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# QUANTUM ENTANGLEMENT OF DIRAC FIELD IN BACKGROUND OF AN ASYMPTOTICALLY FLAT STATIC BLACK HOLE

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The entanglement of the Dirac field in the asymptotically flat black hole is investigated. Unlike the bosonic case in which the initial entanglement vanishes in the limit of infinite Hawking temperature, in this case the entanglement achieves a nonvanishing minimum values, which shows that the entanglement is never completely destroyed when black hole evaporates completely. Another interesting result is that the mutual information in this limit equals to just half of its own initial value, which may be an universal property for any fields.

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## 1 introduction

It is widely accepted that understanding the entanglement in a relativistic framework is not only of interest to quantum information, but also plays an important role in black hole entropy and the black hole information paradox [1, 2, 3, 4]. Many authors have paid attention to the investigation of quantum entanglement in the relativistic framework [5, 6, 7, 8, 9, 10, 11, 12] due to its theoretical importance and practical application. It has been shown that from the perspective of a uniformly accelerated observer, the entanglement is degraded since the fact that the event horizon appears and Unruh effect [13] results in a loss of information for the non-inertial observer [14, 15, 16, 17, 18]. Recently, we discussed the entanglement and teleportation of scalar field in the background of an asymptotically flat static black hole [19]. We found that the degree of entanglement(quantified by the logarithmic negativity) and the fidelity of teleportation are degraded by the Hawking effect [20] with increasing Hawking temperature for the scalar field.

In order to further investigate the character of the entanglement in background of a black hole, in this paper we will study quantum entanglement for the Dirac field in the spacetime of a static and asymptotically flat black hole and to see whether or not special properties exist in this case. We assume that Alice has a detector which only detects mode $|n\rangle_A$ and Bob has a detector sensitive only to mode $|n\rangle_B$, and they share a generically entangled state

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at the same initial point in flat Minkowski spacetime before the black hole is formed. After the coincidence of Alice and Bob, they moves toward an star. The difference is that Alice moves along geodesic while Bob it with constant acceleration. Once Bob is safely hovering outside of the star with constant acceleration, let it collapse to form a black hole. By Birkhoff's theorem [21] this won't change the metric outside of the black hole and therefore won't change Bob's acceleration. Thus, Bob's detector registers only thermally excited particles due to the Hawking effect Bob turns on his detector after the star collapses to form a black hole and check to see whether the quantum state have been excited or not.

The outline of the paper is as follows. In Sec. 2 we discuss the essential features of the background spacetime and the Hawking effect for the Dirac particles as experienced by the observer outside the black hole. In Sec. 3 we analyze the effects of the Hawking temperature on the entanglement between the modes for the different state parameter. We summarize and discuss our conclusions in the last section.

## 2 Vacuum structure and Hawking Radiation for Dirac fields

Before we discuss the nature of entanglement, we wish to review the essential features of quantum field theory in the background of a general asymptotically flat static black hole. The Dirac equation in a general background space-time can be written as [22]

$$[\gamma^a e_a{}^\mu(\partial_\mu + \Gamma_\mu)]\Psi = 0, \tag{1}$$

here $\gamma^a$ are the Dirac matrices, the four-vectors $e_a{}^\mu$ represent the inverse of the tetrad $e^a{}_\mu$ defined by $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$ with $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$, and $\Gamma_\mu = \frac{1}{8}[\gamma^a, \gamma^b] e_a{}^\nu e_{b\nu;\mu}$ are the spin connection coefficients. Throughout this paper we use $G = c = \hbar = \kappa_B = 1$.

