THE IMPACT OF RESOURCE
ON REMOTE QUANTUM CORRELATION PREPARATION

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When Alice and Bob share two pairs of quantum correlated states, Alice can remotely prepare quantum entanglement and quantum discord in Bob's side by measuring the parts in her side and telling Bob the measurement results by classical communication. For remote entanglement preparation, entanglement is necessary. We find that for some shared resources having the same amount of entanglement, when Bell measurement is used, the entanglement remotely prepared can be different, and more discord in the resources actually decreases the entanglement prepared. We also find that for some resources with more entanglement, the entanglement remotely prepared may be less. Therefore, we conclude that entanglement is a necessary resource but may not be the only resource responsible for the entanglement remotely prepared, and discord does not likely to assist this process. Also, for the preparation of discord, we find that some states with no entanglement could outperform entangled states.

Keywords: quantum entanglement; quantum discord; remote preparation.

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1 Introduction

Quantum entanglement and quantum discord are important resources in many quantum information processes\cite{1, 2, 3, 4, 5, 6}. Quantum entanglement is critical in quantum teleportation\cite{7, 8}, quantum computation\cite{3} superdense coding\cite{9} quantum cryptography\cite{10, 11}, testing Bell’s inequalities\cite{12}, etc. The role of quantum discord has also been investigated in recent years\cite{2, 13, 14, 15, 16}. It is thought that discord might increase the performance of deterministic quantum computation with one qubit\cite{17, 18}. Quantum discord is also be considered as the necessary resource for remote state preparation (RSP)\cite{19}. Although separable states, which may possess nonzero limited discord\cite{20}, can be prepared with local operation and classical communication (LOCC)\cite{3}, the states having the maximum discord are Bell states, which cannot be prepared by LOCC. This means that when two parts are spatially separated and the discord wanted is larger than that can be prepared by LOCC, remote dis-
cord preparation might be important. For example, in RSP, the best efficiency is attained by
states with largest discord[19, 21, 22, 23].

The teleportation protocol[7] shows that when sharing one pair of maximally entangled
resource, Alice can teleport an unknown state to Bob by making Bell measurement on her
side and telling Bob the results via classical communication. When the unknown state is an
entangled state, this is entanglement swapping[7, 24, 25, 26], and has been experimentally
realized[27]. Recently, a scheme to remotely prepare an arbitrary amount of multipartite
entanglement is proposed[28].

In this paper, we concentrate on the influence of the resources to the amount of quantum
entanglement and quantum discord remotely prepared. For remote entanglement preparation,
entanglement is necessary. For some resources with the same amount of entanglement, when
Bell measurement is used, we find that the entanglement prepared can be different, and more
discord in the resources actually decreases the amount of entanglement prepared. Moreover,
we find that, for some resources with the same amount of entanglement, some of them can
be used to remotely prepare entanglement while others cannot. Besides, we find that for
some resources with more entanglement, the entanglement remotely prepared may be less.
Therefore, entanglement is a necessary resource but may not be the only resource responsible
for the entanglement remotely prepared, and discord does not likely to assist this process.
Furthermore, we show that for discord preparation, some separable resources can outperform
entangled resources.

2 Theoretical framework

![Diagram of quantum resource preparation](image)

Fig. 1. Initially, Alice and Bob share two pairs of correlated states $\rho_{AB}$ and $\rho_{CD}$. Alice does
measurement $\{E_m\}$ on her qubits A, D of global four-qubit state $\rho_{\text{tot}} = \rho_{AB} \otimes \rho_{CD}$, where $E_m$
is the element of a set of positive operator valued measurement and $\sum_m E_m = I$, and tell Bob
the results by classical communication. When the outcome of the measurement is $m$, the state
prepared in BC is $\rho_{BCm}$.

Alice and Bob share two pairs of quantum correlated resources $\rho_{AB}$ and $\rho_{CD}$. We assume
that parts B and C are spatially separated and Bob does not have the control of the two parts
in his side. Initially, the parts B and C are uncorrelated. To get the two parts correlated,
remote preparation is needed, in which Alice can make a measurement $\{E_m\}$ on her qubits
A, D of global four-qubit state $\rho_{\text{tot}} = \rho_{AB} \otimes \rho_{CD}$, where $E_m$ is the element of a set of positive operator valued measurement and $\sum_m E_m = I$. After the measurement, Alice tell Bob the measurement results by classical communication. Then the two parts in Bob’s side are prepared in state $\rho_{BCm}$, which corresponds to the measurement outcome $m$.

