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CONTROLLED IMPLEMENTATION OF TWO-PHOTON CONTROLLED PHASE GATE WITHIN A NETWORK

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We propose a protocol to controlled implement the two-photon controlled phase gate within a network by using interference of polarized photons. The realization of this protocol is appealing due to the fact that the quantum state of light is robust against the decoherence, and photons are ideal carriers for transmitting quantum information over long distances. The proposed setup involves simple linear optical elements and the conventional photon detectors that only distinguish the vacuum and nonvacuum Fock number states. This can greatly simplify the experimental realization of a linear optical quantum computer.

Keywords: Phase gate; Linear optical elements; Polarization photon

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1 General

In recent years, the development of quantum information processing has shed new light on complexity and communication theory. For example, quantum computation, based on fundamental quantum mechanical principles such as superposition and entanglement, may provide a promising perspective to advance modern computational science [1, 2]. It has been shown that a quantum computational network can be decomposed into one-qubit gate and two-qubit controlled phase-gate [3]. So far, a lot of substantial efforts have been dedicated to the field of quantum computation and a number of significant progresses have been made [4, 5, 6, 7]. One of the most promising cadidates for quantum information processing is a system of trapped ions, first proposed by Cirac and Zoler [4]. In this protocol, system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency; The recent significant progress on the linear optical elements on chip have been made by Xiao *et al.* [6, 7], they have proposed a potential quantum-computer hardware-architecture model on a silicon chip in which the basic cell gate is the atom-photon controlled-phase-flip gate [7].

We all know that in order to enable a quantum computer to work, the individual gate infidelity should be below a certain threshold value, which is about 1% [8]. Though many scientists have been paid much attention to this point, the gate infidelity is far above the threshold. The main source of gate error is the decoherence due to the coupling between the qubit system and the environment. In 2006, Zheng and Guo [9] have proposed a protocol for realizing a tunable phase gate for two atoms. In this protocol, the gate fidelity is not affected by both the atomic spontaneous emission and cavity decay. They claim that their protocol opens a new prospect for realizing quantum gate with the error below the threshold under realistic conditions. Later, by virtue of single-photon interference, Deng *et al.* [10] have presented how to realize a nonlocal N-qubit conditional phase gate. In this protocol, even by taking photon loss into consideration, only the success probability is affected, not fidelity. Zou *et al.* [11] have proposed a linear optical protocol to implement the nondeterministic two-qubit controlled phase gate with the certain success probabiltiy. In view of these protocols [10, 11], we are led to ask if it is possible to controlled implement the controlled phase gate within a network. Unfortunately, up to now, this problem has not been addressed.

Here, in this paper, we present an alternative protocol to answer the above-mentioned questions. That is, we propose a protocol to controlled implement the two-photon controlled phase gate within a network by using linear optical elements and conventional photon detectors. If and only if all controllers agree to collaboration, the two-photon controlled phase gate can be easily realized. The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over a long distance. The present case, however, holds an interesting advantage in that the detectors do not need to distinguish between one and two photons. This can greatly simplify the experimental realization of controlled implementing the two-photon controlled phase gate.

2 The Main Text

We consider implementing two-photon controlled phase gate within a network consisting of an arbitrary $N \geq 3$ remote parties named P_1, P_2, $\ldots$, P_N: a two-photon controlled phase gate between two-photon in one party under control of all the remaining parties. Without loss of generality, we suppose that P_1, P_2, $\ldots$, P_N share two N-photon entangled polarization states in the forms

$$|\Phi\rangle_{12\ldots N} = (\alpha|H\rangle^{\otimes N} + \beta|V\rangle^{\otimes N})_{12\ldots N}, \tag{1}$$

$$|\Psi\rangle_{1'2'\ldots N'} = (\alpha'|H\rangle^{\otimes N} + \beta'|V\rangle^{\otimes N})_{1'2'\ldots N'}, \tag{2}$$

where $|\alpha|^2 + |\beta|^2 = 1$ and $|\alpha'|^2 + |\beta'|^2 = 1$. photons $\{1, 1'\}$, $\{2, 2'\}$, $\ldots$, $\{N, N'\}$ are in possession of P_1, P_2, $\ldots$, P_N, respectively. If we want to implement the controlled phase gate on P_1's photons $(1, 1')$, then, P_2, P_3, $\ldots$,P_N are the controllers.

