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ARE QUANTUM CORRELATIONS SYMMETRIC ?

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We provide operational definition of symmetry of entanglement: An entangled state contains symmetric entanglement if its subsystems can be exchanged (swapped) by means of local operations and classical communication. We show that in general states have asymmetric entanglement. This allows to construct nonsymmetric measure of entanglement, and a parameter that reports asymmetry of entanglement contents of quantum state. We propose asymptotic measure of asymmetry of entanglement, and show that states for which it is nonzero, contain necessarily bound entanglement.

Keywords: symmetry, swap, entanglement measure
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1 Introduction

There has been much work towards qualitative and quantitative description of quantum entanglement [1, 2, 3]. Mostly, because it is a crucial resource for quantum information processing, but also because it is interesting in itself. Despite huge research, devoted to entanglement, one issue has not been touched so far: can entanglement of bipartite systems be asymmetric with respect to interchange of the subsytems? To address this question, one should first define what does it mean to be symmetric/asymmetric. In this paper we shall provide a suitable definition. Then we shall prove, that there exist states, in which entanglement is not symmetric. However, we shall leave open the question, whether this asymmetry can be lifted in asymptotic limit, where small inaccuracies are tolerable.

The issue of asymmetry should manifest also in quantitative description of entanglement, i.e. since entanglement can be asymmetric, there should also exist asymmetric entanglement measures. Since all the measures introduced so far are symmetric with respect to interchange of Alice and Bob, they cannot account for the asymmetry reported by us. We therefore introduce a new measure of entanglement, which is no longer symmetric under Alice-Bob exchange.

Finally, we also propose quantitative description of the degree of asymmetry, i.e. introduce a parameter, which tells us how much asymmetric is the entanglement contained in the given state. As said, all the above results concern so called "exact" regime. Though we do not know, whether in asymptotic regime, where we have many copies of the same state, and the inaccuracies which vanish in the limit of large number of copies are allowed, the asymmetry still exists, we relate its hypothetical presence to bound entanglement. Namely, states which would contain asymmetric entanglement in such regime, must also contain bound entanglement.

2 Definition of symmmetry of entanglement

Let us start with definition of symmetry of entanglement. First if a bipartite state is symmetric with respect to swap of its subsystems, then of course its entanglement contents is symmetric too, no matter how would we define its symmetry. Consider now a state which is not symmetric under swap. Let us note that if the state is separable, then by LOCC operations, we can always swap it. The easiest way is to remove it and created a new state - the swapped version of the original one. Thus any state, which we cannot swap by LOCC must be necessarily entangled. It is reasonable to attribute the impossibility of swapping by LOCC to entanglement contents of the state: in some way it must be non-symmetric.[a] This motivates the following definition.

Definition 1 *Entanglement contained in the state is symmetric, if by LOCC we can produce a swapped version of the state.*

Here by "swapped version" of a state ρ we mean the state sigma given by

$$\sigma = V\rho V \tag{1}$$

where V is unitary operator which swaps the subsystems.[b] We should note here that there are quantities related to entanglement, such as one-way distillable entanglement, that are manifestly non-symmetric [4], and therefore could be used to indicate asymmetry of entanglement. However they are not true entanglement measures in the sense that they can be increased by local operations and classical communication. (see also discussion in [5], chap. XVIII D).

Our definition of symmetry can be supported by the following analogy. Suppose we have a sculpture which consists of hard core (e.g. stone), and a soft surface material (clay). Suppose, that the surface can be easily modified: by adding or removing material from it. Suppose further, that the core cannot be so easily modified, namely, one can flake it off piece by piece, but any disruption is irreversible. The core corresponds to entanglement.

Let us ask now how one can recognize, whether the core has left-right symmetry. One way is to strip it off the clay, and look, whether what is left is already symmetric. In entanglement domain, it would mean that we distill from a state the entanglement in its essence, without any loss, i.e. in such a way that we can return to the initial state by LOCC. One might think, that such essence of entanglement is the pure entanglement, but it is not the case: there are

[a] One could argue, that it is not entanglement but some other property of a state which is responsible for impossibility of swap, but we prefer to call entanglement everything which is in any way not tractable by LOCC. This is similar to the following common view, according to which any function which cannot be increased by LOCC is an entanglement measure.

