CREATING HIGH DIMENSIONAL ENTANGLEMENT
USING MODE-LOCKED LASERS

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We present a new scheme to generate high dimensional entanglement between two photonic systems. The idea is based on parametric down conversion with a sequence of pump pulses generated by a mode-locked laser. We prove experimentally the feasibility of this scheme by performing a Franson-type Bell test using a 2-way interferometer with path-length difference equal to the distance between 2 pump pulses. With this experiment, we can demonstrate entanglement for a two-photon state of at least dimension d=11. Finally, we propose a feasible experiment to show a Fabry-Perot like effect for a high dimensional two-photon state.

Keywords: entangled qudits, parametric down conversion

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1. Introduction

Photonic entanglement is one of the most important tools for quantum communication experiments [1]. Different types of entanglement can be used: for instance the well known polarization entanglement, momentum entanglement or entanglement between photons created in a coherent superposition of emission times, the so-called time-bin entanglement [2]. To date, most of the experiments involve entangled 2-level systems (qubits). However, entanglement in high-dimensional Hilbert spaces has received a lot of attention recently. The study of such states is interesting from a fundamental point of view, because they lead to quantum predictions that differ from classical physics more radically than 2 level-systems and are more resistant to noise[3, 4]. It has also recently been shown that increasing the dimension of the Hilbert space would allow a decrease in the detection efficiency required to close the detection loophole for EPR experiments [5]. Moreover, high dimensional states can be used to increase the noise tolerance of quantum key distribution schemes [6]. Finally, new quantum protocols involving qutrits (d=3) have been recently proposed [7]. One way to reach experimentally larger Hilbert spaces is to use higher order parametric down conversion to generate multiqubit entanglement [8, 9, 10]. In this case, however, the increase of the Hilbert space dimension is accompanied by an increase of the number of particles involved. A more direct way is to use multilevel systems. Entanglement of angular momentum states of photons has been recently suggested and demonstrated in this context [11, 12].

Here we propose a new scheme, allowing in principle to reach entanglement in arbitrarily high

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dimensional Hilbert spaces that is based on time-bin entanglement. One of the advantages of this type of entanglement is that it can be straight-forwardly extended to higher dimension Hilbert spaces by increasing the numbers of possible times of creation. In our previous experiments, time-bin entangled qubits were created by inserting a 2-way unbalanced interferometer between the pulsed laser and the down-converter [2, 13]. Photons were then sent to analyzers, which were equally unbalanced 2-way interferometers. By inserting a d-way interferometer before the downconverter, we could in principle obtain time-bin entangled qudits. In this paper, we present a natural extension of this scheme, in which the possible times of creation of the photon pairs are directly given by the successive pump laser pulses. This allows the creation of quantum states in arbitrarily high dimension Hilbert spaces. A necessary condition is that the different pulses must be coherent with each others, i.e exhibit a constant phase shift. This coherence can be obtained by using a mode-locked pulsed laser. To prove the feasibility of this scheme, we make a first experiment analyzing the created high-dimensional entangled states with a 2-way interferometer, with path-length difference equal to the distance between two laser pulses. The paper is organized as follow: first we explain the principle of high-dimensional time-bin entanglement; then, we describe the experiment and finally we propose a feasible experiment to show Fabry-Perot like behavior for a high-dimensional two-photon state.

2. Principle of high-dimensional time-bin entanglement generated by a sequence of coherent pulses

We present here an intuitive approach of the principle of high dimensional time-bin entanglement generated by a sequence of laser pulses. A parametric down converter (PDC) is pumped by a sequence of pump laser pulses with a fixed phase difference. This sequence can be generated by a mode-locked laser, or, for instance by a single-shot laser followed by a loop made of a beam splitter where one output is connected to one input[†]. Let us assume that we have a train of d pump pulses. Eventually a pump photon creates a photon pair by spontaneous parametric down conversion. Let us label the successive laser pulses by N=1,2,...,j,...d with N=1 at t=0. The time difference between two pulses is Δt. With this notation, the pulse j arrives before the pulse j+1 at the down-converter. We suppose that the probability of creating more than one pair in d pulses is negligible. If a photon pair is created in pulse, or time-bin, j we write the state:

