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ENTANGLEMENT AND NONLOCALITY FOR A MIXTURE OF PAIR-COHERENT STATES

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We consider a bipartite continuous variables quantum mixture coming from phase randomization of a pair-coherent state. We study the nonclassical properties of such a mixture. In particular, we quantify its degree of entanglement, then we show possible violations of Bell's inequalities. We also consider the use of this mixture in quantum teleportation. Finally, we compare this mixture with that obtained from a pair-coherent state by single photon loss.

Keywords: entanglement, nonlocality, quantum information processing

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1. Introduction

Entanglement and nonlocality are nowadays considered as fundamental resources for the quantum information processing [1]. The efficiency of many protocols significantly depends on the degree of entanglement of a state representing the channel shared by two parties [2]. A paradigmatic example of such dependence is provided by the Werner mixture [3]. Beside discrete variable systems, a great attention has been also devoted to the quantum information processing with continuous variables [4]. In such a context the two-mode squeezed vacuum state [5] is mostly exploited since it provides the maximally entangled state in the limit of very large squeezing. However, there exist other candidate states for quantum information processing with continuous variables which are worth studying. Here we introduce a mixture of pair-coherent state and we study its basic properties, i.e., entanglement, nonlocality and teleportation.

Pair-coherent states provide an interesting example of nonclassical (also non Gaussian) states of the two-mode radiation field. They were introduced in Ref.[6], and their properties were extensively studied [7]. However, the practical generation of such states is rather difficult and several models have been explored [8, 9] including trapped systems [10]. But their experimental signature as pure states still remain questionable [11]. Nevertheless, from a foundational point of view, these states play an important role for testing quantum mechanics versus local realism [12].

A pair-coherent state is given by

$$|\Psi\rangle\rangle = \mathcal{N}^{1/2} \int_0^{2\pi} \frac{d\theta}{2\pi} |\zeta e^{i\theta}\rangle_A |\zeta e^{-i\theta}\rangle_B e^{\zeta^2} , \tag{1}$$

where $\mathcal{N} = [I_0(2\zeta^2)]^{-1}$ with I_n the modified Bessel function of order n, and $\zeta \in \mathbb{R}$.The subscripts A, B denote the two parties. The quantum state (1) is potentially generated, from vacuum fields, in the steady state by nondegenerate parametric oscillation [8] as modeled by the following Hamiltonian, in which coupled signal-idler loss dominates over linear single-photon loss,

$$H = i\hbar\mathcal{E}(a^\dagger b^\dagger - ab) + ab\Gamma^\dagger + a^\dagger b^\dagger \Gamma . \tag{2}$$

The $a^\dagger$, a and $b^\dagger$, b are the usual boson creation and destruction operators for two spatially separated systems (field modes) at location A and B respectively. Often, the a and b are referred to as the signal and idler fields, respectively. Furthermore, $\mathcal{E}$ represents a coherent driving source which generates signal-idler pairs, while Γ represents system's reservoir which gives rise to the coupled signal-idler loss. The Hamiltonian preserves the signal-idler photon number difference $a^\dagger a - b^\dagger b$ of which the state (1) is an eigenstate with eigenvalue zero. We note the analogy to the single-mode even and odd coherent superposition states [13], which are generated by the degenerate form of Hamiltonian (2).

The state (1) can also be rewritten as

$$|\Psi\rangle\rangle = \mathcal{N}^{1/2} \sum_{n=0}^{\infty} \frac{\zeta^{2n}}{n!} |n\rangle_A |n\rangle_B , \tag{3}$$

and represents two coherent states having a well defined phase relation although their phase is random. However, this phase relation is undermined by phase diffusion induced, e.g, by the pump $\mathcal{E}$ or by the reservoir Γ [14]. We model this process by introducing phase damping in each mode through the following master equation [5]

$$\dot{\rho} = - \left[a^\dagger a, \left[a^\dagger a, \rho\right]\right] - \left[b^\dagger b, \left[b^\dagger b, \rho\right]\right] , \tag{4}$$