[b] We have used here and throughout the paper the fact that $V = V^\dagger$.

bound entangled states, which are entangled, but no pure entanglement can be distilled. And even from distillable states, one can usually draw pure entanglement only in an irreversible, lossy way. Thus, the situation is as if the clay were partially so strongly bind to the stony core, that while removing it, one would necessarily destroy the stone. Therefore we need some other way to decide symmetry of the core. In the case of quantum states, we can easily decide whether a given state is symmetric as a whole with respect to exchange of subsystems. This, in the case of our sculpture means, that there is a way to check, whether the sculpture as a whole is left-right symmetric.

This gives the following possibility of verifying symmetry of the sculpture: we try to modify the surface in such a way that the new sculpture is a reflected version of the initial one (of course we assume that the sculpture can be modified but cannot be rotated, i.e. it is firmly stuck to the ground). If we can reflect the sculpture in this way, the core must be symmetric. This is illustrated on the figure 1.

a)

b)

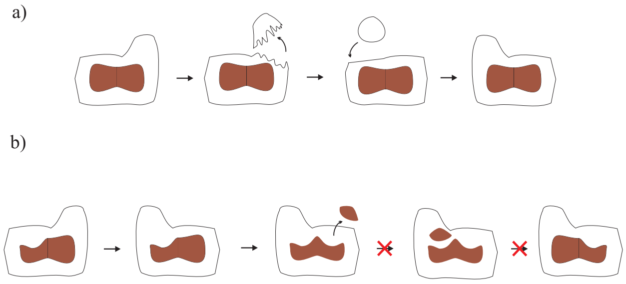

Fig. 1. Impossibility of reflecting the object by modification of the surface implies that the "core" must be asymmetric. a) The core is symmetric, by modifing the surface one can reflect the object b) the core is asymmetric, hence the object cannot be reflected by merely modifying the surface.

This process corresponds to trying swap the state by means of LOCC: by LOCC operations, one is modifying the "surface", and if entanglement itself is symmetric, the state can be swapped, because only the "surface" has to be swapped. But if entanglement is not symmetric, one cannot swap the state by LOCC. And vice versa: if we cannot swap the state, then, since the "surface" is swapable, it must have been entanglement, which was non-symmetric hence it prevented us from swapping the state by LOCC.

3 States with non-symmetric entanglement.

We will now prove that for a large class of states one can swap them by LOCC only when one can swap them by local unitaries. The class consists of all states that have full Schmidt rank [6]. Equivalently, such states can be characterized by the measure of entanglement G-concurrence, denoted as G [7] (see also [8] where similar measures were defined though not proved to be monotoneous under average LOCC operations). The measure for pure state is given by

$$G(\psi_{AB}) = d(\det \rho_A)^{\frac{1}{d}} \tag{2}$$

where ρ_A is reduced density matrix of ψ. For mixed states G is given by

$$G(\rho) = \inf \sum_i p_i G(\psi_i) \tag{3}$$

where infimum is taken over decompositions $\rho = \sum_i |\psi_i\rangle\langle\psi_i|$. (This is standard convex roof procedure [9, 10].) Note that $G(\psi)$ is nonzero if and only if ψ has maximal Schmidt rank. It follows that our class of mixed states is characterized by $G(\rho) > 0$. Thus, we will prove that if $G > 0$, then swapping by LOCC means swapping by product unitary.

In particular, it follows that if state with $G > 0$ has different entropies of subsystems, it cannot be swapped by LOCC, since clearly local unitaries cannot change local entropy. Moreover, for two-qubit states, $G > 0$ iff a state is entangled so that we obtain that any entangled two-qubit state is LOCC swapable iff it is swapable by $U_A \otimes U_B$.

Our main result is contained in the following theorem

Theorem 1 *Consider state ρ acting on $C^d \otimes C^d$, for which $G > 0$ (equivalently, with Schmidt rank equal to d). Then, if such state can be swapped by LOCC, then it can be also swapped by some product unitary operation $U_A \otimes U_B$.*

To prove this theorem we need two lemmas.

Lemma 1 *For any state ρ on $C^d \otimes C^d$, and trace preserving separable operation $\Lambda(\cdot) = \sum_i A_i \otimes B_i(\cdot)A_i^\dagger \otimes B_i^\dagger$ there holds*

$$\sum_i p_i G(\sigma_i) \le \sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} G(\rho) \tag{4}$$

where $\sigma_i = \frac{1}{p_i} A_i \otimes B_i(\rho)A_i^\dagger \otimes B_i^\dagger$, $p_i = \mathrm{Tr}(A_i \otimes B_i(\rho)A_i^\dagger \otimes B_i^\dagger)$.