For a general asymptotically flat static black hole

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2), \tag{2}$$

Eq. (1) can be expressed as

$$-\frac{\gamma_0}{\sqrt{f(r)}}\frac{\partial\Psi}{\partial t} + \sqrt{f(r)}\gamma_1\left[\frac{\partial}{\partial r} + \frac{1}{r} + \frac{1}{4f(r)}\frac{df(r)}{dr}\right]\Psi + \frac{\gamma_2}{r}\left(\frac{\partial}{\partial\theta} + \frac{1}{2}\cot\theta\right)\Psi + \frac{\gamma_3}{r\sin\theta}\frac{\partial\Psi}{\partial\varphi} = 0. \tag{3}$$

If we re-scale $\Psi$ as $\Psi = f(r)^{-\frac{1}{4}}\Phi$ and use an ansatz for the Dirac spinor

$$\Phi = \begin{pmatrix} \frac{i\chi_1^{(\pm)}(r)}{r}\phi^{\pm}_{jm}(\theta,\varphi) \\ \frac{\chi_2^{(\pm)}(r)}{r}\phi^{\mp}_{jm}(\theta,\varphi) \end{pmatrix} e^{-i\omega t}, \tag{4}$$

with spinor angular harmonics

$$\phi^{+}_{jm} = \begin{pmatrix} \sqrt{\frac{j+m}{2j}}Y_l^{m-1/2} \\ \sqrt{\frac{j-m}{2j}}Y_l^{m+1/2} \end{pmatrix}, \qquad (for \;\; j = l + \frac{1}{2}), \tag{5}$$

$$\phi^{-}_{jm} = \begin{pmatrix} \sqrt{\frac{j+1-m}{2j+2}}Y_l^{m-1/2} \\ -\sqrt{\frac{j+1+m}{2j+2}}Y_l^{m+1/2} \end{pmatrix}, \qquad (for \;\; j = l - \frac{1}{2}), \tag{6}$$

the radial Dirac equation can be written as

$$\begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}\begin{pmatrix} \chi_2^{(\pm)} \\ \chi_1^{(\pm)} \end{pmatrix} - \frac{\partial}{\partial r_*}\begin{pmatrix} \chi_2^{(\pm)} \\ \chi_1^{(\pm)} \end{pmatrix} + \sqrt{f(r)}\begin{pmatrix} \frac{K_\pm}{r} & 0 \\ 0 & -\frac{K_\pm}{r} \end{pmatrix}\begin{pmatrix} \chi_2^{(\pm)} \\ \chi_1^{(\pm)} \end{pmatrix} = 0, \tag{7}$$

where we have introduced a tortoise coordinate change $dr_* = dr/f(r)$. We find that the cases for $(+)$ and $(-)$ in the functions $\chi_1^{(\pm)}$ and $\chi_2^{(\pm)}$ can be put together, and then the decoupled equations can be written as[23, 24]

$$\frac{d^2\chi_1}{dr_*^2} + (\omega^2 - V_1)\chi_1 = 0, \tag{8}$$

$$\frac{d^2\chi_2}{dr_*^2} + (\omega^2 - V_2)\chi_2 = 0, \tag{9}$$

with

$$V_1 = \frac{\sqrt{f(r)}|K|}{r^2}\left[|K|\sqrt{f(r)} - \frac{r}{2}\frac{df(r)}{dr} + f(r)\right], \quad \left(K = -j - \frac{1}{2},\ \ j = l - \frac{1}{2}\right); \tag{10}$$

$$V_2 = \frac{\sqrt{f(r)}|K|}{r^2}\left[|K|\sqrt{f(r)} + \frac{r}{2}\frac{df(r)}{dr} - f(r)\right], \quad \left(K = j + \frac{1}{2},\ \ j = l + \frac{1}{2}\right). \tag{11}$$

Solving Eqs. (8) and (9) near the event horizon, we obtain $\chi_1 = \chi_2 = e^{\pm i\omega r_*}$. Hereafter we will use the wavevector $\mathbf{k}$ labels the modes, and for massless Dirac field we have $\omega = |\mathbf{k}|$. All the positive frequency mode in this paper will be defined by use of the corresponding future-directed timelike Killing vector. Thus, for the outside and inside regions of the event horizon, we obtain the positive (fermions) frequency outgoing solutions [25]

$$\Psi_{\mathbf{k}}^{I+} = \frac{1}{r}f(r)^{-\frac{1}{4}}\phi(\theta,\varphi)e^{-i\omega u}, \tag{12}$$