where we have assumed a symmetric, unity, decay rate. The solution of Eq.(4), with the initial condition $\rho_{pair} = |\Psi\rangle\rangle\langle\langle\Psi|$, is

$$\rho(\tau) = \mathcal{N} \sum_{n,m=0}^{\infty} \frac{\zeta^{2n+2m}}{n!m!} \exp\left[-2\tau(n-m)^2\right] |n\rangle_A\langle m| \otimes |n\rangle_B\langle m| , \tag{5}$$

where τ represents the dimensionless time. In the case of $\tau \to \infty$, Eq.(5) reduces to

$$\rho_{rand} = \mathcal{N} \sum_{n=0}^{\infty} \frac{\zeta^{4n}}{(n!)^2} |n\rangle_A\langle n| \otimes |n\rangle_B\langle n| , \tag{6}$$

which shows no phase relation between the two modes.

We are now going to study in detail the quantum channel given by Eq.(5).

2. Mapping on two-qubit system

A simple way to study the nonlocal properties of a bipartite state is to map it into two-qubit system [15], for which separability and nonlocality conditions are known [16, 17]. To this end, we introduce the local spin-$\frac{1}{2}$ operators

$$S_1^{(\alpha)} = \sum_{n=0}^{\infty} \left(|2n\rangle_\alpha\langle 2n+1| + |2n+1\rangle_\alpha\langle 2n|\right) , \tag{7}$$

$$S_2^{(\alpha)} = -i\sum_{n=0}^{\infty} \left(|2n\rangle_\alpha\langle 2n+1| - |2n+1\rangle_\alpha\langle 2n|\right) , \tag{8}$$

$$S_3^{(\alpha)} = \sum_{n=0}^{\infty} (-)^n |n\rangle_\alpha\langle n| , \tag{9}$$

where $\alpha = A, B$ and $|n\rangle_\alpha$ are Fock states. The operators (7)-(9) obey the commutation relation

$$\left[S_i^{(\alpha)}, S_j^{(\beta)}\right] = 2i\delta_{\alpha,\beta}\varepsilon_{ijk}S_k^{(\alpha)} , \tag{10}$$

with ε_{ijk} the totally antisymmetric tensor.

Now, given a state ρ of the bipartite system, we write it through the Hilbert-Schmidt decomposition

$$\rho = \frac{1}{4}\left[I^{(A)} \otimes I^{(B)} + \mathbf{v}^{(A)}\!\cdot\!\mathbf{S}^{(A)} \otimes I^{(B)} + I^{(A)} \otimes \mathbf{v}^{(B)}\!\cdot\!\mathbf{S}^{(B)} + \sum_{n,m=1}^{3} t_{n,m} S_n^{(A)} \otimes S_m^{(B)}\right] , \tag{11}$$

where $\mathbf{v}^{(\alpha)} \equiv (v_1^{(\alpha)}, v_2^{(\alpha)}, v_3^{(\alpha)})$ are vectors in $\mathbb{R}^3$, $v_j^{(\alpha)} = \mathrm{Tr}[\rho S_j^{(\alpha)}]$, while $\mathbf{S}^{(\alpha)} \equiv (S_1^{(\alpha)}, S_2^{(\alpha)}, S_3^{(\alpha)})$ are operator vectors, and $I^{(\alpha)}$ are identity operators. Furthermore, $\mathbf{v}^{(\alpha)}\!\cdot\!\mathbf{S}^{(\alpha)} = \sum_{i=1}^{3} v_i^{(\alpha)} S_i^{(\alpha)}$, and the coefficients $t_{n,m} = \mathrm{Tr}[\rho\, S_n^{(A)} \otimes S_m^{(B)}]$ form the real 3×3 matrix T describing the correlations between the two pseudo-qubits.