$$a_j^\dagger b_j^\dagger \left|0_a, 0_b\right\rangle = \left|0_1, 0_2, ..., 1_j, .., 0_d\right\rangle_a$$
$$\otimes \left|0_1, 0_2, ..., 1_j, .., 0_D\right\rangle_b \equiv \left|j_a, j_b\right\rangle \equiv \left|j, j\right\rangle \tag{1}$$

where $a_j^\dagger$ and $b_j^\dagger$ are the creation operators for the two PDC modes in the time-bin j (in our case, the two modes corresponds to different wavelengths, 1310nm and 1550 nm). In the following, we will use the notation $\left|j, j\right\rangle$ for a photon pair created in time-bin j. As we cannot know in which time bin the photon pair is created, we have after the PDC the d-dimensional

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[†]In this case however, the pump intensity would decrease constantly

entangled state:

$$|\Psi_{PDC}\rangle = \sum_{j=1}^{d} c_j e^{i\phi_j} |j, j\rangle \tag{2}$$

where ϕ_j is the phase difference between two pump pulses and $\sum_{j=1}^{d} c_j^2 = 1$. It must be stressed that this scheme enables in principle the creation of any d-dimensional state of the form (2). Indeed, by inserting an amplitude and/or phase modulator before the down-converter, we could control the desired number of pump pulses (and thus choose the dimension d) and modulate their amplitude and phase (and thus setting the coefficients c_j and ϕ_j in order to generate non-maximally entangled states). In our experiment however, we will consider c_j and ϕ_j as constant.

The creation of high-dimensional time-bin entangled states is relatively straight-forward. A more difficult task is to analyze the created states in order to show high-dimensional entanglement. The more natural way to do it would be to use d-way interferometers. However, building such devices would lead to severe practical difficulties as d increases. Consequently, we decided to use a two-way interferometer as analyzer. Nevertheless, it turns out that this experimental configuration allows us to demonstrate high-dimensional entanglement, as we will explain in the following. The long arm of the two-way interferometer we used introduces a delay Δt with respect to the short one (see fig. 1). This means that a photon travelling through the short arm will remain in the same time-bin while a photon travelling through the long arm will move to the next time-bin. For a photon pair in the time-bin j, the effect of the interferometer can be written as follow :

$$|j, j\rangle \rightarrow |j, j\rangle + e^{2i\delta} |j+1, j+1\rangle$$
$$+ e^{i\delta} |j, j+1\rangle + e^{i\delta} |j+1, j\rangle \tag{3}$$

where δ is the phase shift acquired in the long arm of the interferometer. If two photons are in the same time-bin after the interferometer, they will be detected with a time difference $\tau = 0$, while if they are in different time bins, they will be detected with a time difference $\tau = \Delta t$. In the following, we will consider only the terms leading to a coincidence with $\tau = 0$, because only those terms lead to indistinguishable processes. In practice, this is achieved by selecting a sufficiently small coincidence window. After the interferometer, a state of the form (2) becomes:

$$|\Psi_{PDC}\rangle \rightarrow |\Psi_{int}\rangle = |1, 1\rangle + \sum_{j=2}^{d} |j, j\rangle \left(1 + e^{2i\delta}\right)$$
$$+ e^{2i\delta} |d+1, d+1\rangle \tag{4}$$