$$\Psi_{\mathbf{k}}^{II+} = \frac{1}{r}f(r)^{-\frac{1}{4}}\phi(\theta,\varphi)e^{i\omega u}, \tag{13}$$

where $u = t - r_*$ and $\phi(\theta,\varphi) = \begin{pmatrix} i\phi_{jm}^{\pm}(\theta,\varphi) \\ \phi_{jm}^{\mp}(\theta,\varphi) \end{pmatrix}$ is a 4-component Dirac spinor. The mode $\Psi_{\mathbf{k}}^{I+}$ describes what the Schwarchild observer Bob who hovers outside the asymptotically flat static black hole sees.

Since the black hole modes $\Psi_{\mathbf{k}}^{I+}$ and $\Psi_{\mathbf{k}}^{II+}$ are analytic outside and inside the event horizon respectively, they form a complete orthogonal family. Thus in terms of these basis the field $\Psi_{out}$ can be expanded as [13]

$$\Psi_{out} = \int d\mathbf{k}[\hat{a}_{\mathbf{k}}^{I}\Psi_{\mathbf{k}}^{I+} + \hat{b}_{\mathbf{k}}^{I\dagger}\Psi_{\mathbf{k}}^{I-} + \hat{a}_{\mathbf{k}}^{II}\Psi_{\mathbf{k}}^{II+} + \hat{b}_{\mathbf{k}}^{II\dagger}\Psi_{\mathbf{k}}^{II-}], \tag{14}$$

where $\hat{a}_{\mathbf{k}}^{I}$ and $\hat{b}_{\mathbf{k}}^{I\dagger}$ are the fermion annihilation and antifermion creation operators acting on the state of the exterior region, and $\hat{a}_{\mathbf{k}}^{II}$ and $\hat{b}_{\mathbf{k}}^{II\dagger}$ are the fermion annihilation and antifermion creation operators acting on the state of the interior region of the black hole respectively.

However, the mode Eq.(12) is analytic and complete in outside of the black hole but vanished in the inside region, while Eq.(13) is analytic and complete only in the inside region. In consideration of the completeness of quantization we should find modes complete on all of the whole spacetime. The generalized light-like Kruskal coordinates $U$ and $V$ take the form [11, 19, 4]

$$u = -\frac{1}{\kappa}\ln(-\kappa U), \quad v = \frac{1}{\kappa}\ln(\kappa V), \quad \text{if} \quad \mathrm{r} > \mathrm{r}_+, \tag{15}$$

$$u = -\frac{1}{\kappa}\ln(\kappa U), \quad v = \frac{1}{\kappa}\ln(\kappa V), \quad \text{if} \quad \mathrm{r} < \mathrm{r}_+, \tag{16}$$

where $\kappa$ is the surface gravity of the event horizon determined by $\kappa = f'(r_+)/2$. Noting that $U < 0$ in region I and $U > 0$ in region II and making a analytic continuation for Eqs. (12) and (13), we find a complete basis for positive energy Kruskal modes [25]

$$F^{I+}_{\mathbf{k}} = e^{\pi\omega/2\kappa}\Psi^{I+}_{\mathbf{k}} + e^{-\pi\omega/2\kappa}\Psi^{II-}_{-\mathbf{k}}, \tag{17}$$

$$F^{II+}_{\mathbf{k}} = e^{-\pi\omega/2\kappa}\Psi^{I-}_{-\mathbf{k}} + e^{\pi\omega/2\kappa}\Psi^{II+}_{\mathbf{k}}, \tag{18}$$

which describe what Alice who freely fall into a black hole (Kruskal observer) sees. We can also quantize the Dirac field in the Kruskal spacetime as

$$\Psi_{out} = \int d\mathbf{k}[2\cosh(\pi\omega/\kappa)]^{-1/2}[\hat{c}^I_{\mathbf{k}}F_{\mathbf{k}} + d^{I\dagger}_{\mathbf{k}}F^{I-}_{\mathbf{k}} + \hat{c}^{II}_{\mathbf{k}}F^{II}_{\mathbf{k}} + d^{II\dagger}_{\mathbf{k}}F^{II-}_{\mathbf{k}}]. \tag{19}$$