For the state (5) a straightforward algebra gives

$$v_1^{(A)} = v_1^{(B)} = v_2^{(A)} = v_2^{(B)} = 0 , \tag{12}$$

$$v_3^{(A)} = v_3^{(B)} = \mathcal{N} I_0(2i\zeta^2) , \tag{13}$$

and

$$t_{11} = 2\mathcal{N}\sum_{n=0}^{\infty} \frac{\zeta^{8n+2}}{(2n)!(2n+1)!} e^{-2\tau} , \tag{14}$$

$$t_{22} = -t_{11} , \tag{15}$$

$$t_{33} = 1 , \tag{16}$$

$$t_{ij} = 0 \quad i \neq j . \tag{17}$$

As a consequence of the above equations, the state (5), when mapped into two-qubit, corresponds to the following mixture

$$\rho = p\rho_{pair} + (1-p)\rho_{rand} , \tag{18}$$

where $0 \leq p = e^{-2\tau} \leq 1$. Thus, it describes a real physical channel where correlated pair-coherent states are available with probability p, while the phase relation is disrupted by environmental effects, with probability $1-p$.

3. Entanglement

Let us now quantitatively study the entanglement of the state (18). Several measure of the entanglement have recently been proposed (see, e.g., [18] and references therein), and none of them can be considered as the unique, canonical measure. However, the entanglement of formation [19] plays an important role since it gives the minimal amount of entanglement necessary to create a given density matrix. Furthermore, there exist an analytical expression of this quantity for two-qubit systems [20]. Namely, for a 4×4 density matrix ρ one can defines the flipped state $\tilde{\rho} = O\rho^* O^T$, where ρ^* denotes the complex conjugation, and the orthogonal flipping matrix O contains only four nonzero elements along the antidiagonal: $O_{14} = O_{41} = -O_{23} = -O_{32} = 1$. As a consequence the concurrence [20] results

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \,, \tag{19}$$

where λ_i's are, in decreasing order, the nonnegative square roots of the moduli of the eigenvalues of the non-hermitian matrix $\rho\tilde{\rho}$. Finally, the concurrence determines the entanglement of formation as

$$E(\rho) = h\left(\frac{1}{2}\left[1 + \sqrt{1 - C^2(\rho)}\right]\right) \,, \tag{20}$$

where

$$h(x) = -x \log_2 x - (1-x) \log_2 (1-x) \,, \tag{21}$$

is the Shannon entropy of the two-element partition $\{x, 1-x\}$.

Then, Eqs.(12)-(17) lead to the following λ_i's

$$\lambda_1 = \frac{1}{2}\sqrt{1 - \left[v_3^{(A)}\right]^2} + \frac{1}{2} t_{11} \,, \tag{22}$$

$$\lambda_2 = \frac{1}{2}\sqrt{1 - \left[v_3^{(A)}\right]^2} - \frac{1}{2} t_{11} \,, \tag{23}$$

$$\lambda_3 = \lambda_4 = 0 \,, \tag{24}$$

so that the concurrence results

$$C(\rho) = t_{11} \,. \tag{25}$$

Inserting Eq.(25) into Eq.(20) allows us to evaluate the entanglement of formation.

This quantity is shown in Fig.1 as function of ζ and p. One can see that the state (18) for $p \to 1$ and $\zeta > 2$ reaches the maximum entanglement, while it results separable approximately for $p < 0.1$, or $\zeta \to 0$. It is also worth noting the presence of slight oscillations as function of ζ due to the Bessel function.

4. Nonlocality

As a consequence of the commutation relation (10) the nonlocality of the state (18) can be studied by means of the standard two-qubit Bell inequalities [21]. In particular, the CHSH

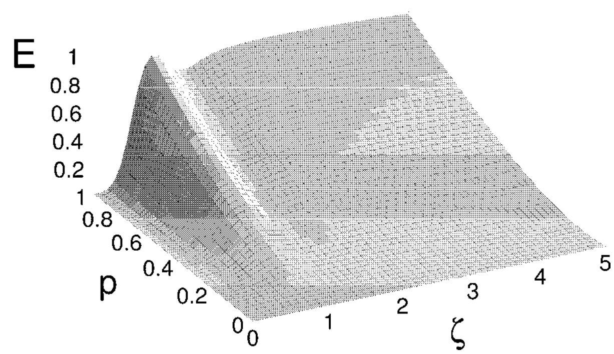

Fig. 1. Degree of entanglement E versus ζ and p.