We see that for all time-bins except the first and the last one we have a superposition of two indistinguishable processes. In principle, the first and the last time bin (i.e photons created in the first pulse travelling through the short arm and photons created in the last pulse travelling through the long arm) can be discarded by using switches. In this case, the coincidence count rate is given by:

$$R_c \sim 1 + V \cos(2\delta) \tag{5}$$

where V is the visibility of the interference pattern given by known experimental imperfections. The maximum visibility in this case is 100%. In practice, if we don't discard the first and the last time-bins, the maximal visibility will be reduced. We can see from eq. 4 that we have 2d different processes that lead to a coincidence with $\tau = 0$. Among these 2d processes, there are always two (the first and the last time-bins) which are completely distinguishable. Hence the maximum visibility is given by:

$$V = \frac{d-1}{d} \tag{6}$$

Consequently, by measuring a given visibility in this experiment, we can prove that *we generate entanglement for a two-photon state of at least dimension* $d = \frac{1}{1-V}$. In our experiment however, we don't have trains of exactly d pulses. Actually, as we pump the PDC directly with a pulsed laser, we have an (almost) infinite number of pulses. Here the dimension d is bounded by the probability of creating one photon pair in d pulses and by the stability of the pump laser cavity.

Another interesting point with this experimental configuration is that the number of possible outcomes is higher than the dimension of the space. We have indeed d+1 time-bins, and for each time-bin we have two possible outcomes (a given photon can take one or the other output of the interferometer). We have thus $2(d + 1) > d$ possible results. This constitutes a generalized quantum measurement, a so-called positive operator-valued measure (POVM) [14].
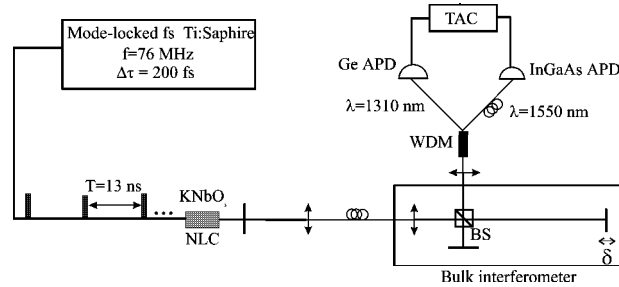

Fig. 1. Experimental setup.

3. Experiment

The experimental setup is shown in figure 1. A Ti-Sapphire femtosecond laser(Mira pumped by a 8W Verdi) at λ_p =710 nm is used to pump a KNbO$_3$ non linear crystal with type-one phase matching. The laser pulses are separated by 13 ns and their duration is approximately 200 fs. The crystal is cut in order to emit non-degenerate photon pairs at telecom wavelength (1310/1550 nm). After the crystal, a filter RG1000 is used to stop the pump beam. The down converted photons are focused into a single mode optical fiber and directed towards a bulk Michelson interferometer consisting of a 50/50 beam splitter (BS) and of 2 mirrors

M_1 and M_2. The physical path-length difference between the two arms corresponds to half the period of the laser, i.e 6.55 ns $\longleftrightarrow$ 1.95 m. It thus introduces a suitable delay so that photons produced by two successive laser pulses interfere at the beam splitter[a] The phase in one arm of the interferometer can be varied with a piezoelectric crystal controlling the position of the mirror. The losses in the interferometer are 12 dB for each wavelength. The output of the interferometer is focused into a single mode optical fiber and the photon are separated deterministically by a wavelength division multiplexer (WDM). The 1310 nm photon is detected with liquid nitrogen cooled Ge avalanche photodiodes (APD) operating in photon counting mode, i.e. reverse biased above the breakdown voltage (the so-called Geiger mode) [16]. The quantum efficiency of such detectors is about 10% for 30 kHz dark counts. Ge APD's are not sensitive at 1550 nm, thus InGaAs APD's are used to detect the 1550 nm photons. In order to remain at a reasonably low dark count level, this kind of APD must be operated in the so called gated mode, i.e active only during a short time window when a photon is expected. We use Peltier cooled Epitaxx APD's, with around 20% quantum efficiency, for a noise probability of $5*10^{-5}$ per nanosecond [17]. In our case, the InGaAs APD is activated by a clic on the Ge APD. Therefore, a suitable optical delay must be inserted before the InGaAs APD, in order to detect the two photons in coincidence. The signals of the two APD's are finally sent to a Time-to-Amplitude Converter (TAC), in order to record the time histogram of the arrival times of the photons. A coincidence window of about 1 ns is selected with a single channel analyzer (SCA) around the desired peak. The coincidence rate is finally recorded as a function of the phase shift δ in the interferometer. Figure 2 shows the histogram of the two photon's arrival time difference, as recorded with the TAC. The central peak corresponds to photons detected in the same time-bin ($\tau = 0$) while the satellite peaks correspond to photons detected in different time-bins. The coincidence window is centered around the interfering peak $\tau = 0$. Figure 3 shows the interference curve, i.e the coincidence rate as a function of the phase shift in the interferometer, for an average pump power of 8mW. The pump power is kept very low, in order to avoid the creation of multiphotons states, that would reduce the visibility [18].

