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MACROSCOPIC DISPLACED THERMAL FIELD AS THE ENTANGLEMENT CATALYST

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We show that entanglement of multiple atoms can arise via resonant interaction with a displaced thermal field with a macroscopic photon-number. The cavity field acts as the catalyst, which is disentangled with the atomic system after the operation. Remarkably, the entanglement speed does not decrease as the average photon-number of the mixed thermal state increases. The atoms may evolve to a highly entangled state even when the photon-number of the cavity mode approaches infinity.

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In quantum mechanics, superposition effects give arise to many striking features. Superpositions of product states of composite systems leads to quantum entanglement, which is an entirely quantum-mechanical effect and results in phenomena that can not be explained in classical terms. On one hand, an entangled state of two or more particle reveals nonlocal structure of quantum theory, providing a basis for the test of quantum mechanics against local hidden variable theories [1,2]. On the other hand, entanglement is an essential ingredient for quantum informatiom processing, such as quantum cryptography [3] and teleportation [4]. The Jaynes-Cummings model (JCM) [5, 6], which describes the interaction of a two-level atom and a single-mode electromagnetic field, is a typical system for producing entanglement. It has been shown that for certain pure initial states, entanglement between the atom and cavity mode oscillates with time [7]. The cavity mode can also act as the catalyst for the synthesis of multiatom entanglement [8]. Over the past few decades, there have been various generalizations of the JCM. One of the typical examples is the Tavis-Cummings model [9], describing the interaction of multiple two-level atoms and the cavity mode.

Recent advances in microwave cavity QED techniques, with Rydberg atoms interacting with a superconducting cavity, allow the test of many interesting quantum effects arising from the interaction of atoms with a quantized cavity field [8]. Up to now, entangled states involving two or three atoms have been produced in experiment [10,11]. In most of the cavity QED experiments, the cavity field is required to be initially in a pure state. Previous research show that the microscopic nature of the field is essential for entangling two or more atoms.

In this paper, we show that maximally entangled states for multiple atoms can be produced via resonant interaction with a cavity field with a macroscopic photon number, showing that a macroscopic system can also act as the entanglement catalyst. Secondly, we show that the entanglement is insensitive to thermal photons. Under certain conditions the atoms are disentangled with the cavity field, which is distinguished from the previous work showing that atom-field entanglement always arises when the field is initially in a thermal state [12]. Thirdly, the atoms are resonant with the cavity mode and thus the entanglement speed is very high. More strikingly, the entanglement speed is independent of both the number of atoms and the mean photon-number of the thermal field, and high entanglement can appear even there exist many thermal photons, which is in contrast with the previous work [13]. Finally, we show that a phase gate between two atoms can be produced with a thermal field, providing a new prospect for quantum information processing in a nonzero temperature environment.

Suppose that the single-mode cavity field is initially in the thermal state

$$\rho_{th} = \frac{1}{\pi \bar{n}} \int e^{-|\alpha|^2/\bar{n}} \left|\alpha\right\rangle \left\langle\alpha\right| d^2\alpha, \tag{1}$$

where $\bar{n}_{th} = 1/(e^{\hbar\omega/k_B T} - 1)$ is the mean photon-number of the thermal field. We first displace the cavity field by an amount α, leading to the density operator $D(\alpha)\rho_{th}D^+(\alpha)$, with $D(\alpha)$ being the displacement operator. We here assume that α is a complex number, i.e., $\alpha = re^{-i\varphi}$. The displacement can be achieved by injecting the cavity a coherent field generated by a source [14]. We consider the resonant interaction of N identical two-level atoms with a single-mode cavity field. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar = 1$)

$$H = \sum_{j=1}^{N} g(a^+ S_j^- + a S_j^+), \tag{2}$$

where $S_j^+ = \left|e_j\right\rangle \left\langle g_j\right|$, $S_j^- = \left|g_j\right\rangle \left\langle e_j\right|$, with $\left|e_j\right\rangle$ and $\left|g_j\right\rangle$ being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators for the cavity mode, and g is the atom-cavity coupling strength. Suppose that the atoms are initially in the state $\left|\phi_0\right\rangle$. Then the initial density operator for the whole system is $\rho_0 = D(\alpha) \left|\phi_0\right\rangle \left\langle\phi_0\right| \otimes \rho_{th} D^+(\alpha)$. The evolution operator of the system is given by $U(t) = e^{-iHt}$. After an interaction time the system evolves to $\rho = U(t)\rho_0 U^+(t)$.