inequality [22] reads

$$B \leq 2\,, \tag{26}$$

with the Bell factor given by

$$\begin{aligned} B \;=\; & \Big|\Big\langle \left(\mathbf{a}\cdot\mathbf{S}^{(A)}\right)\left(\mathbf{b}\cdot\mathbf{S}^{(B)}\right)\Big\rangle + \Big\langle \left(\mathbf{a}'\cdot\mathbf{S}^{(A)}\right)\left(\mathbf{b}\cdot\mathbf{S}^{(B)}\right)\Big\rangle \\ & + \Big\langle \left(\mathbf{a}\cdot\mathbf{S}^{(A)}\right)\left(\mathbf{b}'\cdot\mathbf{S}^{(B)}\right)\Big\rangle - \Big\langle \left(\mathbf{a}'\cdot\mathbf{S}^{(A)}\right)\left(\mathbf{b}'\cdot\mathbf{S}^{(B)}\right)\Big\rangle\Big|\,, \end{aligned} \tag{27}$$

where $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$ are unit vectors in $\mathbb{R}^3$, and the angle brackets denote the averaging over the density matrix (18).

According to Ref. [17], if the sum of the two largest eigenvalues of the matrix $U = T^T T$ is greater than unity, the state (18) violates the inequality (26) for some choices of the vectors $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$. Thus, from Eqs.(14)-(17), the maximal Bell factor results

$$B_{max} = 2\sqrt{t_{11}^2 + t_{33}^2}\,. \tag{28}$$

The quantity (28) is shown in Fig.2. Contrarely to what it is expected, this plot resembles that of Fig.1, that is the violation of the Bell inequality occurs for almost all values of entanglement. This is due to the particular choice of the operators (7)-(9). Furthermore, the violation becomes very large, attaining the maximum value of $2\sqrt{2}$ when $p \to 1$, as opposite to Refs.[12] where, for $p = 1$, measurements on quadrature observables lead to small violations only for $\zeta \approx 1$.

It is worth noting that, although the operators (7)-(9) employed in Eq.(27) are not realistic observables, they could be measured through tomographic techniques [23]. For instance, in the two-mode optical homodyne tomography [24], a complete set of quadrature measurements should be performed. Then, the collected data should be used to reconstruct the joint density operator from which calculate the averages (27). This approach is completely different from those of Ref.[12], where some quadrature distributions were directly involved in the Bell's like inequalities.

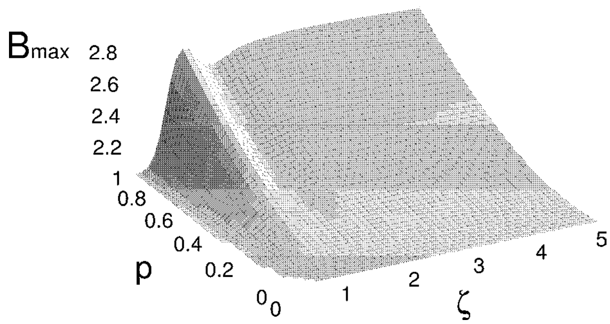

Fig. 2. Maximum attainable value of the Bell operator B_{max} versus ζ and p (only the part exceeding the classical bound $B_{max} = 2$ is shown).

5. Teleportation

The state (18) could be used in the standard quantum teleportation protocol for continuous variable [25]. It consists in measuring the difference $x_- = x_{in} - x_A$ and the sum $y_+ = y_{in} + y_A$ of the orthogonal quadrature components of the input mode in and the reference mode A at Alice's station. Then, the resulting state for the mode B at Bob's site is conditioned by the input state $|\psi\rangle_{in}$ and the measurement result $\alpha = x_- + iy_+$, which ideally define an eigenstate $|\alpha\rangle\rangle_{in,A}$ of modes in and A, that is

$$|\alpha\rangle\rangle_{in,A} = \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} D_{in}(\alpha)|n\rangle_{in}|n\rangle_A \,, \tag{29}$$

where D denotes the displacement operator [5].