From Eqs. (14) and (19) we can easily get the Bogoliubov transformations [26] between the black hole modes and Kruskal modes, then the Kruskal vacuum is found to be a entangled two-mode squeezed state

$$\begin{aligned} |0_{\mathbf{k}}\rangle_K &= \left[(e^{-2\pi\omega/\kappa}+1)^{-\frac{1}{2}} + (e^{2\pi\omega/\kappa}+1)^{-\frac{1}{2}}\hat{a}^{I\dagger}_{\mathbf{k}}\hat{b}^{II\dagger}_{-\mathbf{k}}\right]|0_{\mathbf{k}}\rangle^+_I|0_{-\mathbf{k}}\rangle^-_{II} \\ &= (e^{-2\pi\omega/\kappa}+1)^{-\frac{1}{2}}|0_{\mathbf{k}}\rangle^+_I|0_{-\mathbf{k}}\rangle^-_{II} + (e^{2\pi\omega/\kappa}+1)^{-\frac{1}{2}}|1_{\mathbf{k}}\rangle^+_I|1_{-\mathbf{k}}\rangle^-_{II}, \end{aligned} \tag{20}$$

where $\{|n_{-\mathbf{k}}\rangle^-_{II}\}$ and $\{|n_{\mathbf{k}}\rangle^+_I\}$ are the orthonormal bases for the inside and outside region of the event horizon respectively, and the $\{+,-\}$ superscript on the kets is used to indicate the particle and anti-particle vacua. Due to the Pauli exclusion principle, there are only two allowed states for each mode, $|0_{\mathbf{k}}\rangle_K$ and $|1_{\mathbf{k}}\rangle_K = \hat{c}^{I\dagger}_{\mathbf{k}}|0_{\mathbf{k}}\rangle_K$ for the fermions, and similarly for antifermions. Thus, the only excited state is given by

$$\begin{aligned} |1_{\mathbf{k}}\rangle_K &= \hat{c}^{I\dagger}_{\mathbf{k}}|0_{\mathbf{k}}\rangle_K \\ &= [(e^{-2\pi\omega/\kappa}+1)^{-1}\hat{a}^{I\dagger}_{\mathbf{k}} - (e^{2\pi\omega/\kappa}+1)^{-1}\hat{b}^{II}_{-\mathbf{k}}\hat{a}^{I\dagger}_{\mathbf{k}}\hat{b}^{II\dagger}_{-\mathbf{k}}]|0_{\mathbf{k}}\rangle^+_I|0_{-\mathbf{k}}\rangle^-_{II} \\ &= [(e^{-2\pi\omega/\kappa}+1)^{-1}\hat{a}^{I\dagger}_{\mathbf{k}} + (e^{2\pi\omega/\kappa}+1)^{-1}\hat{a}^{I\dagger}_{\mathbf{k}}\hat{b}^{II}_{-\mathbf{k}}\hat{b}^{II\dagger}_{-\mathbf{k}}]|0_{\mathbf{k}}\rangle^+_I|0_{-\mathbf{k}}\rangle^-_{II} \\ &= \hat{a}^{I\dagger}_{\mathbf{k}}|0_{\mathbf{k}}\rangle^+_I|0_{-\mathbf{k}}\rangle^-_{II} \\ &= |1_{\mathbf{k}}\rangle^+_I|0_{-\mathbf{k}}\rangle^-_{II}. \end{aligned} \tag{21}$$

Hereafter we will refer to the particle mode $\{|n_{\mathbf{k}}\rangle^+_I\}$ simply as $\{|n\rangle_I\}$, and the anti-particle mode $\{|n_{-\mathbf{k}}\rangle^-_{II}\}$ as $\{|n\rangle_{II}\}$ [15].