Remark 1: Similar result (with equality) was obtained for concurrence in [11]. In the proof we will use, in particular, techniques from the proof of monotonicity of convex roof EM's under LOCC [9, 3].

Proof: Consider optimal decomposition $\rho = \sum_j q_j |\psi_j\rangle\langle\psi_j|$, so that $G(\rho) = \sum_j q_j G(\psi_j)$. One finds that

$$\sigma_i = \sum_j \frac{q_j p_i^{(j)}}{p_i} \left(\frac{1}{p_i^{(j)}} X_i |\psi_j\rangle\langle\psi_j| X_i^\dagger \right) \tag{5}$$

$$\equiv \sum_j r_j^{(i)} |\phi_j^{(i)}\rangle\langle\phi_j^{(i)}| \tag{6}$$

where we have denoted $X_i = A_i \otimes B_i$, $p_i^{(j)} = \mathrm{Tr}(X_i|\psi_j\rangle\langle\psi_j|X_i^\dagger)$. The coefficients $r_j^{(i)}$ are probabilities for fixed i and $\phi_j^{(i)}$ are normalized states. We then have

$$\sum_i p_i G(\sigma_i) = \sum_i p_i G(\sum_j r_j^{(i)} |\phi_j^{(i)}\rangle\langle\phi_j^{(i)}|) \le$$

$$\le \sum_{ij} p_i r_j^{(i)} G(\phi_j^{(i)}) = \sum_{ij} q_j G(X_i \psi_j) \tag{7}$$

where we have used convexity of G and the fact that $G(\alpha\rho) = \alpha G(\rho)$ for $\alpha \ge 0$. Now, as shown in [7] $G(A \otimes B\psi) = |\det A|^{\frac{1}{d}} |\det B|^{\frac{1}{d}} G(\psi)$. It follows that

$$\sum_i p_i G(\sigma_i) \le \sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} \sum_j q_j G(\psi_j) =$$

$$\sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} G(\rho) \tag{8}$$

This ends the proof of the lemma. □

The second lemma we need is as follows:

Lemma 2 *For operation Λ from lemma 1 we have $\sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} \leq 1$ with equality if and only if Λ is mixture of product unitary operations.*

Proof: Note that $|\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} = [\det(X_i^\dagger X_i)]^{\frac{1}{d^2}}$ where $X_i = A_i \otimes B_i$. We then have

$$[\det(X_i^\dagger X_i)]^{\frac{1}{d^2}} \leq \frac{1}{d^2} \mathrm{Tr}(X_i^\dagger X_i) \tag{9}$$

as this is actually the inequality between geometric and arithmetic mean of eigenvalues of $X_i^\dagger X_i$ (cf. [12]). It then follows that equality can hold if and only if all eigenvalues are equal i.e. when $X_i^\dagger X_i$ is proportional to identity. Summing up we get

$$\sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} \leq \frac{1}{d^2} \mathrm{Tr} \sum_i (X_i^\dagger X_i) = 1 \tag{10}$$

where used the fact that Λ is trace preserving, so that $\sum_i X_i^\dagger X_i = I$. Equality can hold only when it holds for all terms, which implies that $(A_i \otimes B_i)^\dagger (A_i \otimes B_i)$ is proportional to identity. Hence A_i and B_i are proportional to unitaries. Thus, Λ is mixture of product unitary operations. □

Proof of the theorem 1 : We assume that $G(\rho) > 0$ and that we can swap ρ by LOCC, i.e. $\Lambda(\rho) = V\rho V$. We will now use notation from the lemmas. Thus we assume that $\sum_i p_i \sigma_i = V\rho V$. Using invariance of G under swap, convexity of G and lemma 1 we obtain

$$\begin{aligned} G(\rho) = G(V\rho V) = G(\sum_i p_i \sigma_i) \leq \sum_i p_i G(\sigma_i) \leq \\ \leq \sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} G(\rho) \end{aligned} \tag{11}$$