To evaluate the fidelity of the teleportation protocol, we consider a coherent input state $|\beta\rangle_{in}$, since in this case an upper classical bound has already been established [26]. Then, the state after the Alice's measurement will be

$$\rho_{after} = \frac{1}{P(\alpha)} \; {}_{A,in}\langle\langle\alpha| \Big(\rho \otimes |\beta\rangle_{in}\langle\beta|\Big) |\alpha\rangle\rangle_{in,A} \,, \tag{30}$$

where the probability $P(\alpha)$ of the outcome α is given by

$$P(\alpha) \;=\; \frac{\mathcal{N}}{\pi} e^{-|\alpha-\beta|^2} \sum_{n=0}^{\infty} \frac{(\zeta^2|\alpha-\beta|)^{2n}}{(n!)^3} \,. \tag{31}$$

The final output state is obtained by applying the local transformation $D_B(\alpha)$ at Bob's site after receiving the classical communication (the two values corresponding to the real and imaginary part of α) from Alice. Thus,

$$\rho_{out} = D_B(\alpha)\rho_{after}D_B^{\dagger}(\alpha) \,, \tag{32}$$

Finally, the fidelity conditioned to the value α would be

$$f(\alpha) = \mathrm{Tr}\left[\rho_{out}\rho_{in}\right] = \langle\beta|\rho_{out}|\beta\rangle \,, \tag{33}$$

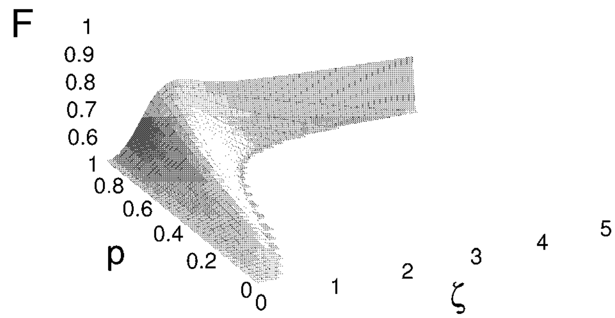

Fig. 3. Average fidelity F versus ζ and p (only the part exceeding the classical bound $F = 0.5$ is shown).

and the average fidelity

$$F = \int d^2\alpha f(\alpha) P(\alpha) \,. \tag{34}$$

Explicitely it results

$$F = \frac{1}{2}\mathcal{N} \sum_{n,m=0}^{\infty} \frac{(n+m)!}{(n!)^2(m!)^2} (\zeta/\sqrt{2})^{(2n+2m)} p^{(n-m)^2} \,. \tag{35}$$

Notice that by integrating overall possible values of α the dependence on the amplitude of the coherent input state disappears.

The quantity (35) is shown in Fig.3. We note that it never reaches the maximum, even when the used quantum channel is maximally entangled. This is due to the fact that the protocol of Ref.[25] has been devised for two-mode squeezed state. Nevertheless, suitable local operations could optimize the protocol also in such a case [27]. Most important is the fact that for $\zeta > 1$ it is almost impossible to have the fidelity above the classical bound while violations of Bell inequality still occur (see Figs.2 and 3). This is in apparent contrast with the behavior of the Werner mixture [3, 28]. However, it could be explained by observing Eqs.(35) and (14) where p appears with different powers. That is, when reducing to two-qubit, the effect of the dephasing is indipendent of the amplitude ζ. Instead, if one tries to exploit the entanglement resource with continuous variable protocols, i.e. by using the whole Hilbert space, the effect of the amplitude ζ can be detrimental.