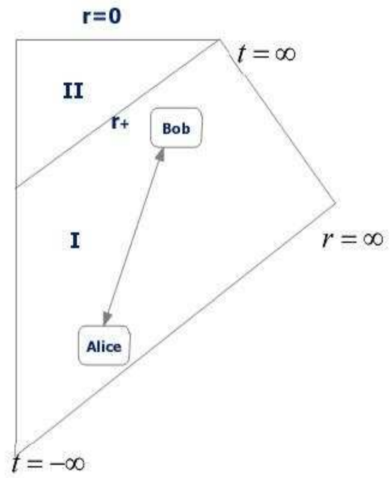


Fig. 1. Penrose diagram for quantum entanglement and teleportation in the spacetime of a static and asymptotically flat black hole.

When Alice travels through the Kruskal vacuum, her detector will detect nothing. However, when Bob travels through the same vacuum, his detector registers the number of particles

$$N_\omega^2 = \frac{1}{e^{2\pi\omega/\kappa}+1} = \frac{1}{e^{\omega/T}+1}, \tag{22}$$

which shows that Bob who hovers out the black hole detects a thermal Fermi-Dirac distribution of particles. Here we have defined the Hawking temperature as $T = f'(r_+)/4\pi$ [27, 28].

## 3 Quantum entanglement for Dirac field

In this section we will discuss how the Hawking temperature affect the degradation of entanglement produced by the Hawking effect for the Dirac field. We assume that Alice has a detector which only detects mode $|n\rangle_A$ and Bob has a detector sensitive only to mode $|n\rangle_B$, and they share a generically entangled state at the same initial point in flat Minkowski spacetime before the black hole is formed. The generically entangled initial state is

$$|\Psi\rangle_{AB} = \sqrt{1-\alpha^2}|0\rangle_A|1\rangle_B + \alpha|1\rangle_A|0\rangle_B, \tag{23}$$

where $\alpha$ is the state parameter which satisfies $|\alpha| \in (0,1)$, $\alpha$ and $\sqrt{1-\alpha^2}$ are the so-called "normalized partners". Note that Bob hovers near the event horizon of the black hole, the states corresponding to mode $|n\rangle_B$ must be specified in the coordinates of the black hole in order to describe what Bob sees in this curved spacetime. Thus, using Eqs. (20) and (21), we can rewrite Eq. (23) in terms of black hole modes for Bob. Since Bob is causally disconnected from the interior region of the black hole, we will take the trace over the states in this region and obtain the mixed density matrix

$$\varrho_{AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1-\alpha^2 & \sqrt{\frac{\alpha^2(1-\alpha^2)}{e^{-\omega/T}+1}} & 0 \\ 0 & \sqrt{\frac{\alpha^2(1-\alpha^2)}{e^{-\omega/T}+1}} & \frac{\alpha^2}{e^{-\omega/T}+1} & 0 \\ 0 & 0 & 0 & \frac{\alpha^2}{e^{\omega/T}+1} \end{pmatrix} \tag{24}$$

with $|mn\rangle = |m\rangle_A |n\rangle_{B,I}$. The partial transpose criterion provides a necessary and sufficient condition for entanglement in a mixed state of two qubits [29]: if at least one eigenvalue of the partial transpose is negative, the density matrix is entangled. The partial transpose $\varrho_{AB}^{T_A}$ is obtained by interchanging Alice's qubits, which yields a negative eigenvalue

$$\lambda_- = \frac{1}{2}\left[\alpha^2(e^{\omega/T}+1)^{-1} - \sqrt{\alpha^4(e^{\omega/T}+1)^{-2} + 4\alpha^2(1-\alpha^2)(e^{-\omega/T}+1)^{-1}}\right].$$

Thus, the state is always entangled for any Hawking temperature $T$.

