 0\\ 0 & 0 & 0 & 4+2e^{-2\gamma t}-6e^{-\gamma t}\end{pmatrix}. \tag{13}$$

Likewise, the reduced density matrices ρ_{13} and ρ_{23} are given by

$$\begin{aligned}\rho_{13} &= tr_2\left(\rho_{evo}\left(W\right)\right)=\rho_{23}=tr_1\left(\rho_{evo}\left(W\right)\right)\\ &= \frac{1}{4}\begin{pmatrix} e^{-2\gamma t} & 0 & 0 & 0\\ 0 & 3e^{-\gamma t}-e^{-2\gamma t} & \sqrt{2}e^{-\gamma t} & 0\\ 0 & \sqrt{2}e^{-\gamma t} & 2e^{-\gamma t}-e^{-2\gamma t} & 0\\ 0 & 0 & 0 & 4+e^{-2\gamma t}-5e^{-\gamma t}\end{pmatrix}. \end{aligned}\tag{14}$$

We plot QD, CC and EOF as a function of γt under the influence of zero temperature reservoir for the density matrices ρ_{12} in Fig. 3(a), and those for ρ_{13} (ρ_{23}) in Fig. 3(c) and Fig. 3(e).

(ii). Next we focus on the case where the three particles are simultaneously infected by an infinite temperature environment. According to the equation (1), the only nonvanishing components of $\rho_{evo}(W)$ can be derived explicitly as

$$\begin{aligned}\chi_{00} &= \frac{1}{8}\left(1+e^{-2\gamma t}-e^{-4\gamma t}-e^{-6\gamma t}\right),\ \chi_{11}=\frac{1}{8}\left(1+e^{-2\gamma t}+e^{-4\gamma t}+e^{-6\gamma t}\right),\\ \chi_{22} &= \chi_{44}=\frac{1}{8}\left(1+e^{-6\gamma t}\right),\ \chi_{33}=\chi_{55}=\frac{1}{8}\left(1-e^{-6\gamma t}\right),\\ \chi_{66} &= \frac{1}{8}\left(1-e^{-2\gamma t}+e^{-4\gamma t}-e^{-6\gamma t}\right),\ \chi_{77}=\frac{1}{8}\left(1-e^{-2\gamma t}-e^{-4\gamma t}+e^{-6\gamma t}\right),\\ \chi_{12} &= \chi_{14}=\sqrt{2}\chi_{24}=\frac{\sqrt{2}}{8}\left(e^{-2\gamma t}+e^{-4\gamma t}\right),\\ \chi_{36} &= \chi_{56}=\sqrt{2}\chi_{35}=\frac{\sqrt{2}}{8}\left(e^{-2\gamma t}-e^{-4\gamma t}\right),\end{aligned}\tag{15}$$

with $\chi_{mn}=\chi_{nm}$. In this case, the explicit expression ρ_{12} is given by

$$\rho_{12}=tr_3\left(\rho_{evo}\left(W\right)\right)=\frac{1}{4}\begin{pmatrix} 1+e^{-2\gamma t} & 0 & 0 & 0\\ 0 & 1 & e^{-2\gamma t} & 0\\ 0 & e^{-2\gamma t} & 1 & 0\\ 0 & 0 & 0 & 1-e^{-2\gamma t}\end{pmatrix}, \tag{16}$$

and ρ_{13} becomes

$$\begin{aligned}\rho_{13} &= tr_2\left(\rho_{evo}\left(W\right)\right)=\rho_{23}=tr_1\left(\rho_{evo}\left(W\right)\right)\\ &= \frac{1}{8}\begin{pmatrix} 2+e^{-2\gamma t}-e^{-4\gamma t} & 0 & 0 & 0\\ 0 & 2+e^{-2\gamma t}+e^{-4\gamma t} & 2\sqrt{2}e^{-2\gamma t} & 0\\ 0 & 2\sqrt{2}e^{-2\gamma t} & 2-e^{-2\gamma t}+e^{-4\gamma t} & 0\\ 0 & 0 & 0 & 2-e^{-2\gamma t}-e^{-4\gamma t}\end{pmatrix}\end{aligned}\tag{17}$$