Since $G(\rho) > 0$ we get $\sum_i |\det A_i|^{\frac{1}{d}} |\det B_i|^{\frac{1}{d}} \geq 1$. Thus in view of lemma 2 we obtain that Λ must be mixture of product unitaries:

$$\Lambda(\rho) = \sum_i p_i U_A^i \otimes U_B^i \rho U_A^{i\dagger} \otimes U_B^{i\dagger} \equiv \sum_i p_i \sigma_i \tag{12}$$

Then the states σ_i have the same von Neumann entropy S as ρ, so that

$$S(\sum_i p_i \sigma_i) = S(V\rho V) = S(\rho) = \sum_i p_i S(\sigma_i) \tag{13}$$

Now, from strict concavity of entropy we obtain that all σ_i's must be the same, so that $V\rho V = U_A^1 \otimes U_B^1 \rho U_A^{1\dagger} \otimes U_B^{1\dagger}$. Thus swap can be performed by local unitary operation. □

3.1 Examples

From the theorem it follows that all entangled two qubit states are swapable, if they are swapable by $U_A \otimes U_B$ Thus any state with subsystems of different spectra is not LOCC swapable, since local unitaries keep local spectra. Exemplary state is mixture of $|01\rangle$ and $a|00\rangle + b|11\rangle$ for $a \neq b$.

Let us see, whether the assumption that $G > 0$ is essential. For higher dimensions there are many states that have $G = 0$. One would be tempted to think that for any entangled state that is LOCC swapable, we can swap it by local unitaries. However, it is not true. Consider state on $C^2 \otimes C^4$ system: being a mixture of $|\psi_+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\psi\rangle = \frac{1}{\sqrt{2}}(|02\rangle + |13\rangle)$. The subsystems have different spectra, so that we cannot swap it by local unitaries. However, the mixture can be reversibly transformed into e.g. ψ_+ by local unitary. Thus it can be swapped.

3.2 Asymmetric entanglement measure

We take any "distance" $\mathcal{D}$ which is continuous, satisfies $\mathcal{D}(\Lambda(\rho)|\Lambda(\sigma)) \leq \mathcal{D}(\rho, \sigma)$ and $\mathcal{D}(\rho, \sigma) = 0$ if and only if $\rho = \sigma$. We consider associated measure $E^{\mathcal{D}}(\rho) = \inf_{\sigma_{sep}} \mathcal{D}(\rho, \sigma_{sep})$ [13] where infimum is taken over all separable states. Consider then a fixed state σ that cannot be swapped by LOCC. Now, our measure is defined as

$$E_\sigma(\rho) = E^{\mathcal{D}}(\sigma) - \inf_{\Lambda} \mathcal{D}(\sigma, \Lambda(\rho)) \tag{14}$$

where infimum is taken over all LOCC operations Λ. Note that for separable states $E_\sigma = 0$, and that by definition it does not increase under LOCC. We have $E_\sigma(\sigma) = E^{\mathcal{D}}(\sigma)$ while $E_\sigma(V\sigma V) < E^{\mathcal{D}}(\sigma)$. To see it note that if we cannot swap a state exactly, then we also cannot swap it with arbitrary good accuracy according to distance satisfying the above conditions. This follows from compactness of set of separable operations. Thus the second term is nonzero.

3.3 Measure of asymmetry of entanglement

We can define a parameter that would report asymmetry of entanglement of a given state.

$$\mathcal{A}_E(\rho) = \inf_{\Lambda} \mathcal{D}(\Lambda(\rho), V\rho V) \tag{15}$$

where infimum is taken over all LOCC operations Λ. Clearly, it is nonzero if and only if a state cannot be swapped by LOCC.

4 Asymptotic regime

So far we have talked about exact transformations. It is interesting to ask if the effect survives limit of many copies, where we allow inaccuracies that vanish asymptotically. We have not been able to answer this question, however we think it is most likely, that even asymptotically, in general one cannot swap states by LOCC.

Under such assumption, we can consider a parameter, which will report *asymptotic symmetry* of entanglement.

To define this parameter we need the notion of optimal transition rate of given state ρ to other state σ denoted as $R(\rho \to \sigma)$ which is the maximal ratio $\frac{m}{n}$ of the transformation $\rho^{\otimes n} \to \sigma' \approx \sigma^{\otimes m}$ via some LOCC map [3].