In the following, Einstein summation convention is used, which are defined as $\sum_i A_i = \sum_{i=1}^3 A_i$. Assume the shared two pair resources are in these two states expressed in the Bloch representation:

\begin{equation}
\rho_{AB} = \frac{1}{4}(I_A \otimes I_B + r_A^i \sigma_A^i \otimes I_B + I_A \otimes r_B^i \sigma_B^i + R_{AB}^{ij} \sigma_A^i \otimes \sigma_B^j),
\rho_{CD} = \frac{1}{4}(I_C \otimes I_D + r_C^i \sigma_C^i \otimes I_D + I_C \otimes r_D^i \sigma_D^i + R_{CD}^{ij} \sigma_C^i \otimes \sigma_D^j),
\end{equation}

where $\sigma's$ ($i = 1, 2, 3$), are pauli operators, $r_A^i = \text{Tr}_{AB}(\rho_{AB} \sigma_A^i \otimes I_B)$, $r_B^i = \text{Tr}_{AB}(\rho_{AB} I_A \otimes \sigma_B^i)$, $R_{AB}^{ij} = \text{Tr}_{AB}(\rho_{AB} \sigma_A^i \otimes \sigma_B^j)$, $R_{CD}^{ij} = \text{Tr}_{AB}(\rho_{CD} \sigma_C^i \otimes \sigma_D^j)$ are defined in the same way. Alice do measurement $\{E_m\}$. For measurement outcome $m$, we note the following expressions as $\text{Tr}_{AD}(E_m I_A \otimes I_D) = C_m$, $\text{Tr}_{AD}(E_m I_A \otimes \sigma_B^i) = M_{Am}$, $\text{Tr}_{AD}(E_m I_A \otimes \sigma_D^j) = M_{Dm}$, $\text{Tr}_{AD}(E_m \sigma_A^i \otimes \sigma_D^j) = M_{mij}$. Note that these expressions are only related to the specific measurement used. The state prepared in Bob’s side reads

\begin{equation}
\rho_{BCm} = \frac{1}{4}(I_B \otimes I_C + r_B^{i_m} \sigma_B^i \otimes I_C + I_B \otimes r_C^{i_m} \sigma_C^i + R_{BCm}^{ij} \sigma_B^i \otimes \sigma_C^j),
\end{equation}

where

\begin{align*}
P_m &= \frac{1}{4}(C_m + r_A^k M_{Am}^k + r_D^k M_{Dm}^k + r_A^k M_{m}^{kl} r_D^{kl}),
\end{align*}

\begin{align*}
r_B^{i_m} &= \frac{1}{4P_m}[(C_m + r_B^k M_{Dm}^k) r_B^i + R_{AB}^{ki} M_{Am}^k + R_{AB}^{kl} M_{m}^{kl} r_D^{ij}],
\end{align*}

\begin{align*}
r_C^{i_m} &= \frac{1}{4P_m}[(C_m + r_A^k M_{Am}^k) r_C^i + R_{CD}^{ik} M_{Dm}^k + r_A^k M_{m}^{kl} R_{CD}^{ij}],
\end{align*}

\begin{align*}
R_{BCm}^{ij} &= \frac{1}{4P_m}[(C_m r_B^i r_C^j + r_B^k M_{Dm}^k R_{CD}^{ik} + M_{Am} R_{AB}^{kl} r_C^j + R_{AB}^{ij} M_{m}^{kl} R_{CD}^{ij})].
\end{align*}

Assume Alice uses the Bell basis to do the measurement, which is

\begin{align*}
E_1 &= |\psi^-\rangle \langle \psi^-|,
E_2 &= |\psi^+\rangle \langle \psi^+|,
E_3 &= |\phi^-\rangle \langle \phi^-|,
E_4 &= |\phi^+\rangle \langle \phi^+|,
\end{align*}

where $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, and $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$. We have $C_m = 1$, $M_{Am}^i = 0$, $M_{Dm}^i = 0$, for $i = 1, 2, 3$ and $m = 1, 2, 3, 4$. $[M_{ij}^i] = \text{diag}\{1, -1, 1\}$, $[M_{ij}^j] = \text{diag}\{1, 1, -1\}$, $[M_{ij}^i] = \text{diag}\{-1, 1, 1\}$, $[M_{ij}^j] = \text{diag}\{1, 1, 1\}$.

