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QUANTUM CLONING OF IDENTICAL MIXED QUBITS

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Quantum cloning of two identical mixed qubits $\rho\otimes\rho$ is studied. We propose the quantum cloning transformations not only for the triplet (symmetric) states but also for the singlet (antisymmetric) state. We can copy these two identical mixed qubits to M ($M \geq 2$) copies. This quantum cloning machine is optimal in the sense that the shrinking factor between the input and the output single qubit achieves the upper bound. The result shows that we can copy two identical mixed qubits with the same quality as that of two identical pure states.

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No-cloning theorem is one of the most fundamental theorems in quantum mechanics and in quantum computation and quantum information[1]. Due to the no-cloning theorm, it is possible for us to design quantum cryptography such as BB84[2], 6-state[3] quantum key distributions and various of their generalizations. The no-cloning theorem is also closely related with no-signaling theorem in quantum mechanics[4].

In case we want to copy a quantum state, we cannot copy it perfectly but approximately[5] or probabilistically[6]. In the past years, much progress has already been made in designing quantum cloning machines for different purposes[1-14], for reviews and references, see [20, 21]. Bužek and Hillery proposed a quantum cloning machine with one qubit input and two qubits output[5]. The quality of the copies is independent of the input state. This quantum cloning machine is called universal quantum cloning machine (UQCM). Later this UQCM was proved to be optimal[14]. For UQCM, the copies are not the same as the input state, but this coping task can always succeed. A different quantum cloning machine was proposed in Ref.[6], while the coping task can succeed with a probability, but if it succeeds, we can always obtain perfect copies. This kind of quantum cloning machine is called probabilistic quantum cloning machine [6]. Other cloning machines such as the asymmetric quantum cloning and the phase-covariant quantum cloning are also studied in the past years [7, 8, 9, 10, 11, 12]. In this paper, we will restrict ourself to the UQCM case.

Bužek and Hillery's UQCM is for one to two case (one input qubit and two output qubits). Gisin and Massar [13] proposed a N to M ($M \geq N$) UQCM and it is also proved to be optimal by different methods[13, 15]. Werner[16] proposed a general N to M UQCM not only for qubit case but also for a general quantum state in d-dimensional system. This quantum cloning machine is realized by symmetric projections and it is proved to be optimal for two different fidelities[16, 17]. Fan *et al* [18] proposed a N to M UQCM following the transformations given in Refs.[5, 13]. This UQCM is optimal for identical pure states and also for quantum states in symmetric subspace[19]. It can be realized by some physical systems like photon stimulated emission[22, 23]. The super-broadcasting of mixed qubit states which is closely related with cloning machine was recently considered in Refs.[24, 25] based on the result of Ref.[26]. The experiments of UQCM were performed in several groups [27, 28, 29, 30].

While considerable works have already been done to study various quantum cloning machines, see recent review papers [20, 21], there are still some simple and basic unsolved problems. The simplest case is perhaps to copy two identical mixed qubits $\rho \otimes \rho$ optimally. Since the UQCM proposed by Fan *et al*[18] only provides the cloning transformations for symmetric input states, we can copy arbitrary identical pure states and a mixed state in symmetric subspace. If the input are two identical mixed qubits, we cannot use this UQCM, since one input state is the singlet state which is not in the symmetric subspace. One may consider to simply use Werner [16] UQCM for this case and do not care about the real input, we can show however that this method does not work. The simplest example is for case 2 to 2 UQCM, actually we do not need to do anything and the cloning is perfect. Here we use this example since all known UQCMs do work for this case given the input is within their working area, i.e., all known UQCMs can copy the input perfectly. We may find for case $\rho \otimes \rho$, the antisymmetric states are simply deleted by the symmetric projection operators by Werner's UQCM. This leads to a result that the output state is different from the input state. Thus we may find: This UQCM is not universal again for this case, or it is not optimal. In this paper, we will consider this problem. And we will give an optimal UQCM which can copy two identical mixed qubits.