Definition 2 *Let $\rho_{AB} \in \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B)$ be an entangled state. Then swap-symmetry is defined for entangled states as follows:*

$$S_{swap}(\rho) = R(\rho \to V\rho V). \tag{16}$$

which is the optimal rate of transition from ρ to $V\rho V$ by means of LOCC.

This quantity is clearly infinite for separable states. However for entangled states it is always finite

Lemma 3: For entangled state ρ we have

$$S_{swap}(\rho) \leq 1 \tag{17}$$

Proof: We apply relation between rates and asymptotically continuous entanglement monotones [3]. Consider two state σ and ρ, and an asymptotically continuous entanglement monotone E. Let us assume that $E^\infty(\sigma) > 0$. Then we have

$$R(\rho \to \sigma) \leq \frac{E^\infty(\rho)}{E^\infty(\sigma)} \tag{18}$$

Here we will take $\sigma = V\rho V$ and E to be entanglement of formation E_F. Regularization of E_F is entanglement cost: $E_F^\infty = E_c$ and it was shown in [14] that it is nonzero for any entangled state. Since $E_c(\rho) = E_c(V\rho V)$ we obtain that $R(\rho \to V\rho V) \leq 1$ which ends the proof. $\square$

We can also design another quantity, which would also report how much asymmetric is entanglement of a given state. To this end let us consider round-trip-travel rate i.e. the optimal rate of transferring state ρ into itself via some other state σ (cf. [15]). It is formally defined as

$$R(\rho \rightleftharpoons \sigma) = R(\rho \to \sigma) R(\sigma \to \rho) \tag{19}$$

where we use convention $0 \cdot \infty = \infty \cdot 0 = 0$. We now define our second quantity:

Definition 3 *The following quantity*

$$S_{sym}(\rho) = \sup_\sigma R(\rho \rightleftharpoons \sigma) \tag{20}$$

where supremum is taken over all symmetric states σ we will call symmetry*.*

Again, using [14] we can get that for any entangled state $S_{sym} \leq 1$. However surprisingly, it turns out that the two quantities are equal:

Theorem 2 *The quantities S_{sym} and S_{swap} are equal to each other*

$$S_{sym} = S_{swap} \tag{21}$$

Proof: To see that $S_{sym} \leq S_{swap}$ consider the protocol achieving S_{sym}

$$\rho^{\otimes n} \to \sigma^{\otimes m} \to \rho^{\otimes k} \tag{22}$$

where $\sigma = V\sigma V$. Since the protocol is optimal, we have $k/n \approx S_{sym}$. In the second stage (transforming σ into ρ) let us exchange roles of Alice and Bob. Then, instead of $\rho^{\otimes k}$ we will obtain $(V\rho V)^{\otimes k}$. Thus the total protocol will simply swap the state with rate k/n. Thus we can swap at least with rate S_{sym} which proves $S_{swap} \geq S_{sym}$.

To prove converse, it is enough to find a symmetric state σ such that $R(\rho \rightleftharpoons \sigma)$ will be equal to S_{swap}. Clearly, instead of symmetric (i.e. swap invariant state) we can choose a state which can be made symmetric by local unitaries. We will take

$$\sigma = \rho \otimes V\rho V \tag{23}$$

It is easy to see that local swaps produce a symmetric state from σ. We will now express $R(\rho \rightleftharpoons \rho \otimes V\rho V)$ in terms of $S_{swap}(\rho)$. To this end consider the following transformation

$$\rho^{\otimes n} \otimes \rho^{\otimes m} \to (V\rho V)^{\otimes m} \otimes \rho^{\otimes m} = \sigma^{\otimes m} \tag{24}$$

where the rate $m/n \approx S_{swap}$ is possible by definition of S_{swap}. Then we consider transformation that returns to the state ρ:

$$\sigma^{\otimes m} = (V\rho V)^{\otimes m} \otimes \rho^{\otimes m} \to \rho^{\otimes k} \otimes \rho^{\otimes m} \tag{25}$$

where again by definition of S_{swap} the rate $k/m \approx S_{swap}$ is possible. Thus the overall round-trip-travel rate vis state σ satisfies

$$R(\rho \rightleftharpoons \sigma) \leq \frac{k+m}{n+m} \approx \frac{S_{swap}+1}{\frac{1}{S_{swap}}+1} = S_{swap} \tag{26}$$

Since S_{sym} is supremum of such rates, we obtain that $S_{sym} \geq S_{swap}$. This ends the proof. □

We thus obtain our asymptotic quantities measuring symmetry/asymmetry.