6. Conclusions

In this paper we have introduced a bipartite mixture of phase randomized pair-coherent state. This mixture could describe realistic quantum channel for information processing. Then we have studied its nonclassical properties, like entanglement and nonlocality, on a two-qubit scale. We have found that it could reach the maximal entanglement and the maximal violation of Bell inequality. Notwithstanding, we have shown that these features do not always guarantee reliable quantum information processing, like teleportation, on infinite dimensional scale (continuous variable).

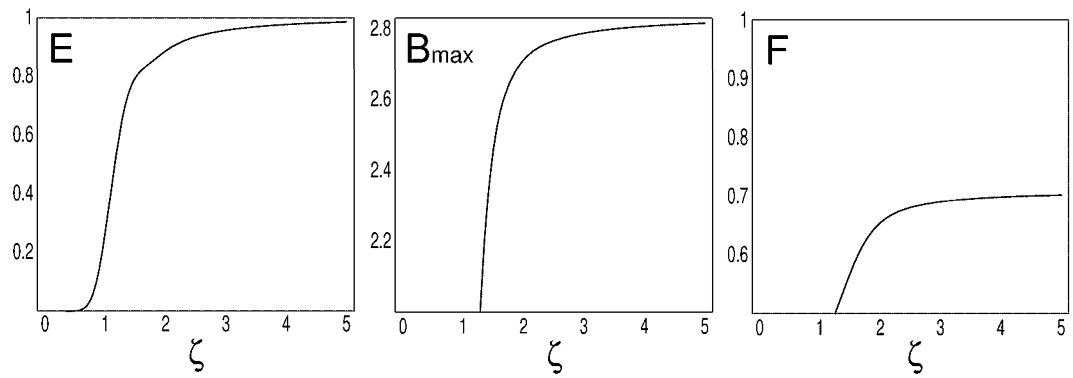

Fig. 4. From left to right, degree of entanglement E, maximum value of the Bell operator B_{max}, and teleportation fidelity F versus ζ.

When trying to use pair coherent states as a realistic channels, another undesirable effect which may occur is the single photon loss [14]. If the channel is symmetric in the two modes, this process can be described by the use of the superoperators [29, 5]

$$\mathcal{J} = a\rho a^\dagger + b\rho b^\dagger \tag{36}$$

$$\mathcal{L} = -\frac{1}{2}\left(a^\dagger a\rho + \rho a^\dagger a\right) - \frac{1}{2}\left(b^\dagger b\rho + \rho b^\dagger b\right) \,, \tag{37}$$

which lead to the following conditional state after the environment has witnessed a photon lost by the system at time $\tau' < \tau$

$$\rho_c(\tau) = 2\mathcal{J} e^{2\mathcal{L}\tau'} \rho(0) \,, \tag{38}$$

with $\rho(0) = \rho_{pair}$. Also in this case we have assumed a unity decay rate, so that τ represents the dimensionless time. The unconditional (normalized) density operator corresponding to Eq.(38) would be

$$\rho(\tau) = \exp\left[2\mathcal{L}\tau\right]\rho(0) + \int_0^\tau d\tau'\, 2\mathcal{J}\exp\left[2\mathcal{L}\tau'\right]\rho(0) \,. \tag{39}$$

Then, in the limit of $\tau \to \infty$, Eq.(39) becomes

$$\rho = \mathcal{N}\Bigg[|0\rangle_A\langle 0| \otimes |0\rangle_B\langle 0| + \sum_{n,m=0}^{\infty} \frac{\zeta^{2(n+m+2)}}{(n+1)!(m+1)!} \frac{\sqrt{(n+1)(m+1)}}{(n+m+2)} \times\Big(|n\rangle_A\langle m| \otimes |n+1\rangle_B\langle m+1| + |n+1\rangle_A\langle m+1| \otimes |n\rangle_B\langle m|\Big)\Bigg] \,. \tag{40}$$

By repeating the procedure of Secs.III, IV and V with the state (40), one can see that in this case the characteristics of the channel remain almost unaltered provided $\zeta > 1$. This is shown in Fig.4.

In conclusion, since the realization of an almost perfect quantum channel based on pair coherent states seems not at the hand, we have provided a study of the tolerance against some subtle noisy effects.

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