We should note the work in Ref.[26] and recent results about the superbroadcasting of mixed states in Refs.[24, 25] which are closely related with quantum cloning of mixed states. Those results are different from our results in this paper. The main difference between our method and the method in Ref.[26] and in Refs.[24, 25] is the following: In Refs.[26, 24, 25] the input identical mixed qubits is first divided into two groups which can be in tensor product form by mixed states purification. One group is the purified state need be cloned and another group is state which contains no information and will not be cloned. While the method in the present paper is that no matter whether the input state contains information or not, all purified states will be cloned. And in this sense, our result is *universal* since the cloning procession does not depend on the input. We remark that both methods in Refs.[26, 24, 25] and in the present paper are reasonable. They can be used for different purposes.

A 2 to 3 UQCM for mixed states.– A mixed state can be copied by the same cloning transformation as we copy a pure state. Thus the simplest non-trivial cloning task of mixed state is to copy *two* identical mixed states. For this aim, we not only need the cloning transformations for triplet states in symmetric subspace but also need a cloning transformation for the singlet state. We consider the UQCM in the sense that the quality of the copies is independent of the input states. Since we consider arbitrary mixed qubits as input, each output state $\rho_{red.}^{(out)}$ and the input ρ should satisfy the scalar form to satisfy the universal condition[15],

$$\rho_{red.}^{(out)} = f\rho + \frac{1-f}{2}I, \tag{1}$$

where f is the shrinking factor, I is the identity. The relationship between each input and output state is just like the input state goes through a depolarizing channel. We can find that the shrinking factor f can describe the quality of the copies. If $f = 1$, the output state is exactly the input state. If it is zero, the input state is completely destroyed, i.e., the output state is a completely mixed state which contains no information. Our aim is to let the cloning machine achieve the maximal shrinking factor. The optimal shrinking factor has already been obtained in Ref.[15] for identical pure input states. It is obvious that the optimal shrinking factor for identical pure states is also an upper bound for identical mixed states. The problem is whether this bound can be saturated or not for the case of two identical mixed qubits, i.e., can we copy identical mixed qubits as the same quality as we copy identical pure states?

To present our result explicitly, we first give the result for 2 to 3 cloning machine, we have 2 input states and 3 copies which may be entangled. We consider ρ to be an arbitrary mixed state

$$\rho = z_0|\uparrow\rangle\langle\uparrow| + z_1|\uparrow\rangle\langle\downarrow| + z_2|\downarrow\rangle\langle\uparrow| + z_3|\downarrow\rangle\langle\downarrow|, \tag{2}$$

with the restriction that this is a density operator. We also use the notations $\chi_0 = |\uparrow\uparrow\rangle$, $\chi_1 = 1/\sqrt{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, $\chi_2 = |\downarrow\downarrow\rangle$, $\chi_3 = 1/\sqrt{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. We propose the following quantum cloning transformations

$$\begin{aligned}
U\chi_0 \otimes R &= \sqrt{\frac{3}{4}}|3\uparrow\rangle \otimes R_\uparrow + \sqrt{\frac{1}{4}}|2\uparrow,\downarrow\rangle \otimes R_\downarrow, \\
U\chi_1 \otimes R &= \sqrt{\frac{1}{2}}|2\uparrow,\downarrow\rangle \otimes R_\uparrow + \sqrt{\frac{1}{2}}|\uparrow,2\downarrow\rangle \otimes R_\downarrow, \\
U\chi_2 \otimes R &= \sqrt{\frac{1}{4}}|\uparrow,2\downarrow\rangle \otimes R_\uparrow + \sqrt{\frac{3}{4}}|3\downarrow\rangle \otimes R_\downarrow, \\
U\chi_3 \otimes R &= \sqrt{\frac{1}{2}}|\widetilde{2\uparrow,\downarrow}\rangle \otimes R_\uparrow + \sqrt{\frac{1}{2}}|\widetilde{\uparrow,2\downarrow}\rangle \otimes R_\downarrow,
\end{aligned} \tag{3}$$