When $r_A^i = r_B^j = 0$, the probability to get result $m$ takes the form $P_m = (\text{Tr}E_m)/4$. Thus, in this case, for Bell measurement, the probability to get each measurement outcome is 1/4.
In this paper, concurrence[30] is used to evaluate the entanglement, which is defined as follows:

\[ C(\rho) = \max \{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \]

where \(\lambda_i\)s are the eigenvalues, in decreasing order, of matrix \(\rho\), and \(\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)\). For maximally entangled states, concurrence equals 1, and for separable states, concurrence equals 0.

Since geometric discord has been proven to have operational meaning in remote quantum state preparation[19] and remote state preparation and remote quantum correlation preparation are closely related, the adjusted geometric discord[14, 31] is used to evaluate discord in \(\rho\), which reads

\[ \tilde{D}_g(\rho) = \min_{\chi \in \Omega_0} \frac{\text{Tr}(\rho - \chi)^2}{\text{Tr}\rho^2}, \]

where \(\Omega_0\) denotes the set of classical states.

### 3 Analysis on the impact of resources

When the shared resources are both singlets \(|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)\), and Alice uses the Bell basis to do the measurement, this is the standard entanglement swapping process[7, 24]. The state prepared in BC has the maximum entanglement and discord for each measurement outcome.

Suppose the resource \(\rho_{AB}\) is separable, i.e., \(\rho_{AB} = \sum_i p_i \rho_A^i \otimes \rho_B^i\), and \(\rho_{CD}\) is any state. For measurement outcome \(m\), the states remotely prepared reads

\[ \rho_{BCm} = \sum_i p_i \rho_B^i \otimes \rho_{Cm}^i\]

where

\[ \rho_{Cm}^i = \frac{\text{Tr}_{AD}(E_m \rho_A^i \otimes \rho_{CD})}{\text{Tr}_{ABCD}(E_m \rho_A^i \otimes \rho_B^i \otimes \rho_{CD})}, \]

\[ \rho_B^i = \frac{p_i \text{Tr}_{ABCD}(E_m \rho_A^i \otimes \rho_B^i \otimes \rho_{CD})}{\text{Tr}_{ABCD}(E_m \rho_A \otimes \rho_{CD})}. \]

This is a separable state. Thus, if either one of the resource pair is separable, the state remotely prepared will be separable. In other words, entanglement is a necessary resource in remote entanglement preparation.

Specifically, for the following resources

\[ \rho_{AB} = \frac{2 - \lambda_1}{16} |\psi^-\rangle \langle \psi^-| + \frac{2 - \lambda_1}{16} |\psi^+\rangle \langle \psi^+| + \frac{2 + \lambda_1}{4} |\phi^-\rangle \langle \phi^-| + \frac{2 - \lambda_1}{8} |\phi^+\rangle \langle \phi^+|, \]

\[ \rho_{CD} = \frac{2 - \lambda_2}{16} |\psi^-\rangle \langle \psi^-| + \frac{2 - \lambda_2}{16} |\psi^+\rangle \langle \psi^+| + \frac{2 + \lambda_2}{4} |\phi^-\rangle \langle \phi^-| + \frac{2 - \lambda_2}{8} |\phi^+\rangle \langle \phi^+|, \]