We observe the predicted sinusoidal behavior, and a sinusoidal fit gives a net visibility of 91 ±6%. The rather high level of noise in the coincidence count rates is due to the dark counts in the detectors. From this measured visibility, we can infer, using eq.6, that *we generated entanglement for a two-photon state of at least dimension d=11.*

4. Two photon Fabry-Perot interferometer

High-dimensional time-bin entanglement can lead to interesting effects. As an example, we propose a feasible experiment that shows a Fabry-Perot like effect for two photon states. This effect is a direct manifestation of high-dimensional entanglement. The idea is to use fiber loops as analyzers, as shown in fig 4.

In this case, down converted photons are separated directly after the down converter and each one is sent to a fiber loop introducing an optical delay Δt equal to the distance between pump pulses, before going to the detector. This fiber loop is made of a fiber coupler where

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[a]Although the present work is a very natural extension of our previous experiments, it is conceptually similar to the interesting idea of Burlakov et al [15], where they use a cw multimode pump and make interference with photon pairs created in two successive longitudinal modes of the laser cavity.
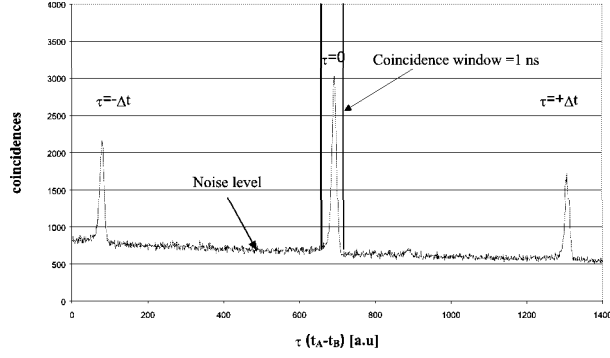
Fig. 2. Histogram of the two photons arrival time difference, as recorded with the TAC. The coincidence window is centered around the $\tau = 0$ peak. The time between two side peaks is given by the period of the laser (t=13 ns). The decreasing of the noise rate is due to afterpulses effects [17].
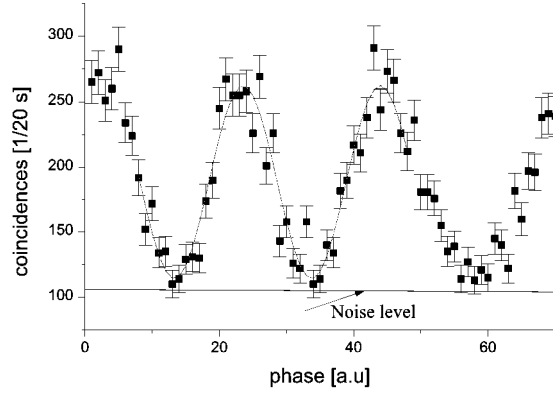

Fig. 3. Coincidence count rate as a function of the phase of the interferometer. Black squares are experimental points. The error bars are the statistical errors. The solid line is a sinusoidal fit, from which we can infer a visibility of 91 ± 6 %.
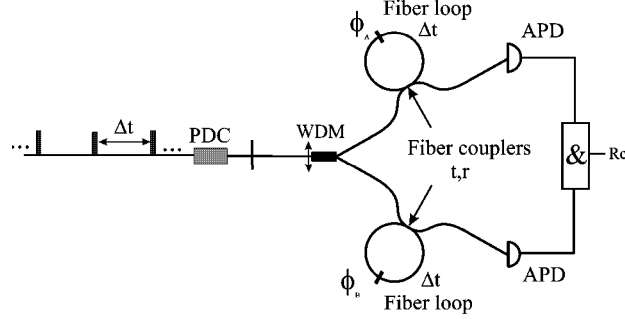
Fig. 4. Setup proposed to show two-photon Fabry-Perot like effects.