where Rs in the r.h.s. are ancillary and blank states, $|2\uparrow,\downarrow\rangle = (|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle)/\sqrt{3}$ is a symmetric state with 2 spins up and 1 spin down, similarly for $|\uparrow,2\downarrow\rangle$. The state $|\widetilde{2\uparrow,\downarrow}\rangle = (|\uparrow\uparrow\downarrow\rangle + \omega|\uparrow\downarrow\uparrow\rangle + \omega^2|\downarrow\uparrow\uparrow\rangle)/\sqrt{3}$ is almost the same as the symmetric state $|2\uparrow,\downarrow\rangle$ but with the phase of $\omega = e^{2\pi i/3}$. $R_\uparrow, R_\downarrow$ are ancillary states and are orthogonal to each other. It can be checked easily that the above relations satisfy the unitary condition. We next show that this quantum cloning machine is universal and optimal in the sense the relation (1) is satisfied and the shrinking factor saturates the optimal bound. We expand the input state $\rho\otimes\rho$ in terms of the 4 basis $\chi_i, i = 0, 1, 2, 3$. By using the cloning transformations (3), tracing out the ancillary states $R_\uparrow, R_\downarrow$, we obtain the output state of 3 qubits. This state is a mixed state and may be entangled. What we are interested is the reduced density operator of each output qubit. One can see that each output qubit is the same from the cloning transformation (3). By some calculations (see the appendix for detail), we find the following relation,

$$\rho_{red.}^{(out)} = \frac{5}{6}\rho + \frac{1}{12}I. \tag{4}$$

Really, our cloning transformation (3) is universal and optimal since the shrinking factor $\frac{5}{6}$ is optimal[15]. This is the first non-trivial quantum cloning of identical mixed qubits. We remark that two identical pure qubits can be expanded in the symmetric subspace, so the first three quantum cloning transformations are enough for identical pure states. For general identical mixed states, the cloning transformation for singlet state is necessary.

General 2 to M ($M > 2$) UQCM.– Next, we shall present our general result of 2 to M cloning. The cloning machine creates M copies out of 2 identical mixed qubits. The quantum cloning transformation is presented as follows:

$$\begin{aligned}
U\chi_0 \otimes R &= \sum_{k=0}^{M-2} \alpha_{0k} |(M-k)\uparrow, k\downarrow\rangle \otimes R_k, \\
U\chi_1 \otimes R &= \sum_{k=0}^{M-2} \alpha_{1k} |(M-1-k)\uparrow, (1+k)\downarrow\rangle \otimes R_k, \\
U\chi_2 \otimes R &= \sum_{k=0}^{M-2} \alpha_{2k} |(M-2-k)\uparrow, (2+k)\downarrow\rangle \otimes R_k \\
U\chi_3 \otimes R &= \sum_{k=0}^{M-2} \alpha_{1k} |(M-1-\widetilde{k)\uparrow}, (1+k)\downarrow\rangle \otimes R_k,
\end{aligned} \tag{5}$$

where

$$\alpha_{jk} = \sqrt{\frac{6(M-2)!(M-j-k)!(j+k)!}{(2-j)!(M+1)!(M-2-k)!j!k!}}, \quad j = 0, 1, 2. \tag{6}$$

As previously, the state $|i\uparrow, j\downarrow\rangle$ is a completely symmetrical state with i spins up and j spins down, the state $|i\,\widetilde{\uparrow, j}\downarrow\rangle$ is almost the same as $|i\uparrow, j\downarrow\rangle$, but each term has a different phase of $\begin{pmatrix} i+j \\ i \end{pmatrix}$-th root of unity so that $|i\uparrow, j\downarrow\rangle$ and $|i\,\widetilde{\uparrow, j}\downarrow\rangle$ are orthogonal to each other. R_k are ancillary states and are orthogonal for different k. We can find that this quantum cloning machine is universal and optimal, see appendix for detailed calculations.

$$\rho_{red.}^{(out)} = \frac{M+2}{2M}\rho + \frac{M-2}{4M}I, \tag{7}$$

where the shrinking factor $(M+2)/2M$ achieves the optimal bound[15]. Thus we show that we can copy two identical mixed qubits as the same quality as we copy two identical pure states.

Discussions and summary.–In summary, we present the quantum cloning transformations (5) which can copy arbitrary two identical mixed qubits. This quantum cloning machine is optimal in the sense the shrinking factor between single input and output qubit achieves the upper bound which is the same as for the pure qubit.