where \(-1 \leq \lambda_1, \lambda_2 \leq 2\), we have \(\tilde{r}_A = \tilde{r}_B = \tilde{r}_C = \tilde{r}_D = 0\), \(R_{AB} = \text{diag}((-2 + 3\lambda_1)/8, (2 + 3\lambda_1)/8, (2 + \lambda_1)/4)\) and \(R_{CD} = \text{diag}((-2 + 3\lambda_2)/8, (2 + 3\lambda_2)/8, (2 + \lambda_2)/4)\). The concurrence of AB and CD are max\{0, \lambda_1/2\}, max\{0, \lambda_2/2\}, respectively. Alice uses Bell basis to do the measurement. As showed before, the probability to get each outcome is the same. For measurement outcome \(m\), we have \(\tilde{r}_{Bm} = 0, \tilde{r}_{Cm} = 0, \) and \(R_{BCm} = \text{diag}((2 + 3\lambda_1)(2 + 3\lambda_2)/8, (2 + \lambda_1)(2 + \lambda_2)/4)\).
3λ_2)M_{11}^{11}/64, (2 + 3λ_1)(2 + 3λ_2)M_{22}^{22}/64, (2 + λ_1)(2 + λ_2)M_{33}^{33}/16}. It can be proved that, for Bell diagonal resources and Bell measurement, the quantum entanglement and discord prepared in BC are the same for each four measurement outcomes. Therefore, the average entanglement and discord are also the same as every outcome. The concurrence in BC is

\[ C = \max\{0, \frac{11λ_1λ_2 + 10λ_1 + 10λ_2 - 20}{64}\}. \]

For these resources, the requirement for preparing nonzero quantum entanglement is

\[ 11λ_1λ_2 + 10λ_1 + 10λ_2 \geq 20. \] (8)

For \( λ_1 = 0.5, \ λ_2 = 0.4 \), the resource pairs are both entangled, but the above inequality is violated. This means that although both of the resource pairs are entangled, the entanglement prepared may be zero.

As shown in Fig. 2 (b), when the entanglement of one pair resource is zero, the discord...
prepared is not, which means that to remotely prepare discord, the shared resources do not need to be entangled.

From the above results, it may seem that the prepared entanglement is determined by the entanglement in the resources. Actually, it turns out not to be so. We investigate this by considering resources having fixed amount of entanglement. We choose the following resources:

\[
\rho_{AB} = \rho_{CD} = \frac{2 + \lambda_1}{4} \left| \psi^- \right\rangle \left\langle \psi^- \right| - \frac{\lambda_1 + 2\lambda_2}{4} \left| \psi^+ \right\rangle \left\langle \psi^+ \right| + \frac{2 + \lambda_1 + 4\lambda_2}{4} \left| \phi^- \right\rangle \left\langle \phi^- \right| - \frac{\lambda_1 + 2\lambda_2}{4} \left| \phi^+ \right\rangle \left\langle \phi^+ \right|,
\]

(9)

where inequality \(0 < \lambda_1 \leq 2\) is required to ensure these states are entangled. For these states to be physical, the relation \(-(2 + \lambda_1)/4 \leq \lambda_2 \leq -\lambda_1/2\) should be satisfied. The concurrence of them is \(\lambda_1/2\). Alice uses Bell basis to do the measurement. The entanglement prepared in BC is

\[
C = \max\{0, \frac{6(\lambda_2 + \frac{1+\lambda_1}{3})^2 + (\frac{1+\lambda_1}{3})^2}{2} - 1\}.
\]

As showed in Fig. 3 (a), when the concurrence of the resources is fixed as 0.6 (\(\lambda_1 = 1.2\)), the concurrence in BC varies with different \(\lambda_2\), and it is always nonzero. It takes the minimal value 0.307 when \(\lambda_2 = -0.73\). In Fig. 3 (b), when the concurrence of the resources is fixed
In Fig. 3 (a), when entanglement in BC increases as \( \lambda \), it is straightforward to think that discord is the other resources. However, it is not like this. For entanglement fixed resources, the discord of them are different, as showed in Fig. 3, it might be straightforward to think that discord is the other resources. However, it is not like this. In Fig. 3 (a), when entanglement in BC increases as \( \lambda \) goes from \(-0.73\) to \(-0.5\), the discord in resources actually decreases. So the discord does not assist in the entanglement preparation. On the contrary, the more discord the resources have, the less entanglement may be prepared. Thus, discord may not be the other resource responsible for the remote entanglement preparation. But for the prepared discord, it decreases as the discord in the resources decreases and increases as the discord in the resources increases. It is likely that discord is a good quantity for predicting the remote discord preparation, but not good for remote entanglement preparation.