The degree of entanglement for the two observers here can be measured by using the logarithmic negativity which serves as an upper bound on the entanglement of distillation [30], defined as $N(\varrho_{AB}) = \log_2 ||\varrho_{AB}^{T_A}||$, where $||\varrho_{AB}^{T_A}||$ is the trace norm of the partial transpose $\varrho_{AB}^{T_A}$. In this case, the logarithmic negativity is found to be

$$\begin{aligned} N(\varrho_{AB}) = \log_2[1 - \alpha^2(e^{\omega/T}+1)^{-1} \\ + \sqrt{\alpha^4(e^{\omega/T}+1)^{-2} + 4\alpha^2(1-\alpha^2)(e^{-\omega/T}+1)^{-1}}\ ]. \end{aligned} \tag{25}$$

Eqs. (24) and (25) shows that due to the finite occupation of the fermionic states, we obtain finite dimensional density matrices and a closed form expression of the logarithmic negativity, which are more easily obtained than their infinite dimensional bosonic counterparts [19].

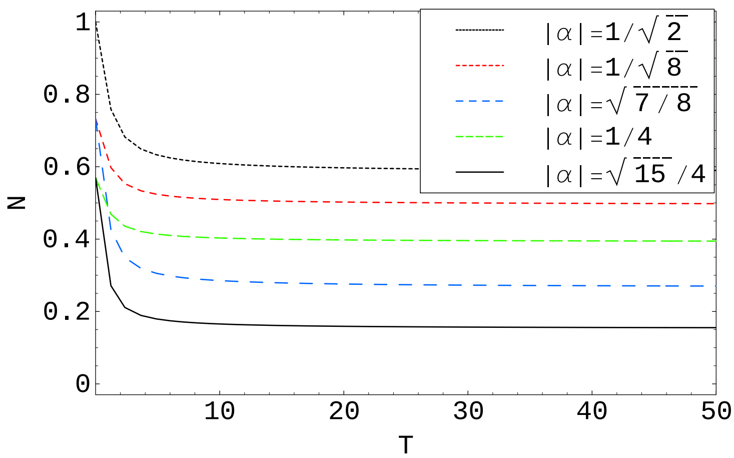


Fig. 2. The logarithmic negativity $N(\varrho_{AB})$ as a function of the Hawking temperature $T$ with the fixed $\omega$ for different $\alpha$.

In Fig. 2 we plot the behavior of the entanglement for different $\alpha$ which shows how the Hawking temperature $T$ would change the properties of the entanglement. The monotonous decrease of $N(\varrho_{AB})$ with increasing $T$ for five different $\alpha$ means that the entanglement is lost due to the thermal fields generated by the Hawking effect. Unlike the behaviors of the scalar case, the same initial entanglement of the Dirac modes for $\alpha$ and $\sqrt{1-\alpha^2}$ will be degraded along two different trajectories and asymptotically reach two differently nonvanishing minimum values in the infinite Hawking temperature limit, which just shows the inequivalence of the quantization for a Dirac field in the black hole and Kruskal spacetimes. It indicates that when black hole evaporates completely, the logarithmic negativity is $N(\varrho_{AB}) = \log_2(1-\alpha^2/2+|\alpha|\sqrt{2-7\alpha^2/4}) \neq 0$, which is different from the scalar case [19, 31, 17]. In Dirac systems the entanglement is still remained in the infinite Hawking temperature limit, means that the quantum information about the entanglement state is not completely disappeared and can be used as a resource for performing certain quantum information processing tasks. This essential difference is caused by the fact that fermions have access to only two quantum levels versus the infinite ladder of excitations available to bosons due to the Pauli exclusion principle.

It is worth mentioning that in Ref. [15], the author found the entanglement of the Dirac field is also not completely destroyed in the infinite acceleration limit, which confirmed that the entanglement character of Dirac field have essential differences to the scalar field in noninertial frames. Further more, the decrease of entanglement of quantum field [17] has a closely relation to the understanding of quantum information in curved spacetime as well as the nature of the information lost in black holes.