Now we turn to an experimental setup for controlled implementing the two-photon controlled phase gate within a network. The kinds of operations to be performed by the controllers P_n $(n = 2, 3, \ldots, N-1)$ and P_N are different. For controllers P_n $(n = 2, 3, \ldots, N-1)$, each of them proceeds as follows. In order to help the sender P_1 to implement the two-photon controlled phase gate, the controller (P_2) first sends his/her photons 2 and $2'$ through a quarter-wave plates (QWP) [12], respectively, whose action is given by transformation $|H\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|V\rangle \rightarrow \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$. After QWP, the state of the channels Eq.(1) and Eq.(2) change into Eq.(3) and Eq.(4), respectively, where Eq.(3) and Eq.(4) are shown as follows:

$$\begin{aligned}|\Phi\rangle_{12\ldots N} &= \frac{1}{\sqrt{2}}\{|H\rangle_2(\alpha|H\rangle^{\otimes N-1}+\beta|V\rangle^{\otimes N-1})_{134\ldots N}\\ &\quad +|V\rangle_2(\alpha|H\rangle^{\otimes N-1}-\beta|V\rangle^{\otimes N-1})_{134\ldots N}\}.\end{aligned}\tag{3}$$

$$\begin{aligned}|\Psi\rangle_{1'2'\ldots N'} &= \frac{1}{\sqrt{2}}\{|H\rangle_{2'}(\alpha'|H\rangle^{\otimes N-1}+\beta'|V\rangle^{\otimes N-1})_{1'3'4'\ldots N'}\\ &\quad +|V\rangle_{2'}(\alpha'|H\rangle^{\otimes N-1}-\beta'|V\rangle^{\otimes N-1})_{1'3'4'\ldots N'}\}.\end{aligned}\tag{4}$$

Then P_2 sends the photons 2 and $2'$ pass through a polarizing beam splitter (PBS) , respectively. The PBS transmits horizontal polarization and reflects vertical ones. At the outputs of PBS we obtain

$$\begin{aligned}|\Phi\rangle_{12\ldots N} &= \frac{1}{\sqrt{2}}\{|H\rangle_2^A(\alpha|H\rangle^{\otimes N-1}+\beta|V\rangle^{\otimes N-1})_{134\ldots N}\\ &\quad +|V\rangle_2^B(\alpha|H\rangle^{\otimes N-1}-\beta|V\rangle^{\otimes N-1})_{134\ldots N}\},\end{aligned}\tag{5}$$

and

$$\begin{aligned}|\Psi\rangle_{1'2'\ldots N'} &= \frac{1}{\sqrt{2}}\{|H\rangle_{2'}^A(\alpha|H\rangle^{\otimes N-1}+\beta|V\rangle^{\otimes N-1})_{1'3'4'\ldots N'}\\ &\quad +|V\rangle_{2'}^B(\alpha|H\rangle^{\otimes N-1}-\beta|V\rangle^{\otimes N-1})_{1'3'4'\ldots N'}\},\end{aligned}\tag{6}$$

where A and B are output paths of PBS, as is shown in Fig.1. In order to realize the two-photon controlled phase gate on photon 1 and photon $1'$, P_2 performs a measurement with the conventional photon detectors, that only distinguish the vacuum and nonvacuum Fock number states. Then, P_2 will inform the controller P_N of his/her measurement result V via a classical communication. If his/her measurement result is $|H\rangle$, $C=0$; if the measurement result is $|V\rangle$, $C=1$. The communication costs is 2 cbit since there are two possible results. After that, All the other controllers expect P_N repeat this process for the photons $\{3,3'\}$,$\{4,4'\}$, …, $\{(N-1),(N-1)'\}$, respectively, and inform P_N of their measurement results C_n ($n=3,4,\ldots,N-1$) via classical communication. The total communication costs are $2(N-2)$ cbits.

For the controller P_N, after receiving the classical information form P_n, he/she carries out the following calculation

$$C_{23\ldots N-1}=C_2\oplus C_3\cdots\oplus C_{N-1},\tag{7}$$

where the $\oplus$ denotes an addition mod 2. If $C_{23\ldots N-1}=0$, the channels Eq.(1) and Eq.(2) will be transformed into

$$|\Phi\rangle_{1N}=\alpha|HH\rangle_{1N}+\beta|VV\rangle_{1N},\tag{8}$$

and

$$|\Psi\rangle_{1'N'}=\alpha'|HH\rangle_{1'N'}+\beta'|VV\rangle_{1'N'}.\tag{9}$$

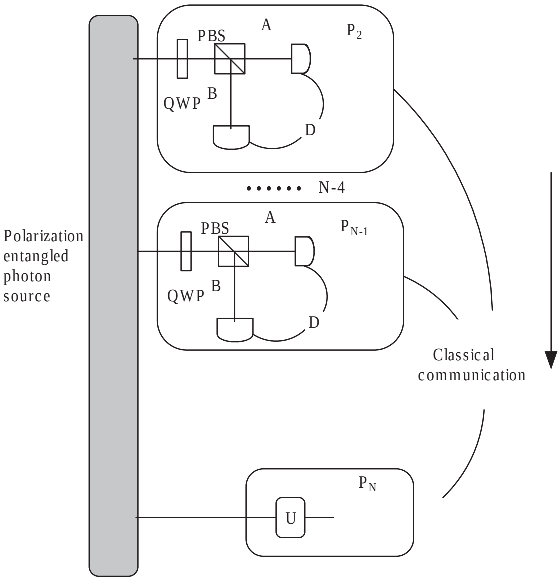

Fig. 1. Schematic diagram for controlled implementing two-photon controlled phase gate within a network. PBS denotes polarization beam splitter, QWP denotes quarter-wave plate, U denotes the operation of p_N, D denotes conventional photon detectors