For the following two resources,

\[
r_{AB}^1 = \rho_{CD}^1 = 0.75 \left| \psi^- \right> \left< \psi^- \right| + 0.25 \left| \phi^- \right> \left< \phi^- \right|,
\]

\[
r_{AB}^2 = \rho_{CD}^2 = 0.775 \left| \psi^- \right> \left< \psi^- \right| + 0.075 \left| \psi^+ \right> \left< \psi^+ \right| + 0.075 \left| \phi^- \right> \left< \phi^- \right| + 0.075 \left| \phi^+ \right> \left< \phi^+ \right|,
\]

the concurrence of them are 0.5 and 0.55, respectively. The concurrence remotely prepared are 0.25 and 0.235, respectively. For resource with more entanglement, the entanglement remotely prepared may be less. This also indicates that although entanglement is necessary for remote entanglement preparation, entanglement itself may not account for all the performance of this process.

Next, we show that for the remote preparation of discord, some separable resources can outperform entangled resources. For separable ones, we choose

\[
\rho_{AB} = \rho_{CD} = \frac{1 - 2 \lambda}{4} \left| \psi^- \right> \left< \psi^- \right| + \frac{1 + \lambda}{2} \left| \phi^- \right> \left< \phi^- \right| + \frac{1}{4} \left| \phi^+ \right> \left< \phi^+ \right|. \tag{10}
\]

For this state to be physical, relation \(-\frac{1}{2} \leq \lambda \leq 0\) should be satisfied. For entangled states, we choose

\[
\rho_{AB} = \rho_{CD} = \frac{2 + \lambda_1}{4} \left| \psi^- \right> \left< \psi^- \right| - \frac{\lambda_1(2 - \lambda_1)}{4} \left| \psi^+ \right> \left< \psi^+ \right|
+ \frac{2\lambda(2 - \lambda) + 2 - \lambda_1}{4} \left| \phi^- \right> \left< \phi^- \right| - \frac{\lambda_1(2 - \lambda_1)}{4} \left| \phi^+ \right> \left< \phi^+ \right|. \tag{11}
\]

For this state to be physical and entangled, relation \(-\frac{1}{2} \leq \lambda_1 \leq 0\) and \(0 < \lambda_1 \leq 2\) should be satisfied. The concurrence of the entangled resources is \(\lambda_1/2\). Alice uses Bell basis to do the measurement.

When the concurrence of the entangled pair is 0.05 (\(\lambda_1 = 0.1\)), the result is showed in Fig. 4 (a). When the concurrence of the entangled pair is 0.25 (\(\lambda > -0.2217\)), the adjusted geometric discord prepared with separable resources will be larger than that with entangled resources. When \(\lambda_1 = 0.5\), the results is showed in Fig. 4 (b). When \(\lambda > -0.1108\), the
Alice uses Bell basis for the measurement. The resources are chosen as entangled states (see Eq. 11) (blue solid line), and separable states (see Eq. 10) (green dashed line). The concurrence of the entangled resources are chosen as (a) 0.05 and (b) 0.25.

Fig. 4. Adjusted geometric discord prepared in BC as the function of parameter of resources $\lambda$. The resources are chosen as entangled states (see Eq. 11) (blue solid line), and separable states (see Eq. 10) (green dashed line). The concurrence of the entangled resources are chosen as (a) 0.05 and (b) 0.25.
adjusted geometric discord prepared with separable resources will be larger than that with entangled resources. Therefore, to remotely prepare discord, entangled resources are not necessary, and separable resources may outperform entangled resources.

4 conclusion

We have investigated the impact of resources, constituted by a pair of quantum correlated states between two distant parties, on remote quantum entanglement and discord preparation. For remote entanglement preparation, entanglement is necessary. By fixing the entanglement of resources, we have shown that the entanglement remotely prepared are different, and more discord in the resource may decrease the entanglement prepared. Also, for some states with the same entanglement, we have shown that some of them can prepare entanglement while others cannot. Moreover, we have shown that for some resources with more entanglement, the entanglement prepared may be less. Based on these results, we conclude that entanglement is necessary for remote entanglement preparation but may not be the only factor responsible for the entanglement remotely prepared, and discord does not likely to assist this process. For remote discord preparation, we have shown that some separable resources can outperform entangled resources. These results could be useful to practical remote quantum entanglement and discord preparation.

This qubit model has been extended to the remote multipartite entanglement preparation[28]. The impact of resource on it can be analyzed in a similar way in this paper.

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