The optimal quantum cloning is closely related with quantum state estimation as presented in Ref.[15]. The optimal quantum state estimation are known for identical pure states and the mixed state with support in symmetric subspace. It is not clear how to make a state estimation for identical mixed states which are not restricted to symmetric subspace. In this paper, when $M \to \infty$, the quantum cloning machine is naturally a realization of the quantum state estimation. Since our cloning transformations work for arbitrary identical mixed qubits (including identical pure states and mixed state with support in symmetric subspace), we actually provide a *universal* and *optimal* state estimation for this case.

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Appendix.–First, we denote $A_{ij} = \chi_i \chi_j^\dagger$. The density operator $\rho \otimes \rho$ can be written as,

$$\begin{aligned}
\rho \otimes \rho &= z_0^2 A_{00} + z_1 z_2 \sqrt{2} A_{01} + z_1^2 A_{02} \\
&+ z_1 z_2 \sqrt{2} A_{10} + (z_0 z_3 + z_1 z_2) A_{11} + z_1 z_3 \sqrt{2} A_{12} \\
&+ z_2^2 A_{20} + z_2 z_3 \sqrt{2} A_{21} + z_3^2 A_{22} \\
&+ (z_0 z_3 - z_1 z_2) A_{33}.
\end{aligned} \tag{8}$$

To do quantum cloning for $\rho \times \rho$, we shall add blank and ancillary state, do unitary transformation U as presented in Eqs.(3,5), then trace out the ancillary state. The output state is written as

$$\rho^{(out)} = Tr_{R(k)} U (\rho \times \rho \otimes R) U^\dagger, \tag{9}$$

where $Tr_{R(k)}$ means tracing out the ancillary state. Since the cloning procedure is linear, we then can study the Eq.(8) term by term. We denote the output state of term A_{ij} as ρ_{ij}. Then the output state $\rho^{(out)}$ is in the same form as $\rho \otimes \rho$ in Eq.(8), the only difference is that we should replace A_{ij} by ρ_{ij}. By using the cloning transformation (5), we have

$$\begin{aligned}
\rho_{ij} &= \sum_{k=0}^{M-2} \alpha_{ik} \alpha_{jk}^* \left(|(M-i-k) \uparrow, (i+k) \downarrow\rangle \right. \\
& \left. \langle (M-j-k) \uparrow, (j+k) \downarrow | \right), \\
& \quad i, j = 0, 1, 2 \\
\rho_{33} &= \sum_{k=0}^{M-2} \alpha_{1k} \alpha_{1k}^* \left(|\widetilde{(M-1-k) \uparrow, (1+k) \downarrow}\rangle \right. \\
& \left. \langle \widetilde{(M-1-k) \uparrow, (1+k) \downarrow} | \right).
\end{aligned} \tag{10}$$

Thus by using the UQCM in Eq.(5), we find explicitly the output state $\rho^{(out)}$.

Since we use the shrinking factor f to quantify the quality of the copies, we need to find the reduced density operator of single qubit of the output state $Tr_{M-1} \rho^{(out)}$. That means $M-1$ qubits are traced out from the output state $\rho^{(out)}$ and the single qubit reduced density operator is obtained. We first consider the diagonal elements of the reduced density operator. From the definition of the symmetric state, we know that the state $|(M-i)\uparrow, i\downarrow\rangle$ can be rewritten as the following form,

$$\begin{aligned} |(M-i)\uparrow, i\downarrow\rangle &= \sqrt{\frac{C^i_{M-i}}{C^i_M}}|\uparrow\rangle|(M-i-1)\uparrow, i\downarrow\rangle \\ &+ \sqrt{\frac{C^{i-1}_{M-1}}{C^i_N}}|\downarrow|(M-i)\uparrow,(i-1)\downarrow\rangle. \end{aligned}$$