We can rewrite the evolution of the density operator as

$$\rho = D(\alpha)U_d(t) \left|\phi_0\right\rangle \left\langle\phi_0\right| \otimes \rho_{th} U_d^+(t) D^+(\alpha), \tag{3}$$

where

$$\begin{aligned} U_d(t) &= D^+(\alpha)U(t)D(\alpha) \\ &= e^{-iH_d t}, \end{aligned} \tag{4}$$

where

$$H_d = \sum_{j=1}^{N} g[(a^+ + \alpha^*)S_j^- + (a + \alpha)S_j^+]. \tag{5}$$

Define the new atomic basis [15,16]

$$|+_{j,\varphi}\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle + e^{i\varphi}|g_j\rangle), \ \ |-_{j,\varphi}\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle - e^{i\varphi}|g_j\rangle). \tag{6}$$

Then we can rewrite H_d as

$$H_d = \sum_{j=1}^{N}\{\frac{g}{2}[e^{-i\varphi}a^{+}(2\sigma_{z,j,\varphi} + \sigma_{j,\varphi}^{+} - \sigma_{j,\varphi}^{-}) + e^{i\varphi}a(2\sigma_{z,j,\varphi} + \sigma_{j,\varphi}^{-} - \sigma_{j,\varphi}^{+})] + 2\Omega\sigma_{z,j,\varphi}\}, \tag{7}$$

where $\sigma_{z,j,\varphi} = \frac{1}{2}(|+_{j,\varphi}\rangle\langle+_{j,\varphi}| - |-_{j,\varphi}\rangle\langle-_{j,\varphi}|)$, $\sigma_{j,\varphi}^{+} = |+_{j,\varphi}\rangle\langle-_{j,\varphi}|$, $\sigma_{j,\varphi}^{-} = |-_{j,\varphi}\rangle\langle+_{j,\varphi}|$, and $\Omega = rg$. We can rewrite $U_d(t)$ as

$$U_d(t) = e^{-i2\Omega\sigma_{z,j,\varphi}t}e^{-iH_it}, \tag{8}$$

where

$$H_i = \sum_{j=1}^{N}\frac{g}{2}[e^{-i\varphi}a^{+}(2\sigma_{z,j,\varphi}+e^{i2\Omega t}\sigma_{j,\varphi}^{+}-e^{-i2\Omega t}\sigma_{j,\varphi}^{-})+e^{i\varphi}a(2\sigma_{z,j,\varphi}+e^{-i2\Omega t}\sigma_{j,\varphi}^{-}-e^{i2\Omega t}\sigma_{j,\varphi}^{+})] \tag{9}$$

Assuming that $\Omega \gg g$, we can neglect the terms oscillating fast. Then H_i reduces to

$$H_i = g(e^{-i\varphi}a^{+} + e^{i\varphi}a)\sigma_{z,\varphi}, \tag{10}$$

where

$$\sigma_{z,\varphi} = \sum_{j=1}^{N}\sigma_{z,j,\varphi}. \tag{11}$$

The Hamiltonian H_i describes a spin-dependent force on the cavity field. It has been shown that this Hamiltonian can be obtained in the ion trap [17,18]. In this case, a collective vibrational mode acts as the bosonic system and the internal degrees of freedom of the ions correspond to the spin system. The spin-dependent force has been used to generate Schrödinger cat states [19] and implement two-qubit phase gates [20] in ion trap experiments. Milburn et al. [17] have proposed a scheme for realizing multi-qubit gates via sequent applications of the Hamiltonian with variable parameter φ. In the ion trap, φ is adjustable via the phases of driving lasers resonant with the sideband transitions. The aim of the following section is to show that sequent spin-dependent forces with controllable parameter φ can be achieved in the atom-cavity system by applying a sequence of displacements and atomic rotations interspersed between periods of evolution of the system. The corresponding spin-dependent displacement along a close path in phase space produces a spin-dependent phase, which can be used to generate Greenberger-Horne-Zeilinger (GHZ) states and implement two-qubit phase gates.