Similarly, under the action of infinite temperature reservoir, QD, CC and EOF for the density matrix ρ_{12} of Eq. (16) versus γt are plotted in Fig. 3(b), while the same three physical quantities for ρ_{13} of Eq. (17) are displayed in Fig. 3(d) and Fig. 3(f). From Fig. 3, one can see that both QD and CC decay asymptotically to zero under the influence of the zero temperature and infinite temperature environments. However, EOF suffers ESD in the zero temperature environment in a finite time $\gamma t = 0.230$ for ρ_{12} (and at $\gamma t = 0.807$ for ρ_{13}) while EOF also undergoes ESD in the infinite temperature environment at $\gamma t = 0.173$ for ρ_{12} (and at $\gamma t = 0.287$ for ρ_{13}). Moreover, the decay of the three physical quantities in infinite temperature environment is more rapidly than that in zero temperature environment. In other words, the influence of infinite temperature reservoir on the three physical quantities of ρ_{12} (and ρ_{13}) is larger than that of zero temperature environment. Furthermore, it can be seen from Fig. 3 that QD is always no less than CC and EOF in the two kinds of environments, which is quite distinct from the cases that the W-class state is exposed to the phase-flip noise in Fig. 1(a, d, h) and to the bit-flip noise in Fig. 1(b, e, i). Comparing Fig. 1 with Fig. 3, we find that QD subject to Pauli noisy environment decays more rapidly in contrast to zero temperature and infinite temperature environments. In addition, we can see from Fig. 3(c,d) and Fig. 3(e,f) that under the same conditions, the dynamics evolution of D_f with respect to the measurement over the first subsystem resembles that of D_s over the second subsystem, similar to the Pauli noise cases studied in section 3.

Now we need to prove whether pairwise QD and EOF for the W-class state $|W\rangle_{123}$ exposed to zero temperature and infinite temperature environments satisfy the monogamic relation or not. Fig. 4 displays the correlation between $E_{12} + E_{13}$ and $D_{12} + D_{13} + S(\rho_{12}) + S(\rho_{13}) - S(\rho_3) - S(\rho_2)$ versus γt. Here, the physical quantities E_{12}, E_{13}, D_{12}, D_{13} and $S(\rho)$ represent the same meaning as the inequality (11), apart from the W-class state subject to different noisy environments. From the figure, we can see that the sum of QD and von Neumann entropy is exactly equal to that of EOF for the initial W-class state corresponding to $t = 0$, however, when $t > 0$ the former is always larger than the latter. Therefore, the monogamic relation is operative in the dissipative and noisy environments. Besides, it can be observed from Fig. 4 that when the W-class state is subject to both zero temperature and infinite temperature reservoirs, $D_{12} + D_{23} - D_{32}$ is always greater than E_{12}, which is quite different from the cases under the influence of Pauli noises.

5 Summary

We have numerically analyzed the dynamics of pairwise quantum discord, classical correlation, and entanglement of formation for the tripartite asymmetric W-class state $|W\rangle_{123} = \frac{1}{2}\left(|100\rangle_{123} + |010\rangle_{123} + \sqrt{2}\,|001\rangle_{123}\right)$ exposed to Pauli noises, zero temperature and infinite temperature environments. We found that the three physical quantities provide independent measures of correlation without simple ordering relations between them. Moreover, EOF decays asymptotically to zero for the phase-flip noise while it undergoes the behavior of ESD for the bit-flip and isotropic noises, as well as the dissipative and noisy environments. Meanwhile, QD always decreases to zero exponentially for all the above investigated Markovian noises. Thus QD is more robust than EOF against decoherence apart from the dephasing noise, which, notably, is only suitable for the W-class state $|W\rangle_{123}$ under the action of the noises discussed in this paper, not valid for all types of W-class state. Furthermore, QD affected by Pauli noise environment decays more rapidly compared with zero temperature and infinite temperature environments. We also found that for the phase-flip noise, CC decreases monotonously until a critical time and then remains constant while QD displays the behavior of very small sudden change at the critical time, which is different from the results of Ref. [35], where there exists the sudden transition between classical and quantum correlation. It should be pointed out that our present work is different from that of Ref. [28], where the investigated tripartite W-type state is symmetric under exchange of subsystems and the Ornstein-Uhlenbeck noise only reduces to the dephasing noise in the Markov limit. In addition, through numerical calculations, we verified that the monogamic relation between pairwise QD and EOF is valid no matter whether the W-class state is subject to the Pauli noise or the dissipative and noisy environments, and revealed that the relation between E_{12} and $D_{12}+D_{23}-D_{32}$ is not definite for the tripartite mixed states.

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