if $C_{23\ldots N-1} = 1$, the channel Eq.(1) and Eq.(2) will be will be transformed into

$$|\Phi\rangle_{1N} = \alpha|HH\rangle_{1N} - \beta|VV\rangle_{1N}, \tag{10}$$

and

$$|\Psi\rangle_{1'N'} = \alpha'|HH\rangle_{1'N'} - \beta'|VV\rangle_{1'N'}. \tag{11}$$

If $C_{23\ldots N-1} = 0$, P_N do nothing on photons (N, N'); otherwise, P_N let photons (N, N') pass though a $\pi/2$-phase shift, respectively, to change the signs of the polarization states V_N and $V_{N'}$. After above operations, P_1 and P_N share the states as follows

$$|\Phi\rangle_{1N} = \alpha|HH\rangle_{1N} + \beta|VV\rangle_{1N}, \tag{12}$$

and

$$|\Psi\rangle_{1'N'} = \alpha'|HH\rangle_{1'N'} + \beta'|VV\rangle_{1'N'}. \tag{13}$$

In the following, we will show how to controlled implement the two-photon controlled phase gate by conventional photon detectors. The schematic representation of the protocol is shown in Fig.2. The controller P_N first let photon N pass through HWP_1, the joint state of the whole system follows that

$$|\Phi\rangle_{1N} \otimes |\Psi\rangle_{1'N'} \quad = \quad \frac{1}{\sqrt{2}}[\alpha|H\rangle_1(|H\rangle_N + |V\rangle_N) + \beta|V\rangle_1(|H\rangle_N - |V\rangle_N)]$$

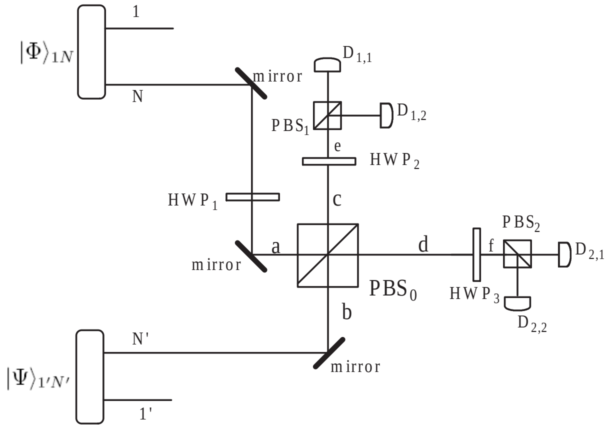

Fig. 2. Experimental setup for realizing two-photon controlled phase gate. The PBS transmits the horizontal polarization and reflects the vertical polarization, and the HWP's action is given by transformation $|H\rangle \rightarrow (1/\sqrt{2})(|H\rangle + |V\rangle)$ and $|V\rangle \rightarrow (1/\sqrt{2})(|H\rangle - |V\rangle)$

$$\otimes [\alpha'|H\rangle_{1'}|H\rangle_{N'} + \beta'|V\rangle_{1'}|V\rangle_{N'}]. \tag{14}$$

As a result, after modes a and b passing through the polarization beam splitter PBS_0, the state of the system becomes

$$\begin{aligned}|\Phi\rangle_{1N} \otimes |\Psi\rangle_{1'N'} &= \frac{1}{\sqrt{2}}[\alpha\alpha'|H\rangle_1|H\rangle_d|H\rangle_c|H\rangle_{1'} + \alpha\beta'|H\rangle_1|V\rangle_c|V\rangle_d|V\rangle_{1'} \\ &\quad +\beta\alpha'|V\rangle_1|H\rangle_d|H\rangle_c|H\rangle_{1'} - \beta\beta'|V\rangle_1|V\rangle_c|V\rangle_d|V\rangle_{1'}] \\ &= \frac{1}{\sqrt{2}}[\alpha'|H\rangle_d|H\rangle_c|H\rangle_{1'}(\alpha|H\rangle_1 + \beta|V\rangle_1) \\ &\quad +\beta'|V\rangle_d|V\rangle_c|V\rangle_{1'}(\alpha|H\rangle_1 - \beta|V\rangle_1)],\end{aligned} \tag{15}$$