Define the symmetrical state $|\Phi_{k,\varphi}\rangle$ with k atoms being in the state $|-_{j,\varphi}\rangle$, i.e., the well known Dicke state [21]. Applying the collective atomic operator $\sigma_{z,\varphi}$ to the Dicke state $|\Phi_{k,\varphi}\rangle$, we obtain

$$\sigma_{z,\varphi}|\Phi_{k,\varphi}\rangle = (N/2 - k)|\Phi_{k,\varphi}\rangle. \tag{12}$$

We first assume that $\varphi = 0$, i.e., $\alpha = r$. We now assume that each atom is initially in the state $|e_j\rangle$. $|e_j\rangle$ can be rewritten as

$$|e_j\rangle = \frac{1}{\sqrt{2}}(|+_{j,0}\rangle + |-_{j,0}\rangle). \tag{13}$$

Then the initial state for the N atoms can be written as a Bloch state [22]

$$|\phi_0\rangle = \frac{1}{\sqrt{2^N}} \sum_{k=0}^{N} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} |\Phi_{k,0}\rangle . \tag{14}$$

Using Eqs.(3), (8), (10), (12), and (14), we obtain evolution of the system after an interaction time τ

$$\begin{aligned} \rho_1 \quad = \quad & \frac{1}{2^N} \sum_{k=0}^{N} \sum_{k'=0}^{N} e^{2i(k-k^{'})\Omega\tau} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} \begin{pmatrix} N \\ k^{'} \end{pmatrix}^{1/2} |\Phi_{k,0}\rangle \left\langle \Phi_{k^{'},0} \right| \\ & \otimes D(r) D[-i(N/2-k)g\tau] \rho_{th} D^{+}[-i(N/2-k^{'})g\tau] D^{+}(r), \end{aligned} \tag{15}$$

The resonant interaction of the atom with the strongly displaced thermal field results in the spin-dependent displacement operator on the cavity mode.

We then displace the cavity mode by an amount $-r + ir$ and perform the rotation $|g_j\rangle \rightarrow i\,|g_j\rangle$, which leads to

$$\begin{aligned} \rho_1^{'} \quad = \quad & \frac{1}{2^N} \sum_{k=0}^{N} \sum_{k'=0}^{N} e^{2i(k-k^{'})\Omega\tau} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} \begin{pmatrix} N \\ k^{'} \end{pmatrix}^{1/2} \left|\Phi_{k,\pi/2}\right\rangle \left\langle \Phi_{k^{'},\pi/2} \right| \\ & \otimes D(ir) D[-i(N/2-k)g\tau] \rho_{th} D^{+}[-i(N/2-k^{'})g\tau] D^{+}(ir). \end{aligned} \tag{16}$$

After an interaction time τ, we obtain

$$\begin{aligned} \rho_2 \quad = \quad & \frac{1}{2^N} \sum_{k=0}^{N} \sum_{k'=0}^{N} e^{4i(k-k^{'})\Omega\tau} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} \begin{pmatrix} N \\ k^{'} \end{pmatrix}^{1/2} \left|\Phi_{k,\pi/2}\right\rangle \left\langle \Phi_{k^{'},\pi/2} \right| \\ & \otimes D(ir) D[(N/2-k)g\tau] D[-i(N/2-k)g\tau] \rho_{th} \\ & D^{+}[-i(N/2-k^{'})g\tau] D^{+}[(N/2-k)g\tau] D^{+}(ir). \end{aligned} \tag{17}$$

After the field displacement and atomic rotation, the resonant interaction yields a second spin-dependent displacement perpendicular to the first one in phase space. This is due to the fact that the total displacement before the second resonant interaction is just perpendicular to that before the first resonant interaction.

We repeat the procedure for two more times. During the two cycles the displacements are $-r - ir$ and $r - ir$, respectively. The final state of the system is

$$\begin{aligned} \rho_f \quad = \quad & \frac{1}{2^N} \sum_{k=0}^{N} \sum_{k'=0}^{N} e^{8i(k-k^{'})\Omega\tau} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} \begin{pmatrix} N \\ k^{'} \end{pmatrix}^{1/2} \left|\Phi_{k,-\pi/2}\right\rangle \left\langle \Phi_{k,-\pi/2} \right| \\ & \otimes D(-ir) D[-(N/2-k)g\tau] D[i(N/2-k)g\tau] D[(N/2-k)g\tau] \\ & D[-i(N/2-k)g\tau] \rho_{th} D^{+}[-i(N/2-k^{'})g\tau] D^{+}[(N/2-k^{'})g\tau] \\ & D^{+}[i(N/2-k^{'})g\tau] D^{+}[-(N/2-k^{'})g\tau] D^{+}(-ir). \end{aligned} \tag{18}$$