Definition 4 *The quantity $S_{sym} = S_{swap}$ we will call symmetry of entanglement, and will denote by $\mathcal{S}_E^{as}$. The quantity $\mathcal{A}_E^{as} = 1 - \mathcal{S}_E^{as}$ we will call asymmetry of entanglement.*

Thus, entanglement in a given state is not symmetric when $\mathcal{A}_E^{as} > 0$. We will now argue that states with nonsymmetric entanglement must possess bound entanglement, i.e. for such state distillable entanglement is strictly smaller than entanglement cost $E_D < E_c$. Thus asymptotic asymmetry brings irreversibility. The reason is obvious, reversibility in distillation-creation process means that we can go reversibly from ρ to ρ through maximally entangled state which is symmetric state. Thus $\mathcal{S}_E^{as} = 1$ in such case. We have

Theorem 3 *For entangled states, we have*

$$\frac{E_D}{E_c} \leq \mathcal{S}_E^{as} \leq 1 \tag{27}$$

Equivalently we have

$$\frac{E_b}{E_c} \geq \mathcal{A}_E^{as} \tag{28}$$

where $E_b = E_c - E_D$.

Proof: The optimal rate $R(\rho \rightleftharpoons \psi_+)$ where $\psi_+ = \frac{1}{2}(|00\rangle + |11\rangle)$ is given by

$$R(\rho \rightleftharpoons \psi_+) = \frac{E_D}{E_c} \tag{29}$$

Since maximally entangled state is symmetric, this is rate of a particular protocol of round-trip-travel from ρ to ρ via symmetric state. Thus it is no greater than $\mathcal{S}_E^{as}$ which is supremum of rates over such protocols. □

From this theorem it follows that $\mathcal{S}_E^{as}$ is nonzero for distillable states.

5 Concluding remarks.

We have proposed an operational definition of asymmetry of entanglement, and provide examples of states containing asymmetric entanglement. We also propose a quantitative measure of asymmetry of entanglement for a single copy of quantum state. This proposition is not unique. Other candidate can be the infimum of distance from the set of single copy LOCC swapable states. It seems that the lower bound on this measure in terms of G-concurrence can be found.

We also conjecture that entanglement can be asymmetric in asymptotic regime of many copies i.e. that there exist states with $\mathcal{S}_E^{as} < 1$. This would imply a nice correspondence. Namely existence of bound entanglement can be viewed as "time asymmetry", hence we would have that "space asymmetry" of entanglement would imply its "time asymmetry" [c]. Moreover one could then ask if V can increase E_D of some distillable states i.e. if $E_D(\rho \otimes V\rho V) > E_D(\rho^{\otimes 2})$.

If however $\mathcal{S}_E^{as} = 1$ for all states one would have that certain nontrivial task can be achieved via LOCC. Namely one would get the answer to the question what Alice and Bob should do if by accident they got $\rho_{BA}^{\otimes n}$ instead of $\rho_{AB}^{\otimes n}$. Moreover a nice correspondence between transposition and swap would hold. As we have mentioned, like $I \otimes T$ is not physical, the operation $I \otimes V$ can not be implemented by means of LOCC i.e. it is not physical with respect to this class of operations. Although transposition is not completely positive it can be performed on a *known* state, as it is positive. If then $\mathcal{S}_E^{as} = 1$ for all states i.e. all states would be swapable, then V like T could be performed on a *known* state (in this case via LOCC operations).

Note that still there are many states which have $\mathcal{S}_E^{as} = 1$ because they are swap invariant. It is then tempting to develop a scheme of symmetry of entanglement with respect to certain group G of unitary transformations (see in this context [16]). That is $G-$symmetry of a state would be maximal rate of distillation of states which are invariant under actions of G.

As a generalization of our approach one can consider the asymmetry of general quantum correlations by restricting class of allowed operations to so called *closed* LOCC operations [17]. In such case also certain separable states may exhibit asymmetry. Moreover in analogy to asymmetry of entanglement one can also quantify asymmetry of private (cryptographic) correlations.

Finally, we note that quite recently other interesting investigations of notion of exchange of subsystems and swap symmetry have been independently developed [18, 19, 20].

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