Since it is a symmetric state, each single qubit reduced density operator is the same. It is written as

$$\begin{aligned} & Tr_{M-1}|(M-i)\uparrow, i\downarrow\rangle\rangle\langle(M-i)\uparrow, i\downarrow| \\ = & \frac{C^i_{M-i}}{C^i_M}|\uparrow\rangle\langle\uparrow| + \frac{C^{i-1}_{M-1}}{C^i_N}|\downarrow\rangle\langle\downarrow| \\ = & \frac{M-i}{M}|\uparrow\rangle\langle\uparrow| + \frac{i}{M}|\downarrow\rangle\langle\downarrow|. \end{aligned} \tag{11}$$

With the help of the results in (6), we know the single qubit reduced density operator of $\rho_{ii}, i = 0, 1, 2$ is

$$\begin{aligned} Tr_{M-1}\rho_{ii} = \sum_{k=0}^{M-2} |\alpha_{ik}|^2 & \left(\frac{M-i-k}{M}|\uparrow\rangle\langle\uparrow| \right. \\ & \left. + \frac{i+k}{M}|\downarrow\rangle\langle\downarrow| \right) \\ = \sum_{k=0}^{M-2} & \frac{6(M-2)!}{(2-i)!i!(M+1)!}\frac{(M-i-k)!(i+k)!}{(M-2-k)!k!} \times \\ & \times \left(\frac{M-i-k}{M}|\uparrow\rangle\langle\uparrow| + \frac{i+k}{M}|\downarrow\rangle\langle\downarrow| \right). \end{aligned} \tag{12}$$

Explicitly, we have the following results:

$$\begin{aligned} Tr_{M-1}\rho_{00} &= \frac{3M+2}{4M}|\uparrow\rangle\langle\uparrow| + \frac{M-2}{4M}|\downarrow\rangle\langle\downarrow|, \\ Tr_{M-1}\rho_{11} &= \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|), \\ Tr_{M-1}\rho_{22} &= \frac{M-2}{4M}|\uparrow\rangle\langle\uparrow| + \frac{3+2M}{4M}|\downarrow\rangle\langle\downarrow|. \end{aligned} \tag{13}$$

The calculations for case ρ_{33} are different from the case ρ_{11} since we have phases for each term in state $|\widetilde{(M-1-k)}\uparrow, (1+k)\downarrow\rangle$. But by careful analyzing, we find that these phases do not change the single qubit reduced density operator, and we have

$$Tr_{M-1}\rho_{33} = Tr_{M-1}\rho_{11} = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|). \tag{14}$$

Finally, let's study the off-diagonal elements of the reduced density operator of $\rho^{(out)}$. We have the following results:

$$\begin{aligned} Tr_{M-1}\rho_{ii+1} = \sum_{k=0}^{M-2} \alpha_{ik}\alpha^*_{i+1k} Tr_{M-1}|(M-i-k)\uparrow, \\ (i+k)\downarrow\rangle\langle(M-i-1-k)\uparrow,(i+1+k)\downarrow| \\ = \sum_{k=0}^{M-2} \alpha_{ik}\alpha^*_{i+1k}\frac{\sqrt{(M-i-k)(i+k+1)}}{M}|\uparrow\rangle\langle\downarrow| \\ = \frac{6}{M^2(M^2-1)}\frac{\sqrt{(2-i)(1+i)}}{(2-i)!(1+i)!}\times \\ \times\sum_{k=0}^{M-2}\frac{(M-i-k)!(i+k+1)!}{k!(M-2-i)!}|\uparrow\rangle\langle\downarrow|. \end{aligned} \quad (15)$$

For cases $i=0,1$, we have

$$Tr_{M-1}\rho_{01} = Tr_{M-1}\rho_{12} = \frac{\sqrt{2}(M+2)}{4M}|\uparrow\rangle\langle\downarrow|. \quad (16)$$

Similarly, we find

$$Tr_{M-1}\rho_{10} = Tr_{M-1}\rho_{21} = \frac{\sqrt{2}(M+2)}{4M}|\downarrow\rangle\langle\uparrow|. \quad (17)$$

Summarize all of these results together, we have

$$\rho^{(out)}_{red.} = Tr_{M-1}\rho^{(out)} = \frac{M+2}{2M}\rho + \frac{M-2}{4M}I. \quad (18)$$

This is the result presented in Eq.(7).

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