We can rewrite ρ_f as

$$\rho_f = |\phi_f\rangle \langle\phi_f| \otimes D(-ir)\rho_{th} D^+(-ir) \tag{19}$$

where

$$\begin{aligned} |\phi_f\rangle &= \frac{1}{\sqrt{2^N}} \sum_{k=0}^{N} e^{8ik\Omega\tau} e^{2i[(N/2-k)g\tau]^2} \begin{pmatrix} k \\ N \end{pmatrix}^{1/2} \left|\Phi_{k,-\pi/2}\right\rangle \\ &= \frac{1}{\sqrt{2^N}} \sum_{k=0}^{N} e^{8ik\Omega\tau - 2ikN(g\tau)^2} e^{2ik^2(g\tau)^2} \begin{pmatrix} k \\ N \end{pmatrix}^{1/2} \left|\Phi_{k,-\pi/2}\right\rangle . \end{aligned} \tag{20}$$

We here have discarded the common phase factor $e^{iN^2(g\tau)^2/2}$. With the choice $2(g\tau)^2 = \pi/2$ we obtain [23,24]

$$\begin{aligned} |\phi_f\rangle &= \frac{1}{\sqrt{2^{N+1}}} [e^{i\pi/4} \prod_{j=1}^{N} (\left|+_{j,-\pi/2}\right\rangle + e^{8i\Omega\tau - iN\pi/2} \left|-_{j,-\pi/2}\right\rangle) \\ &\quad + e^{-i\pi/4} \prod_{j=1}^{N} (\left|+_{j,-\pi/2}\right\rangle - e^{8i\Omega\tau - iN\pi/2} \left|-_{j,-\pi/2}\right\rangle)]. \end{aligned} \tag{21}$$

Since the state

$$(\left|+_{j,-\pi/2}\right\rangle + e^{8i\Omega\tau - iN\pi/2} \left|-_{j,-\pi/2}\right\rangle)/\sqrt{2}$$

is orthogonal to

$$(\left|+_{j,-\pi/2}\right\rangle - e^{8i\Omega\tau - iN\pi/2} \left|-_{j,-\pi/2}\right\rangle)/\sqrt{2},$$

$|\phi_f\rangle$ is a N-particle maximally entangled state, or a GHZ state [2]. The average photon-number of the displaced thermal state depends upon the amount of the initial displacement: $\bar{n} = \bar{n}_{th} + |\alpha|^2$. The entanglement persists in the classical limit $|\alpha|^2 \to \infty$.

The entanglement speed is independent of both the number of atoms and the mean photon-number of the thermal field. The strongly displaced JCM evolution operator produces a displacement conditional on the atomic state. The cavity field is displaced along the sides of a square, whose length depends upon the state of the atomic system. After the operation, the atomic system is disentangled with the cavity field, but acquires a phase conditional on the displace path [17,18,20], leading to the entanglement. The macroscopic thermal field acts as the entanglement catalyst.

We note the idea can be generalized to realize geometric phase gates for two atoms with a thermal field. For the two-atom case, the above mentioned displacements, rotations, and resonant interactions leads to the transformation:

$$\begin{aligned} |+_{1,0}\rangle |+_{2,0}\rangle &\to e^{i2(g\tau)^2} \left|+_{1,-\pi/2}\right\rangle \left|+_{2,-\pi/2}\right\rangle , \\ |+_{1,0}\rangle |-_{2,0}\rangle &\to \left|+_{1,-\pi/2}\right\rangle \left|-_{2,-\pi/2}\right\rangle , \\ |-_{1,0}\rangle |+_{2,0}\rangle &\to \left|-_{1,-\pi/2}\right\rangle \left|+_{2,-\pi/2}\right\rangle , \\ |-_{1,0}\rangle |-_{2,0}\rangle &\to e^{i2(g\tau)^2} \left|-_{1,-\pi/2}\right\rangle \left|-_{2,-\pi/2}\right\rangle . \end{aligned} \tag{22}$$