where we only consider the events that there exist photons in both modes c and d. The probability of getting the state is 50%. After above operation, P_N sends modes c and d pass through the HWP_2 and HWP_3, respectively. So, Eq.(15) becomes

$$\begin{aligned}|\Phi\rangle_{1N} \otimes |\Psi\rangle_{1'N'} &= \frac{1}{2\sqrt{2}}[\alpha'(|H\rangle_f + |V\rangle_f)(|H\rangle_e + |V\rangle_e)|H\rangle_{1'}(\alpha|H\rangle_1 + \beta|V\rangle_1) \\ &\quad +\beta'(|H\rangle_f - |V\rangle_f)(|H\rangle_e - |V\rangle_e)|V\rangle_{1'}(\alpha|H\rangle_1 - \beta|V\rangle_1)].\end{aligned} \tag{16}$$

If conventional photon detectors $D_{1,1}$ and $D_{2,1}$ ($D_{1,2}$ and $D_{2,2}$) detect photons, we can obtain the state of photon 1 and phton $1'$ as follow

$$\zeta_{11'} = \alpha'|H\rangle_{1'}(\alpha|H\rangle_1 + \beta|V\rangle_1) + \beta'|V\rangle_{1'}(\alpha|H\rangle_1 - \beta|V\rangle_1)]; \tag{17}$$

If conventional photon detectors $D_{1,1}$ and $D_{2,2}$ ($D_{1,2}$ and $D_{2,1}$) detect photons, we can obtain the state of photon 1 and phton $1'$ as follow

$$\zeta'_{11'} = \alpha'|H\rangle_{1'}(\alpha|H\rangle_1 + \beta|V\rangle_1) - \beta'|V\rangle_{1'}(\alpha|H\rangle_1 - \beta|V\rangle_1)], \tag{18}$$

which can be transformed into Eq.(17) by applying a $\pi/2$-phase shifter to change the sign of the polarization state $|V\rangle_{1'}$. Eq.(17) demonstrate the controlled implementation of the two-photon controlled phase gate.

3 Conclusion

Up to now, we have proposed a protocol to controlled implement two-photon controlled phase gate within a network by using linear optical elements and conventional photon detectors. The protocol developed here shows that if one of the N parties P_x ($x \in \{1, 2, 3, \ldots, N\}$) wants to implement two-photon controlled phase gate on his/her two-photon, then, the remaining $N-1$ parties P_{N-1} are controllers. If and only if all controllers agree to collaboration, the arbitrary two-photon controlled phase gate can be realized. In terms of practical applicability, some technical difficulties with the present protocol should be pointed out. Firstly, to realize the generation protocol, N-party should share two N-photon polarization-entangled states Eq.(1) and Eq.(2), but, in practice, it is difficulty to realize N-party share two N-photon polarization-entangled states. Secondly, different attenuations are introduced due to the lack of variable beam splitter, which reduce the probability of success in practice. Thirdly, from Eq.(15) we know that the protocol is based on the interference of photons from modes N and N'. This requires that they must arrive simultaneously at the PBS_0 to an accuracy of a fraction of the coherent length. This requirement is also met in multiphoton experiments [13, 14]. Fortunately, this problem has been solved by locking the pathlength. Fourthly, the protocol needs the deterministic entangled-photon states as input source. From ref.[15] we know that an encoding circuit based on postselection strategy has been demonstrated experimentally [16], where parametric down-conversion was used to generate an entangled state of photons. But the states are created at random times, so that such a source cannot be used in the present scheme. Recently developed quantum dot techniques [17] might eventually provide a triggered source of entangled photon states. So, these problem needs further investigation.

We now give a brief discussion on the experimental feasibility of protocol with the current experimental technology. Firstly, The realization of this protocol is appealing due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over a long distance. The present case, however, holds an interesting advantage in that the detectors are conventional photon detectors that only distinguish the vacuum and nonvacuum Fock number states. It is not necessary to distinguish between one and two photons. This can greatly simplify the experimental realization of controlled implementing the two-photon controlled phase gate within a network. Secondly, what we used consists of linear optical elements [18, 19], which have been widely used to entangled photons [20]. In particular, the similar optical setups have been used to successfully prepare W (GHZ) states of photons in experiment [21] ([13]). Therefore, our protocol might be realized in near future.

In summary, we have presented a protocol for controlled implementing two-photon controlled phase gate within a network by using linear optical elements such as PBS, QWP, and conventional photon detectors. The distinct advantages of our protocol are that the detectors are conventional photon detectors and it is not necessary to distinguish between one and two photons. The result shows that for such a protocol, there is still a certain probability of successful controlled implement two-photon controlled phase gate within a network. Though, in terms of practical applicability, our demonstration of the protocol still has some limitations, we believe that with the development of technology in experiment it may be possible to implement the protocol with ease.

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