The mutual information, which can be used to estimate the total amount of correlations between any two subsystems of the overall system [32], is defined as $I(\varrho_{AB}) = S(\varrho_A)+S(\varrho_B)-S(\varrho_{AB})$, where $S(\varrho) = -Tr(\varrho\log_2\varrho)$ is the entropy of the density matrix $\varrho$. For the Dirac modes, it is found to be

$$\begin{aligned} I(\varrho_{AB}) &= \mathcal{F}[1-\alpha^2(e^{\omega/T}+1)^{-1}]+\mathcal{F}[\alpha^2(e^{\omega/T}+1)^{-1}]-\mathcal{F}(1-\alpha^2) \\ &\quad -\mathcal{F}[1-\alpha^2(e^{-\omega/T}+1)^{-1}]-\mathcal{F}[\alpha^2(e^{-\omega/T}+1)^{-1}]-\mathcal{F}(\alpha^2), \end{aligned} \tag{26}$$

where $\mathcal{F}(x) = x\log(x)$. Note that the initially mutual information is $I_i = -2[\mathcal{F}(\alpha^2)+\mathcal{F}(1-\alpha^2)]$ for vanishing Hawking temperature.

The properties of the mutual information are shown in Fig. 3. It demonstrates that the mutual information becomes smaller as the Hawking temperature increases, and again we find that the degradation is dependent of $\alpha$ for Dirac field. In the infinite Hawking temperature limit $T\to\infty$, i.e., the black hole evaporates completely, the mutual information converges to $I_f = -[\mathcal{F}(\alpha^2)+\mathcal{F}(1-\alpha^2)]$, which is just half of $I_i$. This behavior is same as that for the scalar case [19], so we may suggest that $I_f = \frac{1}{2}I_i$, which is independent of $\alpha$, is a universal property for any fields.

## 4 summary

We have discussed the entanglement and teleportation for Dirac field in the background of an asymptotically flat black hole. It is found that just like the scalar case [19], the same initial entanglement of the Dirac modes for the state parameter $\alpha$ and its “normalized partners” $\sqrt{1-\alpha^2}$ also degraded by the Hawking effect with increasing Hawking temperature $T$ along two different trajectories, which just shows the inequivalence of the quantization for a Dirac field in the black hole and Kruskal spacetimes. However, unlike the bosonic case in which the

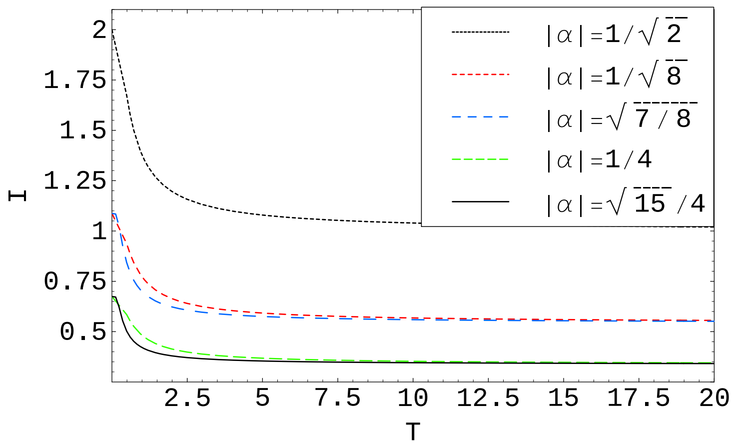


Fig. 3. Mutual information $I(\varrho_{AB})$ of the Dirac modes versus Hawking temperature $T$ with the fixed $\omega$ for different $\alpha$.

initial entanglement vanishes in the limit of infinite Hawking temperature, in this case the entanglement achieves a nonvanishing minimum values, which shows that the entanglement is never completely destroyed when black hole evaporates completely for Dirac modes in this limit. This essential difference is caused by the fact that fermions have access to only two quantum levels versus the infinite ladder of excitations available to bosons due to the Pauli exclusion principle. This essential difference means that in the Dirac case Alice and Rob always share some entanglement which can in principle be used as a resource for performing certain quantum information processing tasks. It is interesting to note that when black hole evaporates completely, the mutual information equals to just half of the initially mutual information, which is independent of $\alpha$ and the type of field.


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