one output is connected to one input. The coupler has a transmission amplitude t and a reflection amplitude r with $t^2 + r^2 = 1$. By convention, a photon transmitted remains in the same fiber. Similarly to the 2-way interferometer, we detect only photons in the same time-bin, i.e with $\tau = \tau_A - \tau_B = 0$ (photons having traveled the same path length from the PDC to the detectors). The photons passing in the loop aquire a phase ϕ_A and ϕ_B where the phase is defined as the difference of optical delay between 2 pump pulses and the loop. The probability of coincidences between Alice and Bob is given by :

$$
\begin{aligned}
P_{coinc} \\
= \left| t^2 + r^2 e^{i(\phi_A+\phi_B)} r^2 + r^2 e^{i(\phi_A+\phi_B)} t^2 e^{i(\phi_A+\phi_B)} r^2 + ... \right|^2 \\
= \left| t^2 + r^4 \sum_{n \geq 0} t^{2n} e^{i(n+1)(\phi_A+\phi_B)} \right|^2 \\
= \left| t^2 + \frac{r^4 e^{i(\phi_A+\phi_B)}}{1 - t^2 e^{i(\phi_A+\phi_B)}} \right|^2
\end{aligned}
\tag{7}
$$

Where we made the hypothesis that $t_A = t_B = t$ and $r_A = r_B = r$. Figure 5 shows a plot of the coincidence count rate as a function of the sum of the two phases $\phi_A + \phi_B$, computed with (7). We see that the interference pattern is periodic, but no longer sinusoidal as in the traditional case. The shape and the visibility of the interference pattern varies strongly with the probability of transmission in the beam splitter. The maximum visibility V =1 is obtained for a transmission probability of $t^2 = 1/3$.

5. Conclusion

We presented a new scheme to generate high-dimensional time-bin entanglement between two photons. The method is based on parametric down conversion with a sequence of pump pulses with a fixed phase difference. The necessary coherence between pump pulses can be obtained by using a mode-locked laser. This scheme enables in principle the generation of any desired
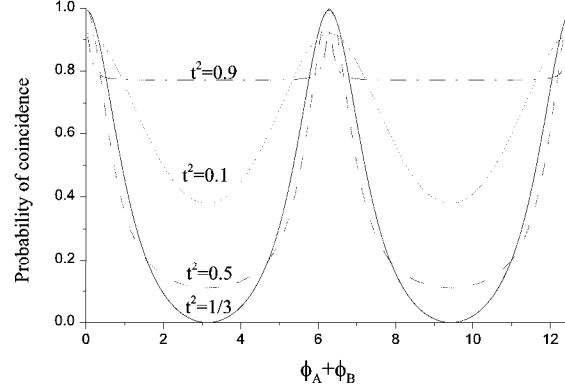
Fig. 5. Probability of coincidence as a function of the sum of the phases $\phi_A + \phi_B$, plotted for different transmission probability t^2. We see that the interference has a visibility of 1 for $t^2 = 1/3$.

two-photon high-dimensional state by setting the number, the amplitude and the phase of the pump pulses. We proved experimentally the feasibility of such a scheme by analyzing the high-dimensional entangled state with a 2-way interferometer. The measured visibility of 91 ± 6 % allows us to demonstrate entanglement for a two-photon state of at least dimension d=11. We also proposed a feasible experiment to show a Fabry-Perot like effect for 2-photons states, as a direct consequence of high dimensional time-bin entanglement. In conclusion, we can say that, with this method, creating high-dimensional entanglement is relatively easy. A more difficult and challenging task will be to use high-dimensional entanglement in quantum protocols.

Acknowledgements

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