We here have discarded the trival single-qubit phase shifts, which can be absorbed into next single-qubit operations. Setting $2(g\tau)^2 = \pi/2$ and performing the rotation $|g_j\rangle \to i\,|g_j\rangle$ we obtain the phase gate

$$\begin{aligned}
\left|+_{1,0}\right\rangle\left|+_{2,0}\right\rangle &\rightarrow i\left|+_{1,0}\right\rangle\left|+_{2,0}\right\rangle, \\
\left|+_{1,0}\right\rangle\left|-_{2,0}\right\rangle &\rightarrow \left|+_{1,0}\right\rangle\left|-_{2,0}\right\rangle, \\
\left|-_{1,0}\right\rangle\left|+_{2,0}\right\rangle &\rightarrow \left|-_{1,0}\right\rangle\left|+_{2,0}\right\rangle, \\
\left|-_{1,0}\right\rangle\left|-_{2,0}\right\rangle &\rightarrow i\left|-_{1,0}\right\rangle\left|-_{2,0}\right\rangle.
\end{aligned} \tag{23}$$

The combination of this gate and the single-qubit phase shifts $\left|+_{1,0}\right\rangle \rightarrow -i\left|+_{1,0}\right\rangle$ and $\left|-_{2,0}\right\rangle \rightarrow i\left|-_{2,0}\right\rangle$ corresponds to two-qubit π-phase gate.

We now show how the gate is robust against parameter fluctuations. Suppose that the two atoms are initially in the state

$$|\phi_0\rangle = \frac{1}{2}(\left|+_{1,0}\right\rangle + \left|-_{1,0}\right\rangle)(\left|+_{2,0}\right\rangle + \left|-_{2,0}\right\rangle). \tag{24}$$

The phase gate of Eq. (23) produces the maximally entangled state

$$|\phi_f\rangle = \frac{1}{2}[\left|+_{1,0}\right\rangle (i\left|+_{2,0}\right\rangle + \left|-_{2,0}\right\rangle) + \left|-_{1,0}\right\rangle (\left|+_{2,0}\right\rangle + i\left|-_{2,0}\right\rangle)]. \tag{25}$$

If the condition $2(g\tau)^2 = \pi/2$ is not exactly satisfied, the final state is

$$\left|\phi_f^{'}\right\rangle = \frac{1}{2}[\left|+_{1,0}\right\rangle (e^{i2(g\tau)^2}\left|+_{2,0}\right\rangle + \left|-_{2,0}\right\rangle) + \left|-_{1,0}\right\rangle (\left|+_{2,0}\right\rangle + e^{i2(g\tau)^2}\left|-_{2,0}\right\rangle)]. \tag{26}$$

The fidelity is given by

$$F = \left|\langle\phi_f|\ \phi_f^{'}\rangle\right|^2 = \frac{1}{4}\{1 + \sin[2(g\tau)^2]\}^2 + \frac{1}{4}\cos^2[2(g\tau)^2]. \tag{27}$$

Set $2(g\tau)^2 = 0.55\pi$. Then the fidelity is about 0.99.

In microwave cavity QED experiments [10,25], two or more atoms are simultaneously sent through a cavity. For the Rydberg atoms with principal quantum numbers 50 and 51, the radiative time is $T_r = 3 \times 10^{-2}s$, and the coupling constant is $g = 2\pi \times 25kHz$ [10]. Thus the interaction times of atoms with the cavity field are $t = 4\tau = 2\sqrt{\pi}/g = 2.26 \times 10^{-5}s$. In the case $N = 3$ the decoherence time of the atomic system is $T_r^{'} = T_r/3 = 1 \times 10^{-2}s$. The decoherence time of the superposition of different components in the nonzero temperature heat bath is $T_c^{'} = T_c/(1 + 2\ \bar{n}_{th})d^2$, where d is the distance between the components in phase space. During the interaction, the distance between the coherent components is on the order of $g\tau \sim 1$. Very recently, a ultrahigh fineness Fabry-Perot resonator with a damping time $T_c = 0.13s$ has been built [25]. Set $\bar{n}_{th} = 5$. Then, the decoherence time for the cavity field is about $T_c^{'} \sim 6.19 \times 10^{-3}s$. The infidelity induced by the decoherence is about $t/T_r + t/T_c^{'} = 0.591 \times 10^{-2}$.

In conclusion, we have shown that maximally entangled states for multiple atoms can be induced by a cavity field initially in a thermal state. The entanglement appears in the macroscopic limit. The time for the appearance of maximal entanglement is independent of both the number of atoms and the mean photon-number of the thermal field. The quantum phase gate for two atoms can also be produced via interaction with the displaced thermal field. The macroscopic thermal field acts as the catalyst for producing entanglement and quantum information processors.

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